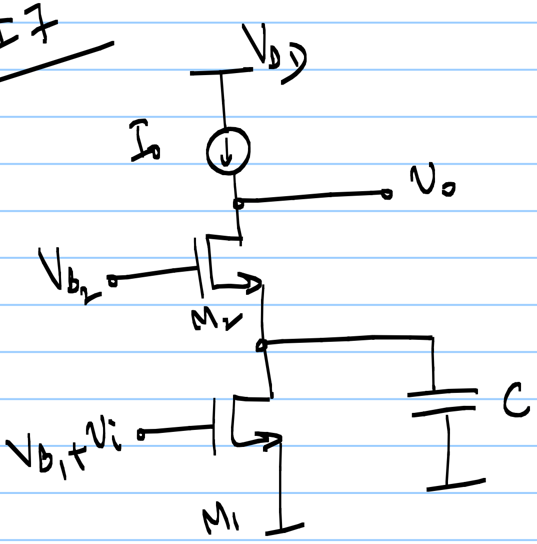
26/9/23

Lec 6

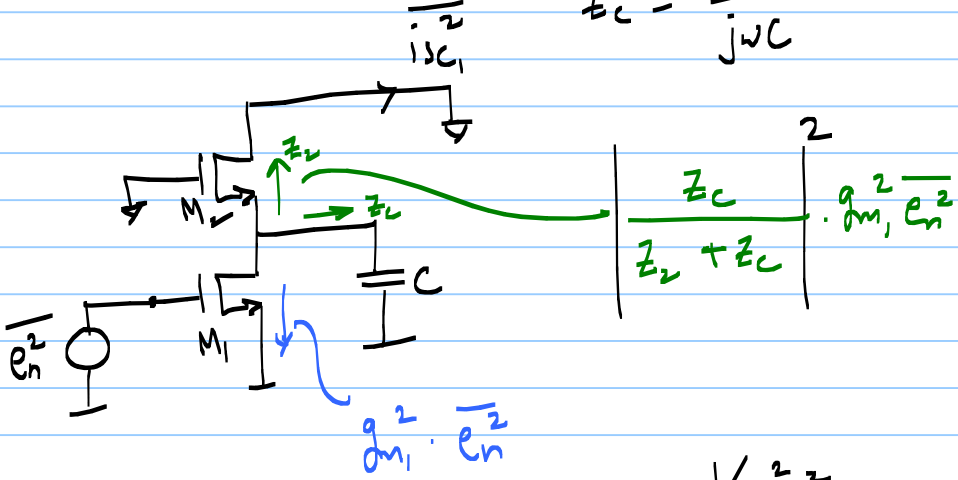
Example #7



1) case with $\overline{e_n^2}$

$Z_2 = \frac{1}{g_{m_2}}$

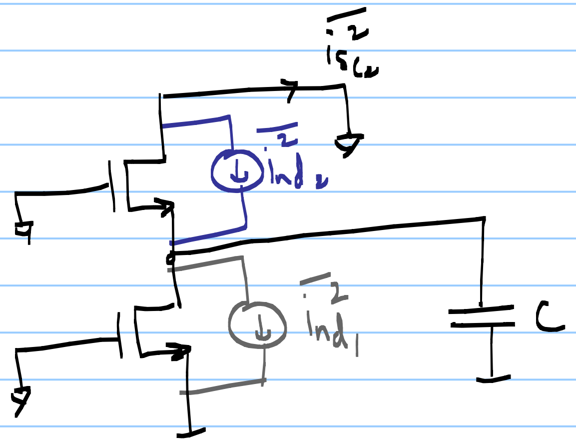
$Z_c = \frac{1}{j\omega C}$



$$\overline{i_{sc_1}^2} = g_{m_1}^2 \overline{e_n^2} \cdot \left|\frac{Z_c}{Z_2+Z_c}\right|^2 = g_{m_1}^2 \cdot \overline{e_n^2} \cdot \frac{1/\omega^2 C^2}{\frac{1}{g_{m_2}^2} + \frac{1}{\omega^2 C^2}}$$

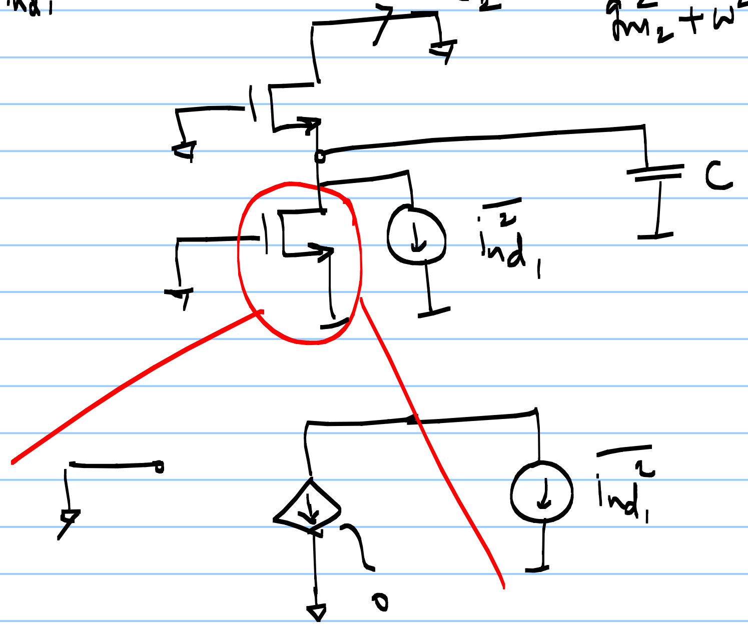
$$\overline{i_{sc_1}^2} = \frac{g_{m_1}^2 \cdot g_{m_2}^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{e_n^2}$$

2) case with $\overline{i_{nd_1}^2}$ & $\overline{i_{nd_2}^2}$

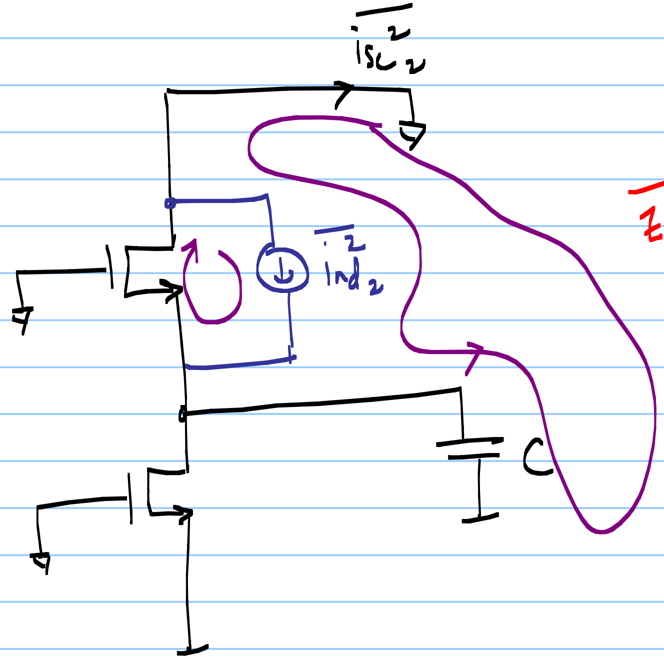


a) $\overline{i_{nd_1}^2}$

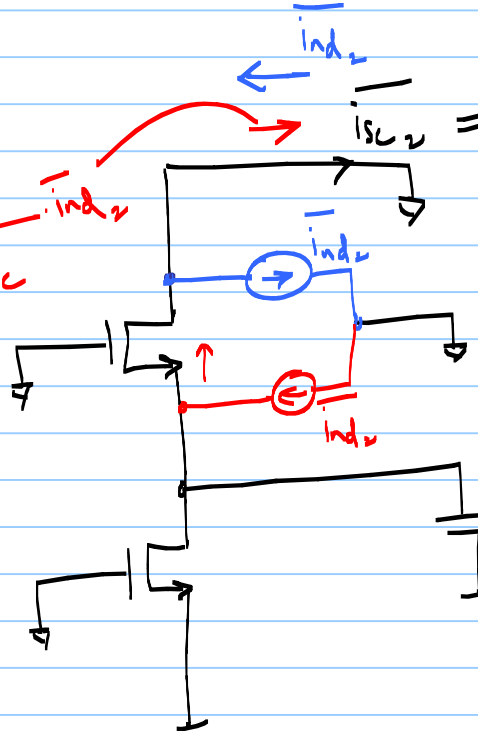
$$\overline{i_{sc_2}^2} = \frac{g_{m_2}^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{i_{nd_1}^2}$$



b) noise from M_2



$\frac{Z_c}{Z_2+Z_c}\cdot\overline{i_{nd_2}}$



$$\overline{i_{sc_2}} = \frac{Z_c}{Z_2+Z_c}\cdot\overline{i_{nd_2}} - \overline{i_{nd_2}}$$

$$= \frac{Z_2}{Z_2+Z_c}\cdot\overline{i_{nd_2}}$$

$$\overline{i_{sc_2}^2} = \left|\frac{Z_2}{Z_2+Z_c}\right|^2\cdot\overline{i_{nd_2}^2}$$

$$= \left|\frac{1/g_{m_2}}{1/g_{m_2}+\frac{1}{j\omega C}}\right|^2\cdot\overline{i_{nd_2}^2}$$

$$\overline{i_{sc_2}^2} = \frac{w^2 C^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{i_{nd_2}^2}$$

Total $\overline{i_{sc_2}^2}$:

$$\overline{i_{sc_2, tot.}^2} = \frac{g_{m_2}^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{i_{nd_1}^2} + \frac{w^2 C^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{i_{nd_2}^2}$$

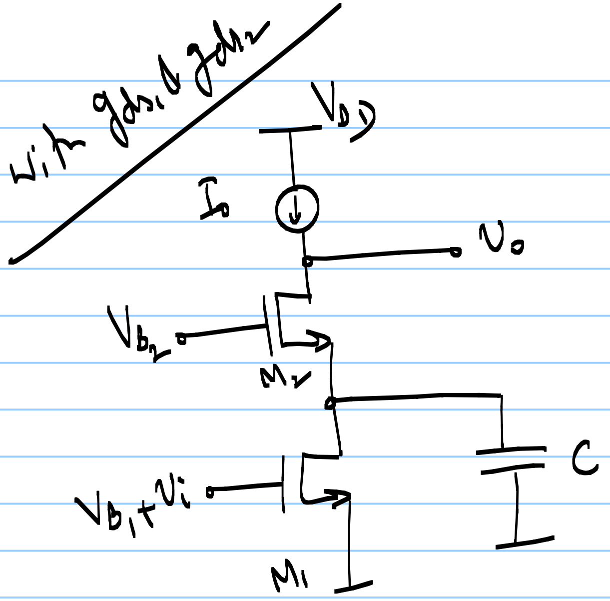
set $\overline{i_{sc_1}^2} = \overline{i_{sc_2}^2}$

$$\frac{g_{m_1}^2 \cdot g_{m_2}^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{e_n^2} = \frac{g_{m_2}^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{i_{nd_1}^2} + \frac{w^2 C^2}{g_{m_2}^2 + w^2 C^2} \cdot \overline{i_{nd_2}^2}$$

$$\overline{e_n^2} = \frac{\overline{i_{nd_1}^2}}{g_{m_1}^2} + \frac{w^2 C^2}{g_{m_1}^2 g_{m_2}^2} \cdot \overline{i_{nd_2}^2}$$

$$\boxed{\frac{\overline{e_n^2}}{\Delta f} = \frac{8kT}{3\, g_{m_1}} + \frac{8kT \cdot w^2 C^2}{3\, g_{m_1}^2\, g_{m_2}}}$$

with g_{ds_1} & g_{ds_2}



$$\frac{\overline{e_n^2}}{\Delta f} = \frac{8kT}{3 g_{m_1}} + \frac{8kT}{3 g_{m_1}^2} \cdot g_{m_2} \frac{\left[r_{ds_2}^2 + \omega^2 c^2 r_{ds_1}^2 r_{ds_2}^2 \right]}{r_{ds_1}^2 + g_{m_2}^2 r_{ds_1}^2 r_{ds_2}^2}$$

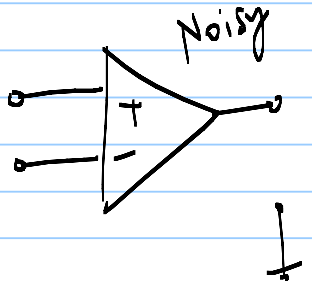
$$\frac{\omega^2 c^2 + g_{ds_1}^2}{g_{m_2}^2 + g_{ds_2}^2}$$

?

Next class

Noise of Differential circuits

opamp



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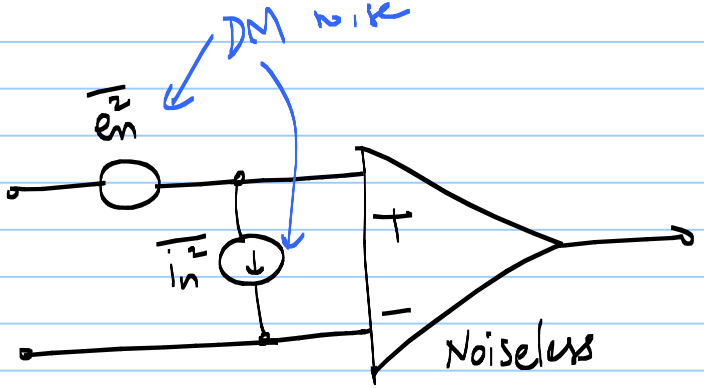
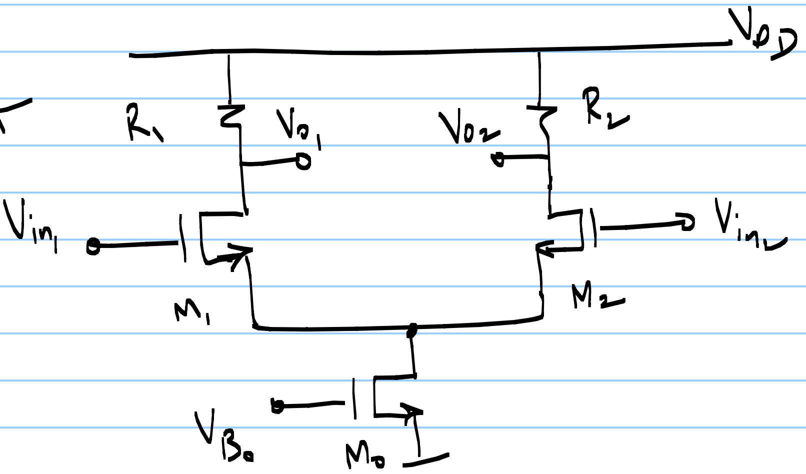


Example #8

$M_1 \equiv M_2$

$R_1 \equiv R_2 \equiv R$

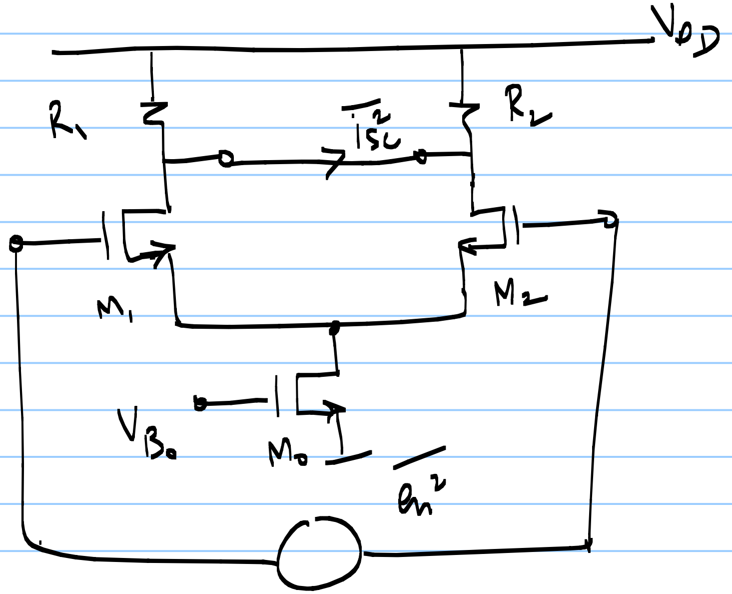


DM : $V_{in_1} = +v_i/2$ | $V_{o_1} = +v_o/2$

$V_{in_2} = -v_i/2$ | $V_{o_2} = -v_o/2$

CM : $V_{in_1} = V_{in_2} = v_i$ | $V_{o_1} = V_{o_2} = v_o$

1) case with $\overline{e_n^2}$



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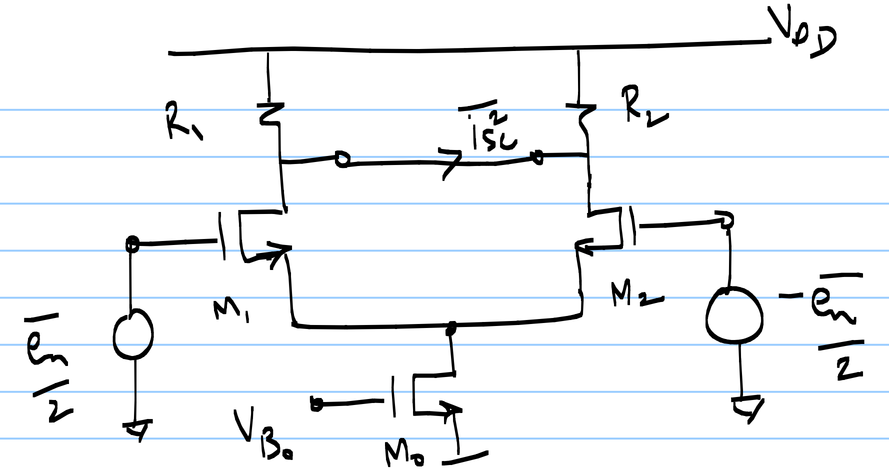

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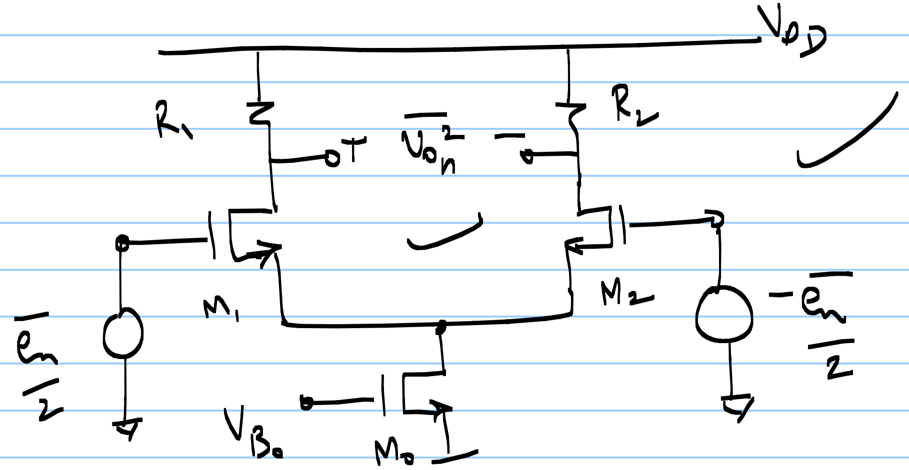
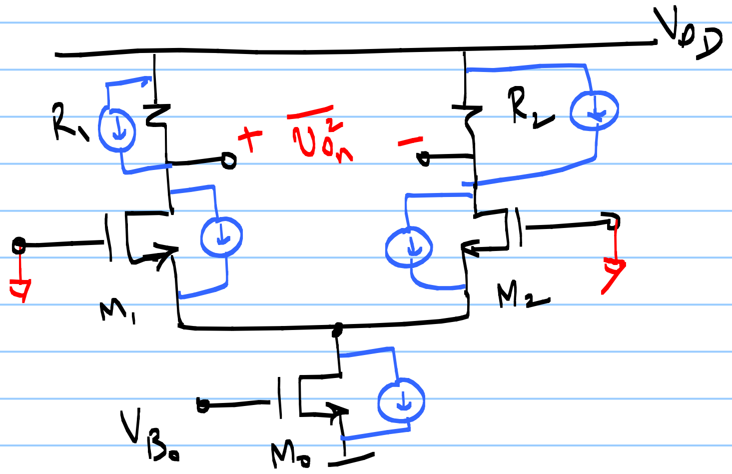


$$\overline{v_{on}} = \overline{v_{o+}} - \overline{v_{o-}} = g_{m_1}R_1 \cdot \frac{\overline{e_n}}{2} - \left(g_{m_2}R_2 \cdot \frac{-\overline{e_n}}{2}\right)$$

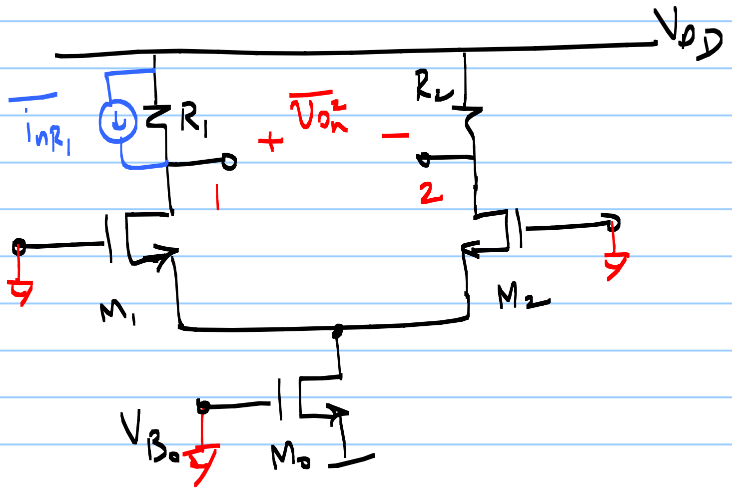
$$\overline{v_{in}} = g_{m_1}R \cdot \overline{e_n} \Rightarrow \overline{v_{on}^2} = g_{m_1}^2 R^2 \overline{e_n^2}$$

2) Case with noise sources

assume $r_{ds} = \infty$, $g_{ds} = 0$



a) $\overline{i_{nR_1}^2}$



* For noise Analysis :

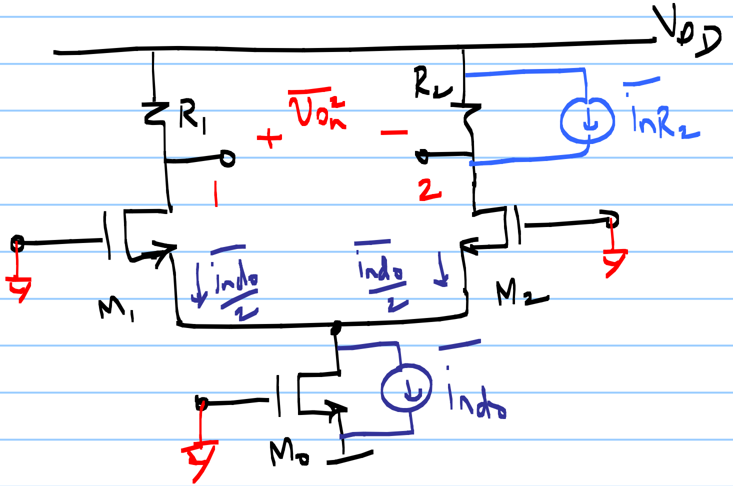
DO NOT USE ANY SYMMETRY ARGUMENTS

$$\overline{v_{on_1}} = \overline{i_{nR_1}} \cdot R_1 \quad ; \quad \overline{v_{on_2}} = 0$$

$$\overline{v_{on}} = \overline{v_{on_1}} - \overline{v_{on_2}} = \overline{i_{nR_1}} \cdot R_1$$

$$\overline{v_{on}^2} = \overline{i_{nR_1}^2} \cdot R_1^2 = 4kTR_1\Delta f$$

b) R_2



$$\overline{v_{on}} = \overline{v_{on_1}} - \overline{v_{on_2}}$$

$$= 0 - \overline{i_{nR_2}} \cdot R_2$$

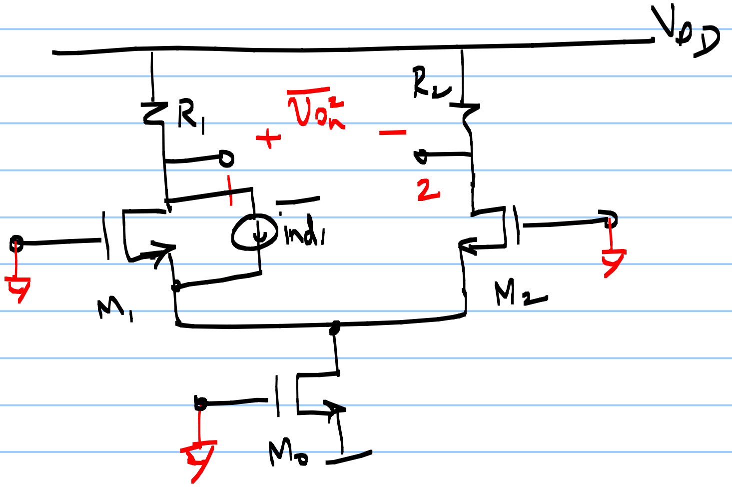
$$\overline{v_{on}^2} = \overline{i_{nR_2}^2} \cdot R_2^2 = 4kT R_2 \cdot \Delta f$$

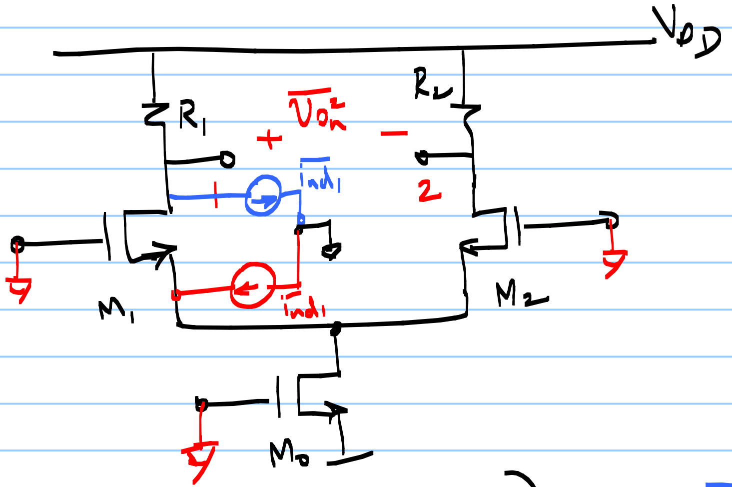
c) M_0 – $\overline{i_{nd_0}^2}$

$$\overline{v_{on_1}} = -\frac{\overline{i_{nd_{0_2}}}}{2} \cdot R_1 \quad ; \quad \overline{v_{on_2}} = -\frac{\overline{i_{nd_0}}}{2} \cdot R_2 \quad \left\{ \text{CM noise} \right\}$$

$$\overline{v_{on}} = \overline{v_{on_1}} - \overline{v_{on_2}} = 0$$

d) Noise of M_1





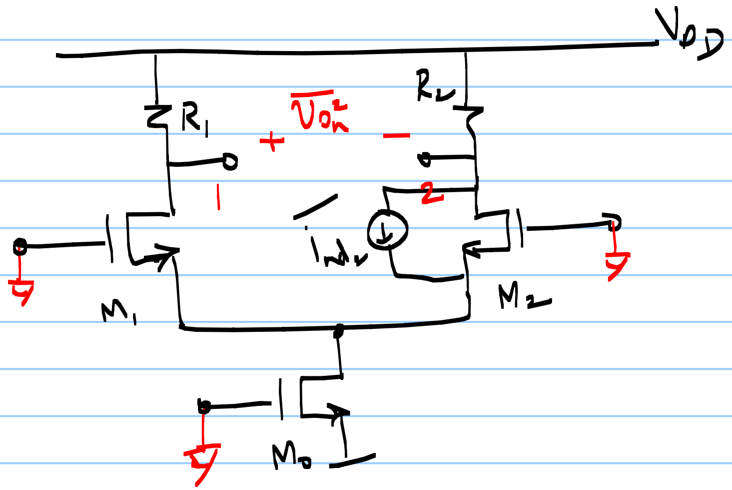
(i) $\overline{i_{nd_1}}$: $\overline{v_{on_1}} = -\overline{i_{nd_1}} R_1$; $\overline{v_{on_2}} = 0$ $\Rightarrow$ $\overline{v_{on}} = -\overline{i_{nd_1}} R_1$

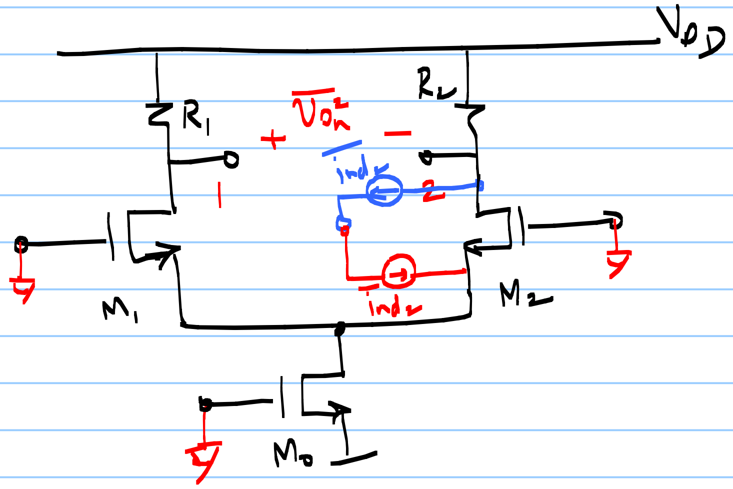
(ii) $\overline{i_{nd_1}}$: $\overline{v_{on_1}} = \frac{\overline{i_{nd_1}}}{2} \cdot R_1$; $\overline{v_{on_2}} = \frac{\overline{i_{nd_1}}}{2} \cdot R_2$ $\Rightarrow$ $\overline{v_{on}} = 0$

$$\overline{v_{on}} = \overline{v_{on}} + \overline{v_{on}}$$
$$= -\overline{i_{nd_1}} R_1$$

$$\overline{v_{on}^2} = \overline{i_{nd_1}^2} \cdot R_1^2$$

e) Noise of M_2





$\underline{\overline{i_{nd2}}}$ $\overline{V_{on1}} = 0$; $\overline{V_{on2}} = -\overline{i_{nd2}} \cdot R_2$

$\underline{\overline{i_{nd2}}}$ $\overline{V_{on1}} = \frac{\overline{i_{nd1}}}{2} \cdot R_1$; $\overline{V_{on2}} = \frac{\overline{i_{nd2}}}{2} \cdot R_2$

$\overline{V_{on}} = \overline{i_{nd2}} \cdot R_2$

$\overline{V_{on}^2} = \overline{i_{nd2}^2} \cdot R_2^2$

$$\overline{V_{on}^2} = \overline{i_{nR_1}^2} \cdot R_1^2 + \overline{i_{nR_2}^2} \cdot R_2^2 + \overline{i_{nd1}^2} \cdot R_1^2 + \overline{i_{nd2}^2} \cdot R_2^2$$

28/9/23

Lec 7

$$\overline{V_{on}^2} = \overline{i_{nR_1}^2}\cdot R_1^2 + \overline{i_{nR_2}^2}\cdot R_2^2 + \overline{i_{nd_1}^2}\cdot R_1^2 + \overline{i_{nd_2}^2}\cdot R_2^2$$

$$\overline{V_{on}^2} = g_{m_1}^2 R^2 \overline{e_n^2}$$

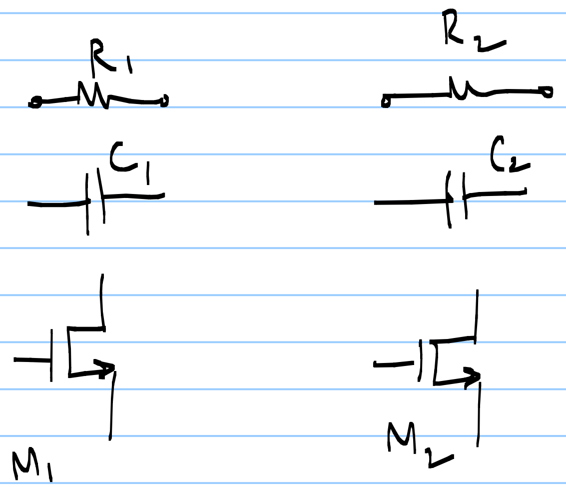
$$g_{m_1}^2 \cancel{R^2}\, \overline{e_n^2} = \overline{i_{nR_1}^2}\cdot \cancel{R^2} + \overline{i_{nR_2}^2}\cdot \cancel{R_2^2} + \overline{i_{nd_1}^2}\cdot \cancel{R_1^2} + \overline{i_{nd_2}^2}\cdot \cancel{R_2^2}$$

$$\overline{e_n^2} = \frac{1}{g_{m_1}^2}\left\{\frac{4kT}{R}\cdot\Delta f + \frac{4kT}{R}\Delta f + \frac{8kT}{3}g_{m_1} + \frac{8kT}{3}g_{m_1}\right\}$$

$$\boxed{\frac{\overline{e_n^2}}{\Delta f} = \frac{8kT}{g_{m_1}^2 R} + \frac{16kT}{3g_{m_1}}}$$

← Input Refered noise voltage squared density of Diff. Amp.

Mismatch



expect $R_1 = R_2 = R$

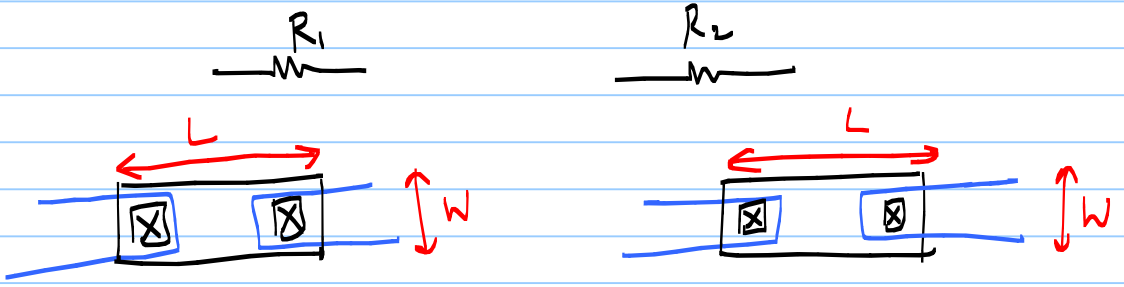
" $C_1 = C_2 = C$

" $M_1 = M_2$

Nominally identical but may not be exactly the same

Mismatch
- Systematic
  - design ✓
  - layout ✓
  - processing ✓
  - * mostly non-zero mean distributions
- Random
  - * statistical phenomenon (beyond your control)
  - * expect zero-mean distribution

Systematic Mismatch



Sheet resistance $= \frac{\rho}{t}$

$$R = (\#\text{ of squares}) \times (\text{Sheet res.})$$

$$\#\text{ of squares} = L/W$$

We want $R_1 = R_2 = R$

* Choose same type of Res.

* Choose same L & same W

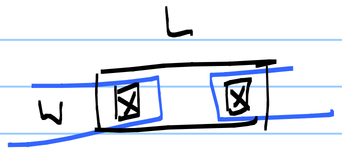
second order effects

e.g. (1) current density effects (crowding)

(2) parasitic resistance, capacitance etc.

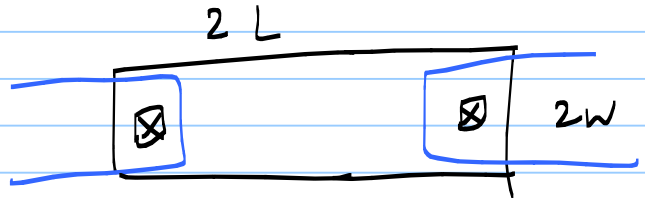
* Choose same environment

e.g. proximity effects



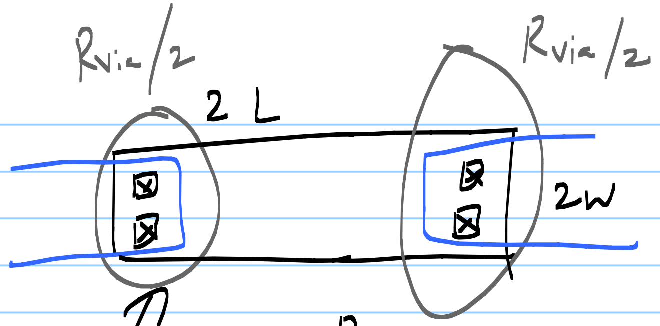
$R_1 = R$

actual $R' = R + 2R_{via}$

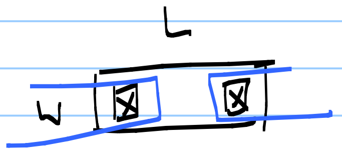


R_2 ✗

or



R_2 ✗



✓

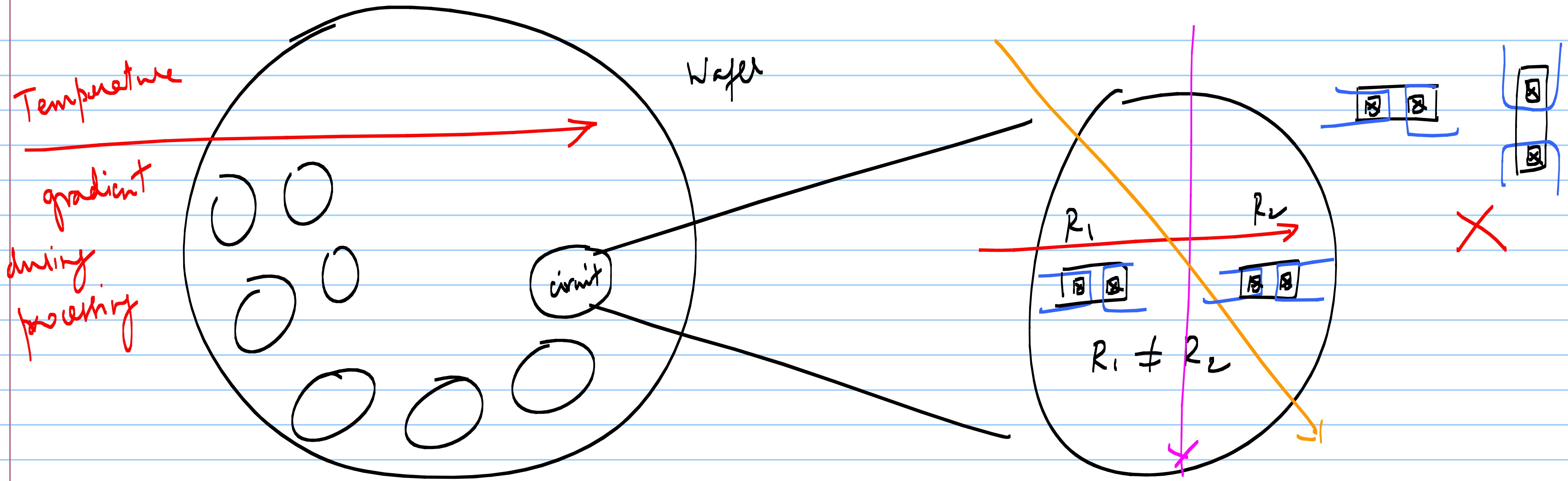
$R_2' = R + 2R_{via}$

$R + 2R_{via}$

$R + R_{via}$

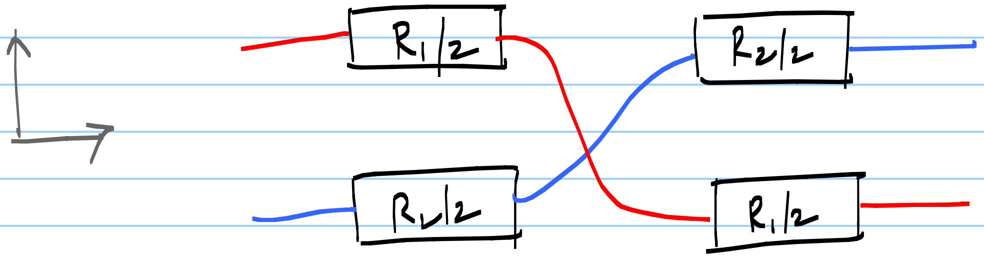
$\overset{a}{\boxtimes} b$ ← for a via or contact, a & b are fixed.

Res/via is known



* Keep R_1 & R_2 as close to each other as possible in layout

"Common Centroid" Layout

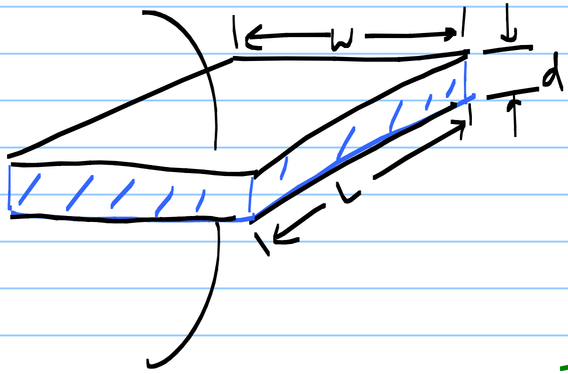


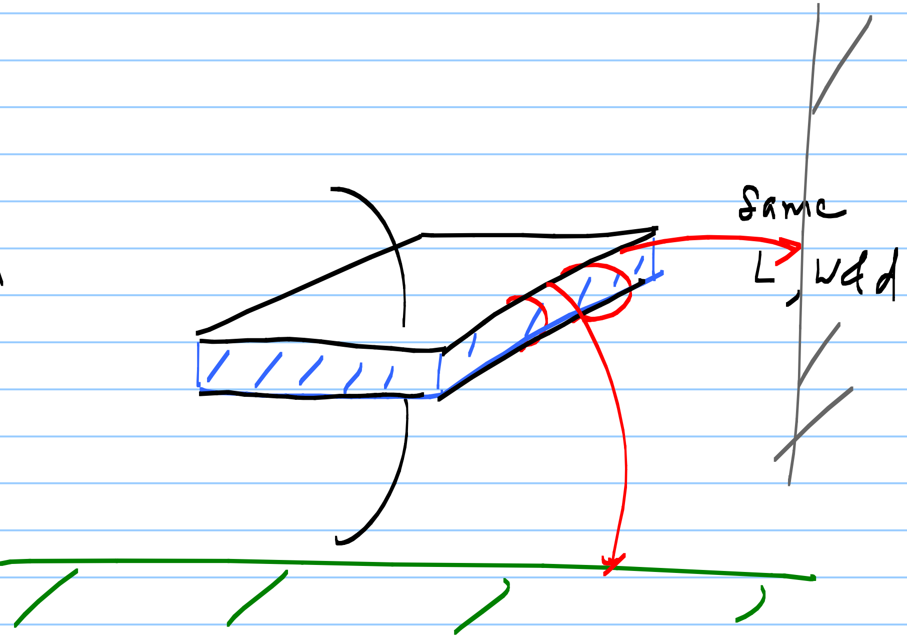
Capacitors

$C_1 = C$

$C_2 = C$

$$C = \frac{\epsilon_0 \epsilon_r (W \cdot L)}{d}$$

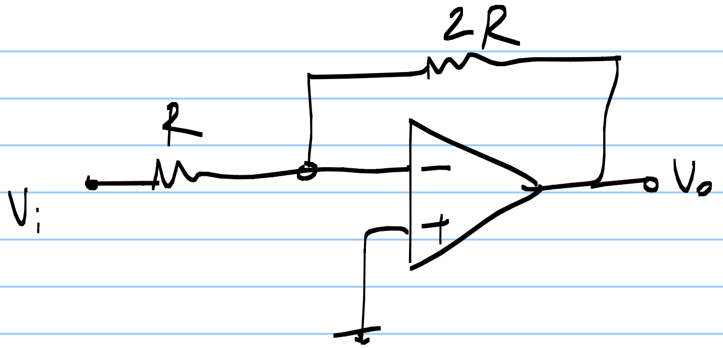




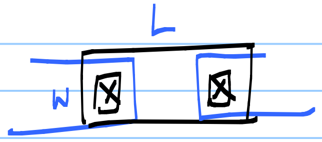
$$\boxed{R_2 = nR_1}$$

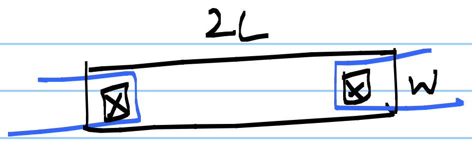
$$C_2 = nC_1$$

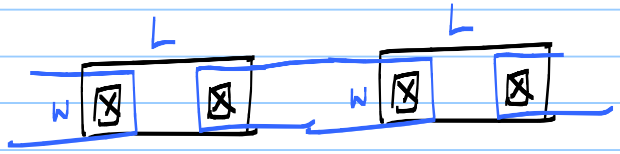
$$M_2 = nM_1$$



CLG = 2







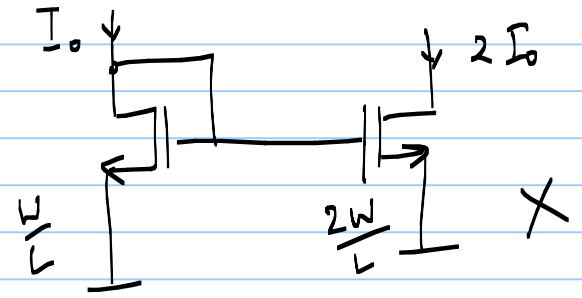
$R_1 = R + 2R_{via}$

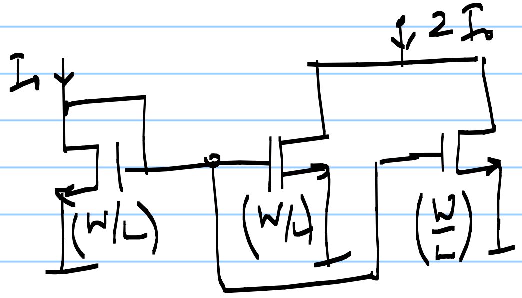
$R_2 = 2R + 2R_{via}$ ✗

$R_2 \neq 2R_1$

$R_2' = R + 2R_{via} + R + 2R_{via}$

$= 2R + 4R_{via} = 2R_1'$ ✓



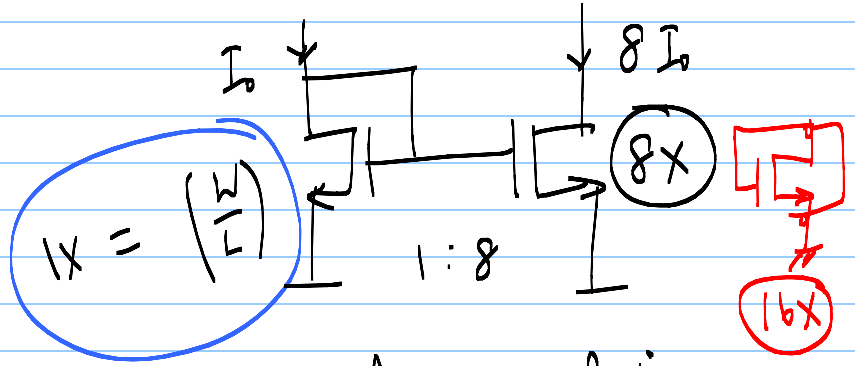


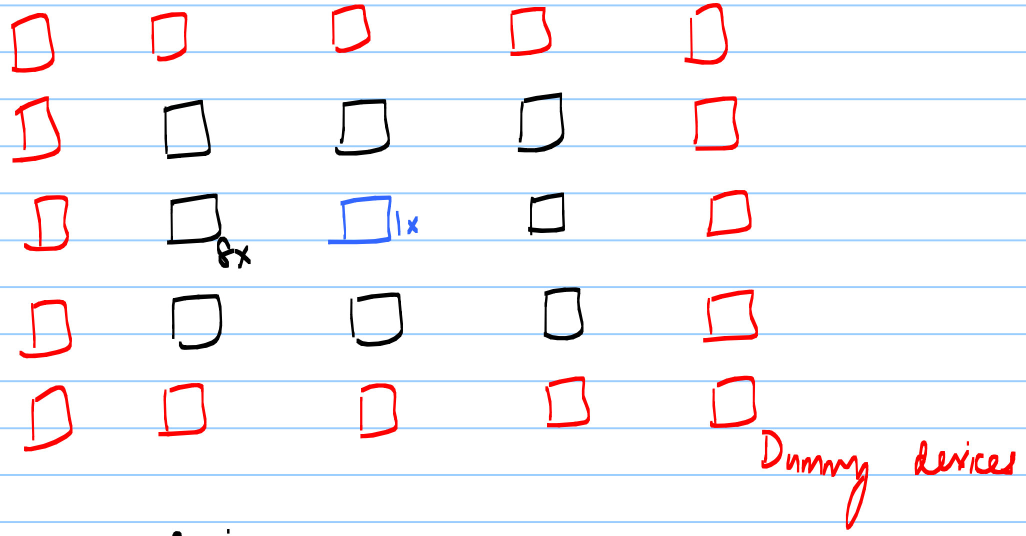
You might have $V_T = f(W)$

$\lambda = f(W)$ etc.

3/10/23

Lec 8

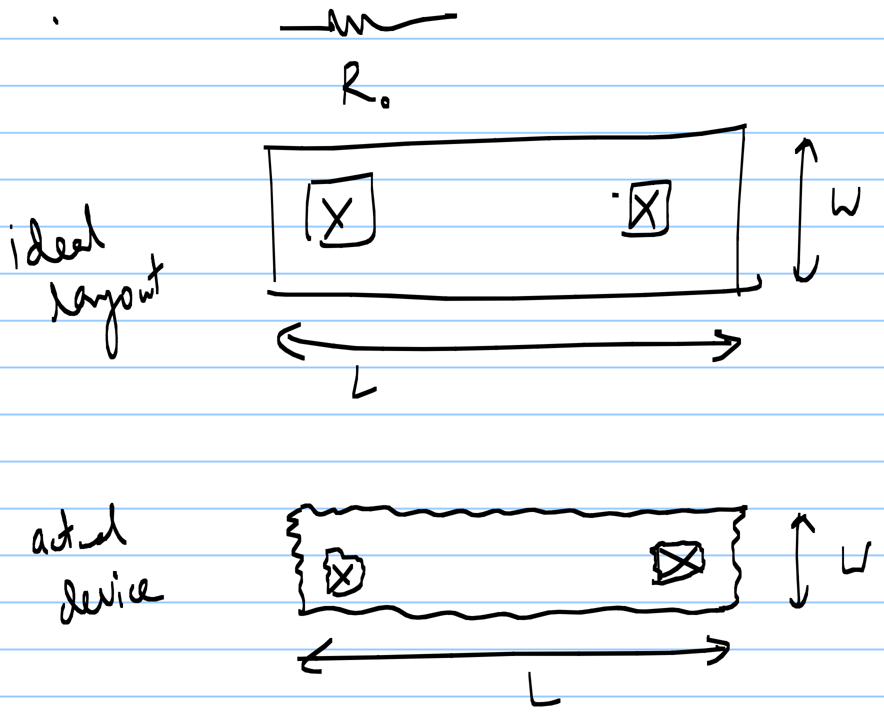




Array of devices

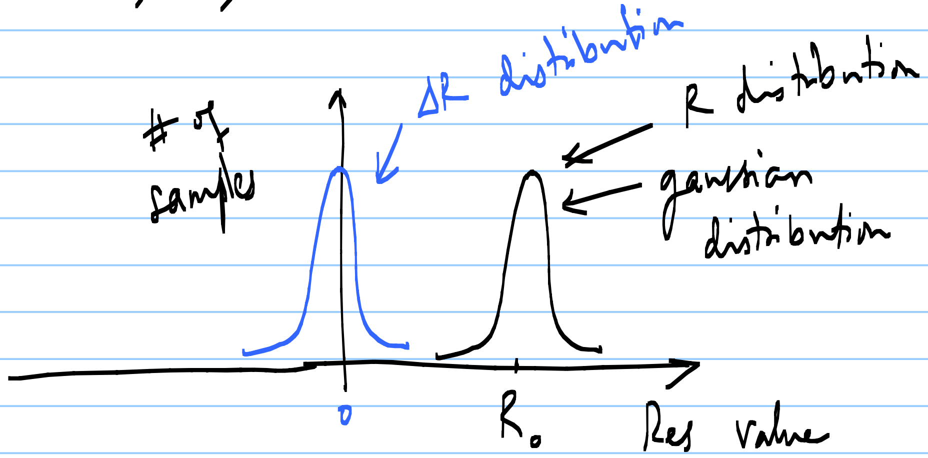
Same environment for every CM device

Random mismatch



Random imperfections in device

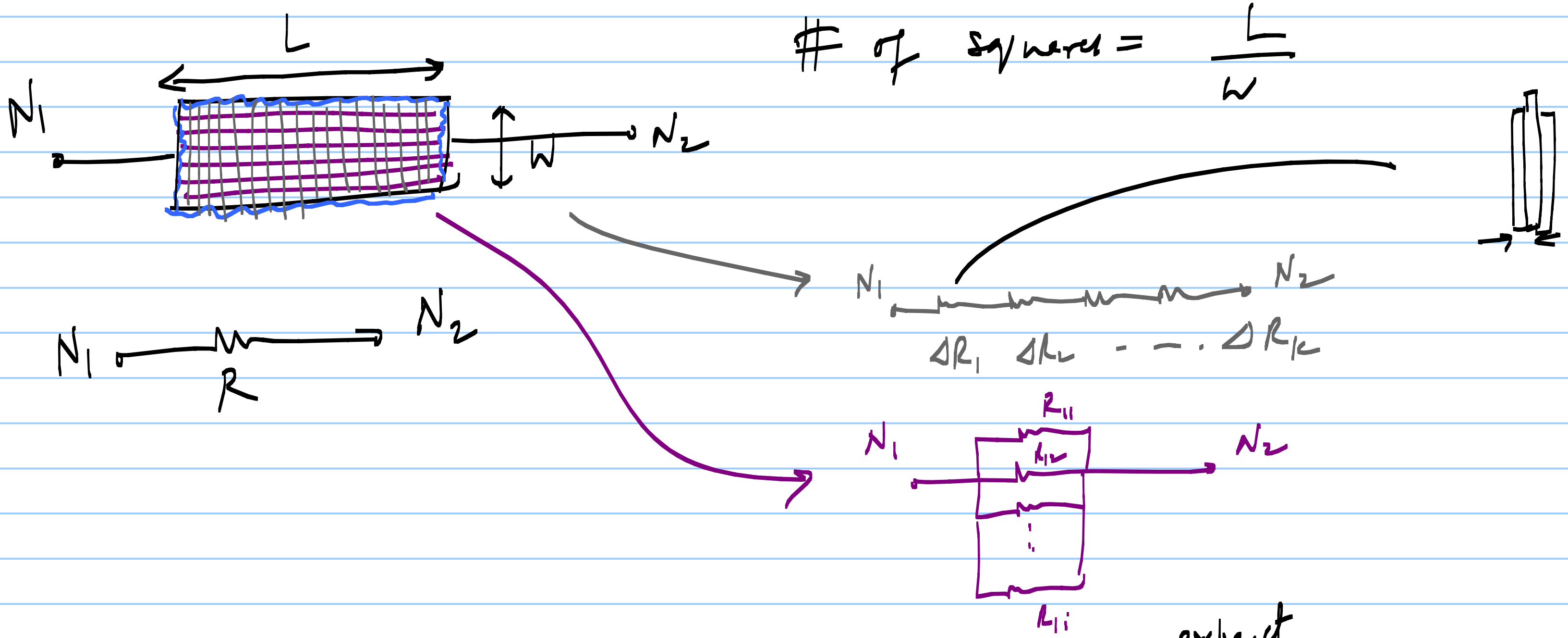
R, C, MOSFET etc.

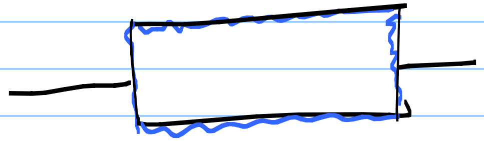


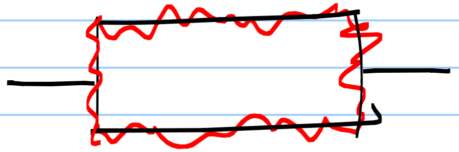
$R = R_o + \Delta R$

↳ described by σ^2

$$\# \text{ of squares} = \frac{L}{W}$$







expect

$$\sigma_R < \sigma_R$$

empirical factor

$$\sigma_R \propto \frac{1}{\sqrt{W_R \cdot L_R}}$$

In general : $\sigma \propto \frac{1}{\sqrt{Area}}$

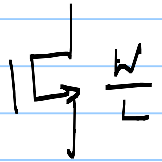
$$\boxed{\sigma_R = \frac{A_R}{\sqrt{W_R \cdot L_R}}}$$

<u>Extend to other components</u>

$$\boxed{\sigma_C = \frac{A_C}{\sqrt{W_C \cdot L_C}}}$$

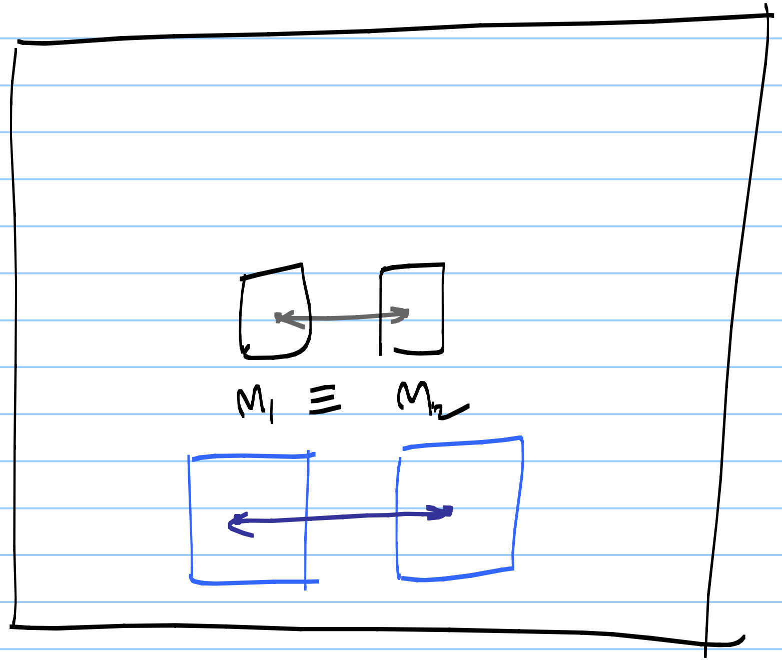
cap

$$\boxed{\begin{aligned} \sigma_{V_T} &= \frac{A_{V_T}}{\sqrt{W \cdot L}} \\ \sigma_\beta &= \frac{A_\beta}{\sqrt{W \cdot L}} \end{aligned}}$$



Low mismatch ⟷ High area

$(\beta = \mu C_{ox})$

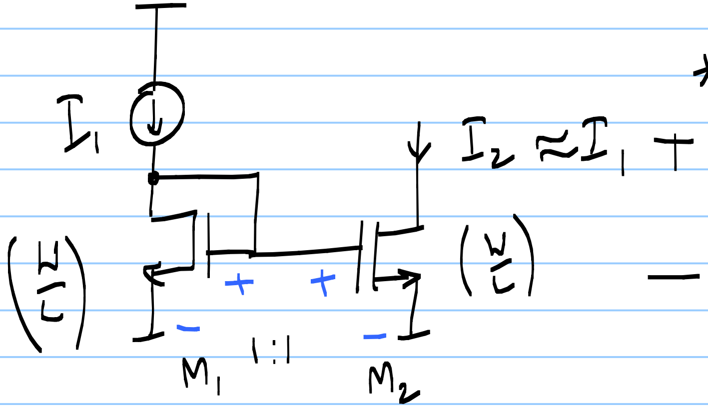


$$I_D = \mu C_{ox} \left(\frac{W}{L}\right)\left(V_{GS}-V_t\right)^2$$

$$\left.\begin{array}{l} W \rightarrow nW \\ L \rightarrow nL \end{array}\right\} I_D \text{ stays the same}$$

choose n so that σ is acceptable

Example #1



* No systematic mismatch

* $V_{T_1} \neq V_{T_2}$ only V_T mismatch

$$V_{T_2} = V_{T_1} + \underbrace{\boxed{\Delta V_T}}_{\text{small}}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T\right)^2$$

$$\frac{\partial I_D}{\partial V_T} = -\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T\right) = -g_m$$

$$\Delta I_D = \frac{\partial I_D}{\partial V_T}\cdot \Delta V_T = -g_m \Delta V_T$$

$$\left.\begin{array}{l}\text{relative}\\ \text{change}\end{array}\right\} = \frac{\Delta I_D}{I_D} = \frac{-g_m \Delta V_T}{I_D}$$

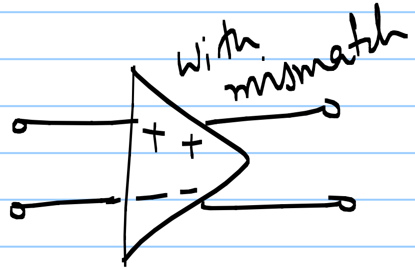
$$\sigma^2_{\frac{\Delta I}{I}} = \frac{g_m^2}{I_D^2} \cdot \sigma_{V_T}^2$$

$$g_m^2 = \left(\frac{2 I_D}{V_{GS} - V_T}\right)^2$$

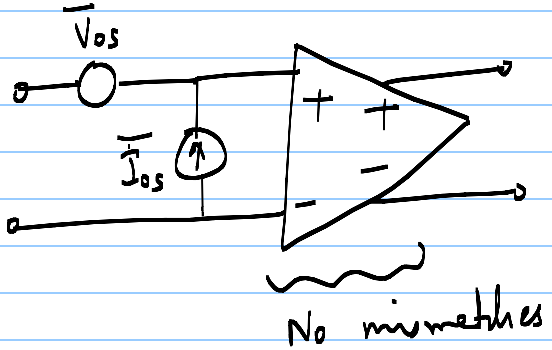
$$\boxed{\sigma^2_{\frac{\Delta I}{I}} = \frac{4}{V_{Dsat}^2} \sigma_{V_T}^2}$$

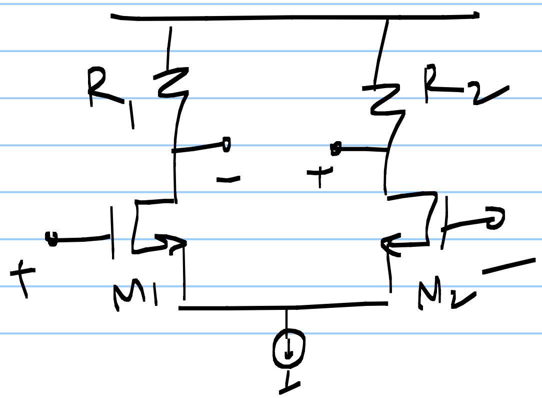
larger V_{Dsat} ⟷ smaller $\frac{\Delta I}{I}$

Mismatch in Differential Amplifiers



$\equiv$





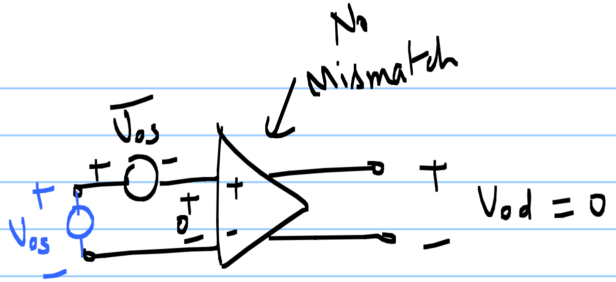
$R_1 \equiv R_2$

$M_1 \equiv M_2$

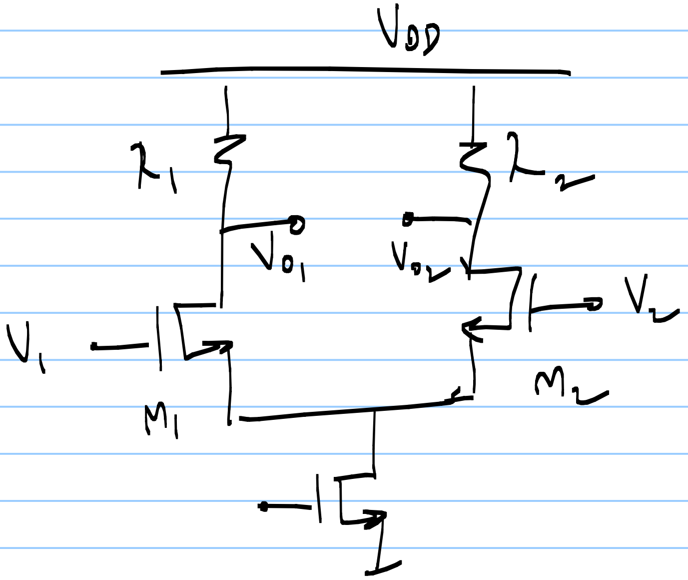
* No systematic mismatch

↳ $\overline{V_{os}}$ & $\overline{I_{os}}$ have zero mean

* $\overline{I_{os}} = 0$ for MOS opamps when input is applied @ gate



$\overline{V_{os}}$ = voltage to be applied @ input so that $V_{od} = 0$

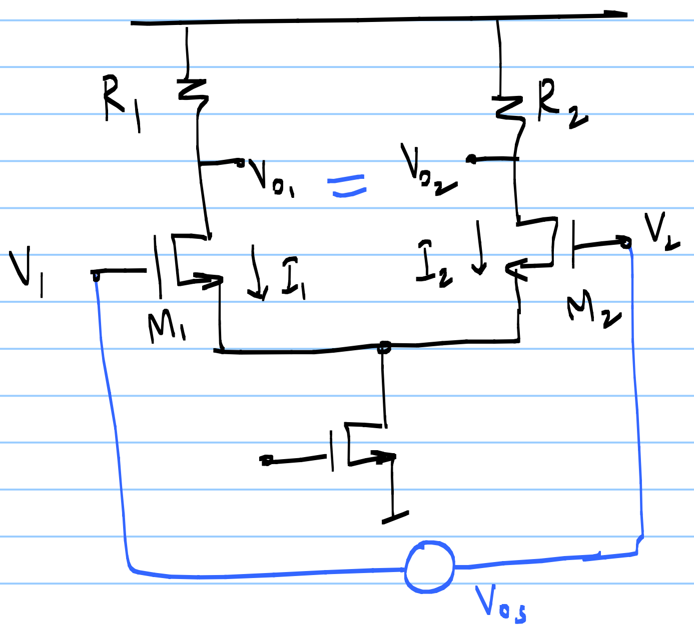


$\overline{V_{os}} = V_1 - V_2$ that makes $V_{o1} - V_{o2} = 0$

5/10/2023

Lec 9

Mismatch in differential amplifier



$R_1 \neq R_2$

$V_{T_1} \neq V_{T_2}$

$\left(\frac{W}{L}\right)_1 \neq \left(\frac{W}{L}\right)_2$

Small mismatches

$$R = \frac{R_1 + R_2}{2}$$

$$\Delta R = \frac{R_1 - R_2}{2}$$

$$V_T = \frac{V_{T_1} + V_{T_2}}{2}$$

$$\Delta V_T = V_{T_1} - V_{T_2}$$

$$\left(\frac{W}{L}\right) = \frac{(W/L)_1 + (W/L)_2}{2}$$

$$\Delta\left(\frac{W}{L}\right) = \frac{(W/L)_1 - (W/L)_2}{2}$$

$$\left.\begin{array}{l} V_{o_1} = V_{o_2} \\ V_{DD} - I_1 R_1 = V_{DD} - I_2 R_2 \end{array}\right\} I_1 R_1 = I_2 R_2$$

$$I = \frac{I_1 + I_2}{2} \quad ; \quad \Delta I = \frac{I_1 - I_2}{2}$$

$$I_1 = I + \Delta I$$

$$I_2 = I - \Delta I$$

$$R_1 = R + \Delta R \quad ; \quad R_2 = R - \Delta R$$

$$V_{T_1} = V_T + \frac{\Delta V_T}{2} \quad ; \quad V_{T_2} = V_T - \frac{\Delta V_T}{2}$$

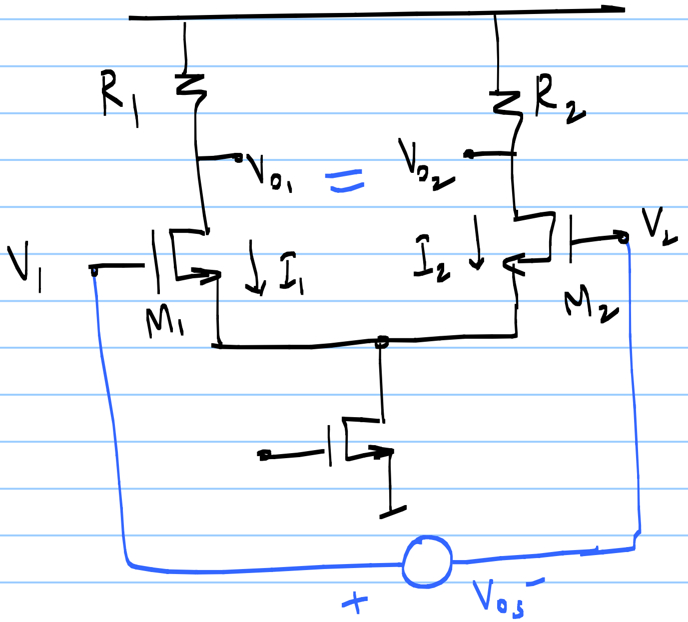
$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right) + \Delta\left(\frac{W}{L}\right) ; \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right) - \Delta\left(\frac{W}{L}\right)$$

$$I_1 R_1 = I_2 R_2 \Rightarrow (I + \Delta I)(R + \Delta R) = (I - \Delta I)(R - \Delta R)$$

$$\cancel{IR} + I \Delta R + \Delta I R + \cancel{\Delta I \Delta R} = \cancel{IR} - \Delta I R - I \Delta R + \cancel{\Delta I \Delta R}$$

$$\cancel{2} \Delta I R = - \cancel{2} I \Delta R$$

$$\boxed{\frac{\Delta I}{I} = \frac{-\Delta R}{R}}$$



$$V_{os} = V_1 - V_2 = V_{GS_1} - V_{GS_2}$$

$$= \left[V_{T_1} + \sqrt{\frac{2 I_1}{\mu C_{ox}\left(\frac{W}{L}\right)_1}}\right] - \left[V_{T_2} + \sqrt{\frac{2 I_2}{\mu C_{ox}\left(\frac{W}{L}\right)_2}}\right]$$

$$= \Delta V_T + \sqrt{\frac{2}{\mu C_{ox}}}\left[\sqrt{\frac{I_1}{(W/L)_1}} - \sqrt{\frac{I_2}{(W/L)_2}}\right]$$

$$V_{os} = \Delta V_T + \sqrt{\frac{2}{\mu C_{ox}}}\left[\sqrt{\frac{I+\Delta I}{\left(\frac{W}{L}\right)+\Delta\frac{W}{L}}} - \sqrt{\frac{I-\Delta I}{\left(\frac{W}{L}\right)-\Delta\left(\frac{W}{L}\right)}}\right]$$

$$= \Delta V_T + \underbrace{\sqrt{\frac{2I}{\mu C_{ox}\left(\frac{W}{L}\right)}}}_{\text{nom. } V_{Dsat}}\left[\sqrt{\frac{1+\Delta I/I}{1+\frac{\Delta(W/L)}{(W/L)}}} - \sqrt{\frac{1-\Delta I/I}{1-\frac{\Delta(W/L)}{(W/L)}}}\right]$$

$$= \Delta V_T + V_{Dsat}\left[\left(1+\frac{\Delta I}{2I}\right)\left(1-\frac{\Delta(W/L)}{2(W/L)}\right) - \left(1-\frac{\Delta I}{2I}\right)\left(1+\frac{\Delta(W/L)}{2(W/L)}\right)\right]$$

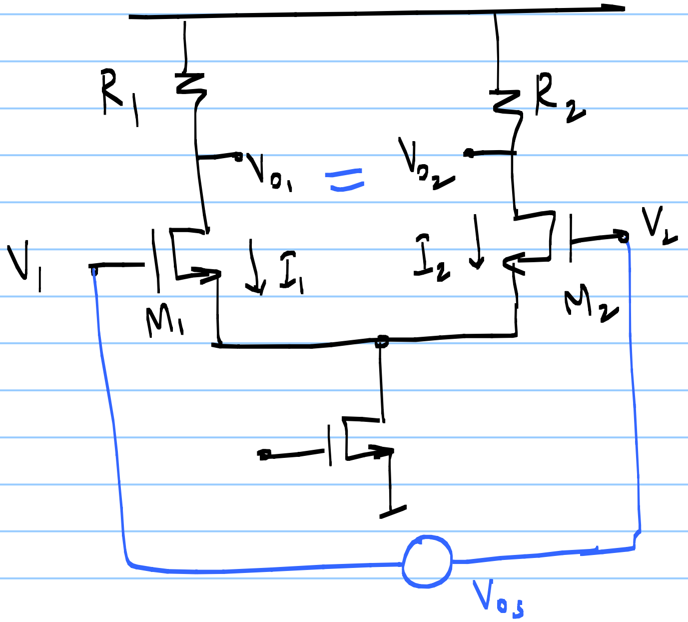
$$V_{os} = \Delta V_T + V_{Dsat}\left[\cancel{1} + \frac{\Delta I}{2I} - \frac{\Delta(W/L)}{2(W/L)} - \cancel{\frac{\Delta I \cdot \Delta(W/L)}{4I(W/L)}}\right.$$

$$\left.\cancel{-1} + \frac{\Delta I}{2I} - \frac{\Delta(W/L)}{2(W/L)} + \cancel{\frac{\Delta I \cdot \Delta(W/L)}{4I(W/L)}}\right]$$

$$= \Delta V_T + V_{Dsat}\left(\frac{\Delta I}{I} - \frac{\Delta(W/L)}{(W/L)}\right)$$

$$V_{os} = \Delta V_T + V_{Dsat}\left(-\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right)$$

$$\sigma_{OS}^2 = \sigma_{V_T}^2 + V_{Dsat}^2 \left[\sigma_{\Delta R/R}^2 + \sigma_{\frac{\Delta W/L}{W/L}}^2 \right] \checkmark$$

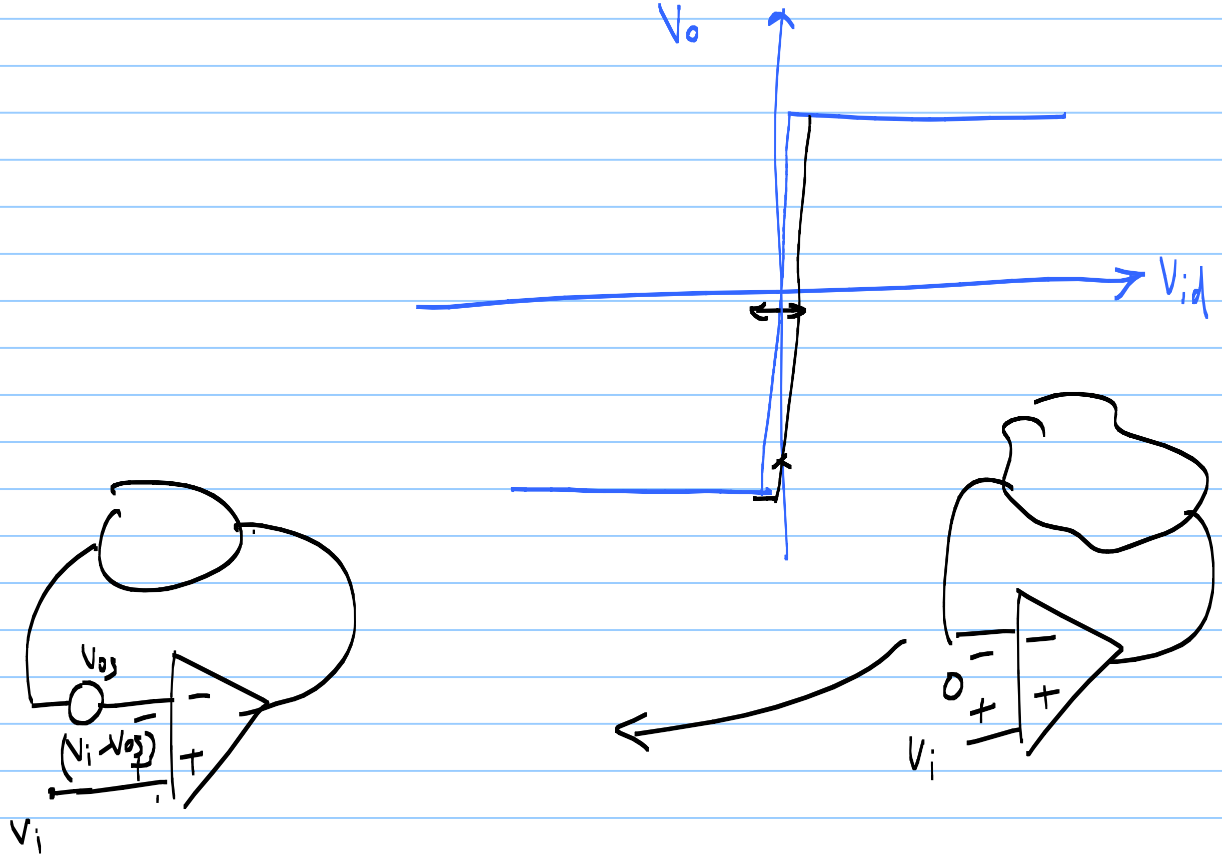


only $V_{T_{1,2}}$ mismatch

$V_{os} = \Delta V_T$

$$V_{os} = V_{GS_1} - V_{GS_2}$$

$$= \Delta V_T + \underbrace{\Delta V_{Dsat}}_{0}$$


Vo
Vid
Vos
(Vi-Vos)
Vi
0
Vi

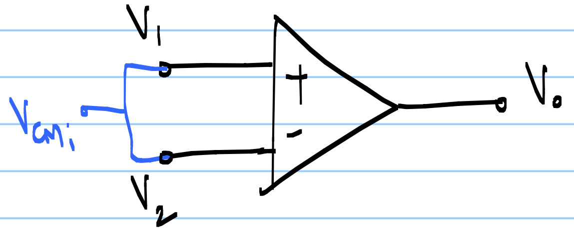
10/10/23

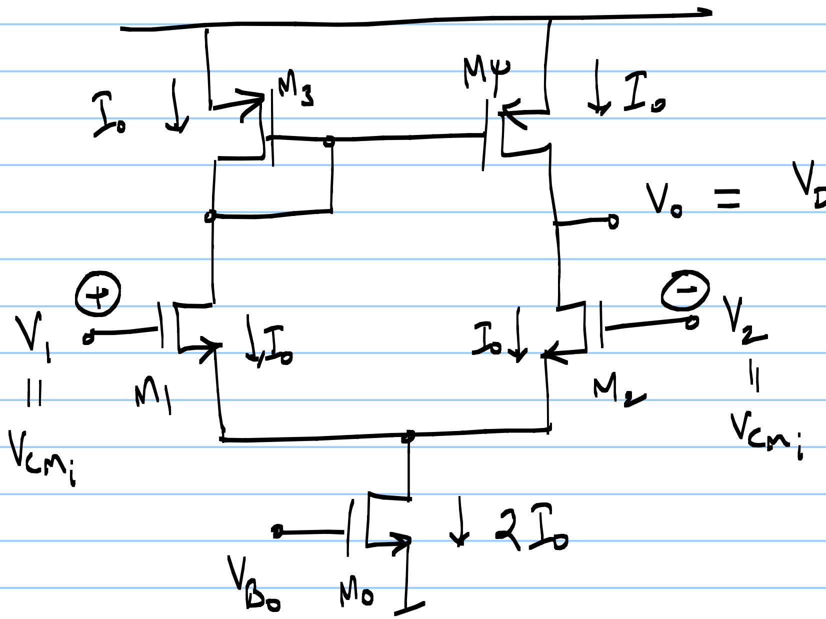
Lec 10

One stage opamp

$V_{GS_1} = V_{GS_2}$ | $V_{SG_3} = V_{SG_4}$

$I_1 + I_2 = 2I_0$ | $I_1 = I_3$; $I_2 = I_4$





$= V_{DD} - V_{SG_3}$ if

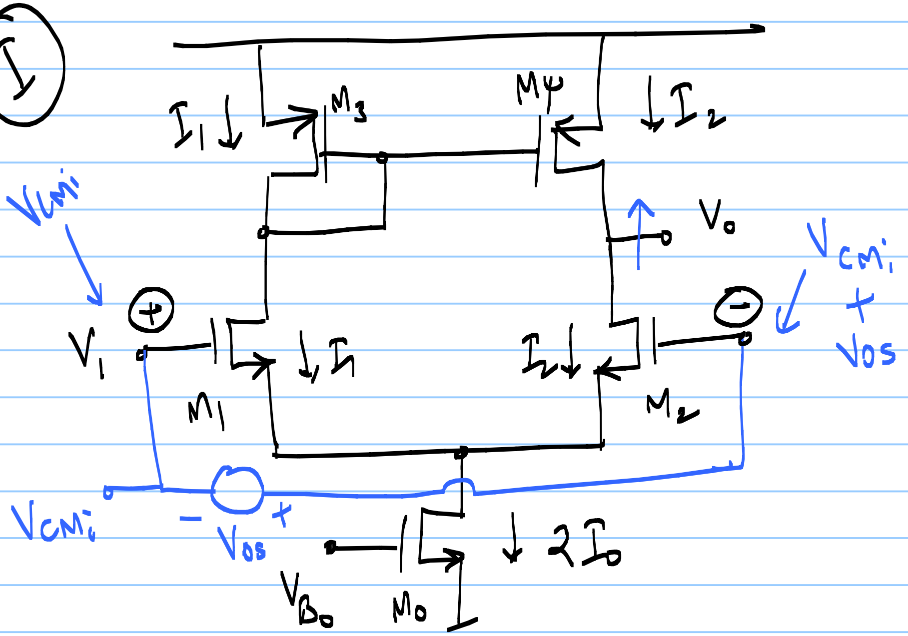
$M_3 \equiv M_4$

$M_1 \equiv M_2$

If $M_1 \equiv M_2$ & $M_3 \equiv M_4$ $\Big\}$ $V_o = V_{DD} - V_{SG_3} = V_{oref}$

If $M_1 \not\equiv M_2$ or/and $M_3 \not\equiv M_4$ $\Big\}$ $V_o \neq V_{DD} - V_{SG_3}$

Ⅰ



Assume mismatch only in V_{T_1} & V_{T_2}

$V_{T_1} = V_T$; $V_{T_2} = V_T + \Delta V_{T_1}$

Apply $V_1 - V_2 = V_{os}$ so that $V_o = V_{DD} - V_{SG_3}$

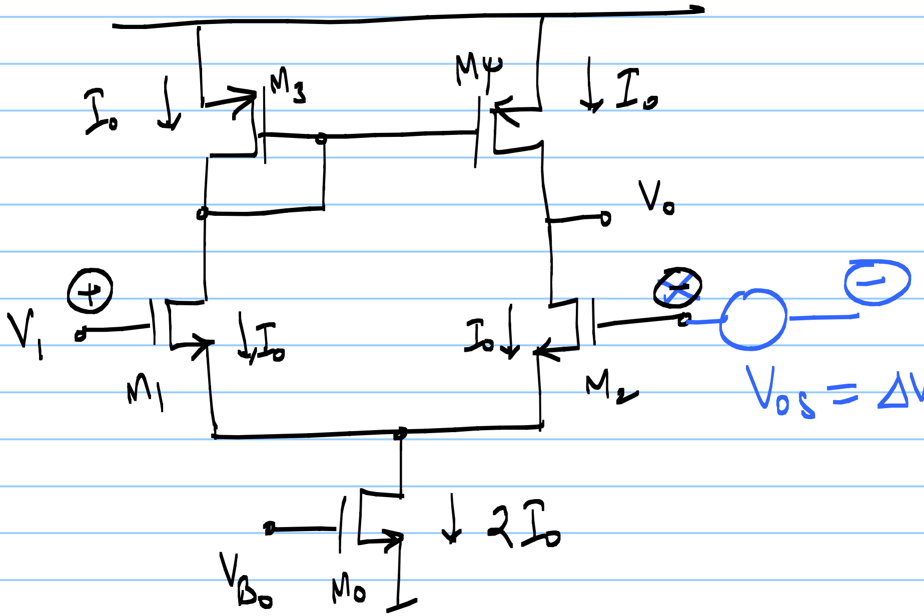
$= V_{o_{ref}}$

$$\left.\begin{array}{l} I_1 = I_0 + g_{m_1}\Delta V_{T_1}/2 \\ I_2 = I_0 - g_{m_1}\Delta V_{T_1}/2 \end{array}\right\} \quad I_1 > I_2$$

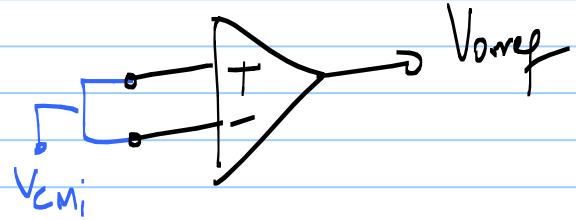
$V_o > V_{o_{ref}}$

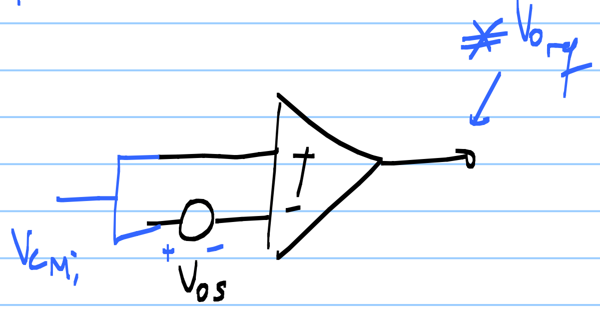
If $V_{os} = \Delta V_{T_1} \Rightarrow I_1 = I_2 = I_0$

$\longrightarrow \sigma_{os}^2 = \sigma_{V_{T_1}}^2$

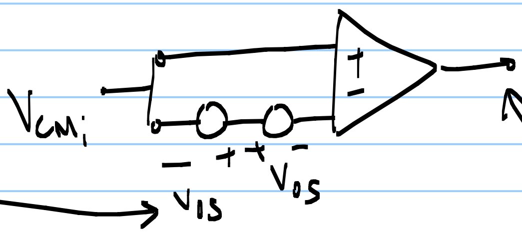


$V_{os} = \Delta V_{T_{1,2}}$



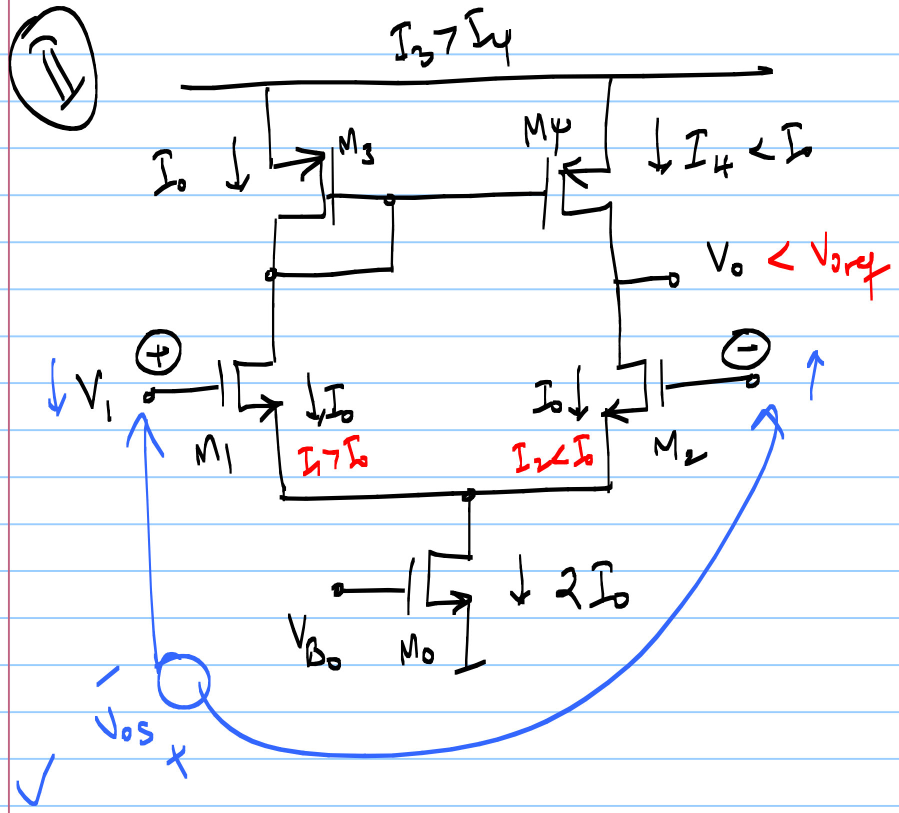


input refered offset



additional applied voltage

$= V_{oref}$



assume $V_{T_3} = V_{T_p}$; $V_{T_4} = V_{T_p} + \Delta V_{T_3}$

$M_1 \equiv M_2$

$V_o < V_{oref}$

$I_1 = I_3$

$I_2 = I_4$

$\boxed{I_1 + I_2 = 2I_o}$

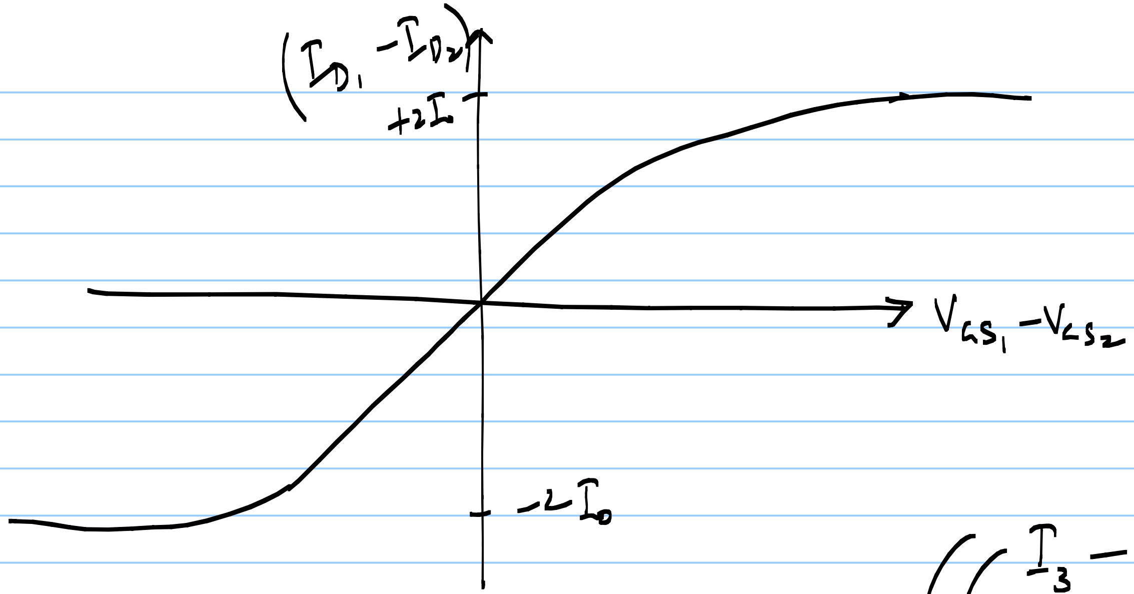
$I_1 = I_3 > I_o$

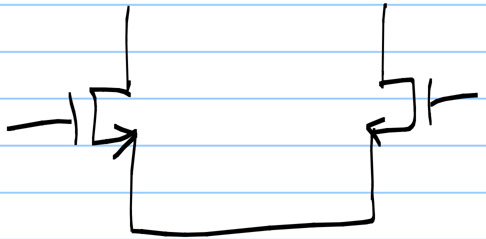
$I_2 = I_4 < I_o$

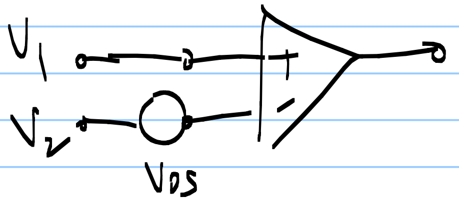
$I_2 = I_4 = I_o - g_{m_3}\Delta V_{T_3}/2$

$I_1 = I_3 = I_o + g_{m_3}\Delta V_{T_3}/2$

$I_1 - I_2 = I_3 - I_4 = g_{m_3}\Delta V_{T_3}$

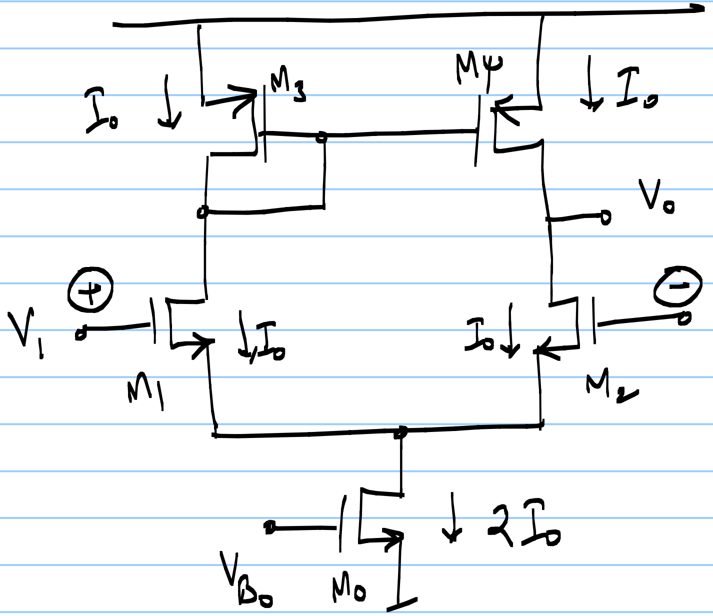






$$\left(\left(\begin{array}{l} I_3 - I_4 = g_{m_3}\Delta V_{T_3} \\ \\ I_1 - I_2 = g_{m_1} V_{os} \end{array}\right.\right.$$

$$V_{os} = \frac{g_{m_3}}{g_{m_1}}\Delta V_{T_3} \Bigg) \longrightarrow \sigma_{os}^2 = \left(\frac{g_{m_3}}{g_{m_1}}\right)^2 \sigma_{V_{T_3}}^2$$



Both $\left(\begin{array}{ll} V_{T_1} = V_{T_n} \; ; & V_{T_2} = V_{T_n} + \Delta V_{T_1} \\ V_{T_3} = V_{T_p} \; ; & V_{T_4} = V_{T_p} + \Delta V_{T_3} \end{array} \right.$

$$\sigma_{os}^2 = \sigma_{V_{T_1}}^2 + \underline{\left(\frac{g_{m_3}}{g_{m_1}}\right)^2 \sigma_{V_{T_3}}^2}$$

low offset from M_3, M_4 → ↓ g_{m_3}

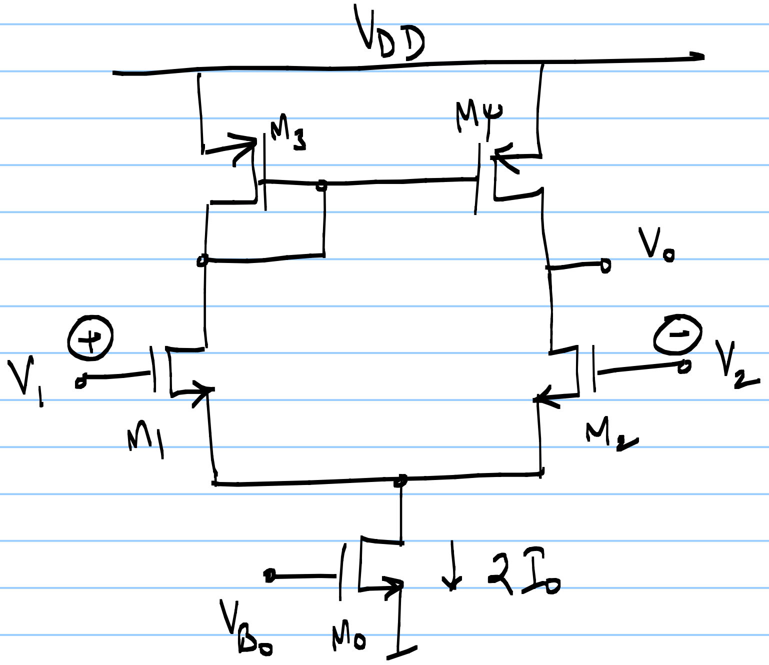
↑ g_{m_1}

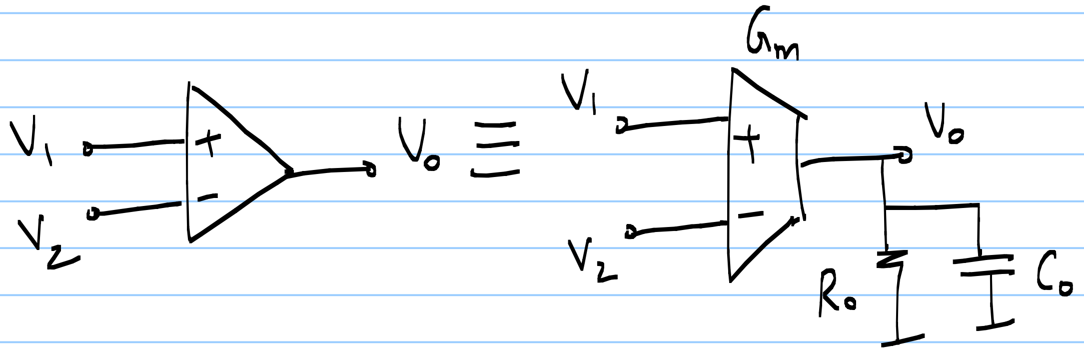
(similar for noise) ←

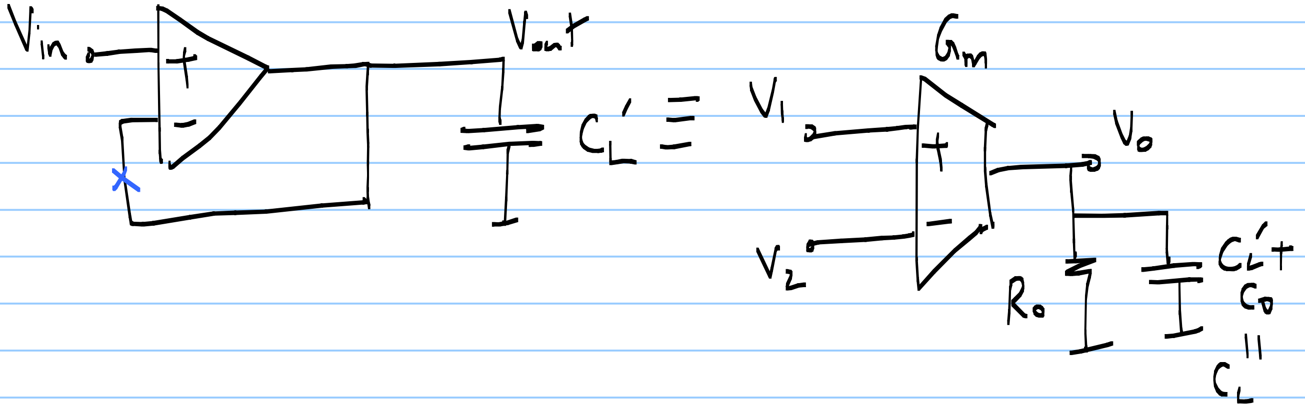
12/10/2023

Lec 11

One-stage opamp datasheet







1) DC gain $= A_o = G_m R_o = g_{m_1}\left(r_{ds_4} \| r_{ds_2}\right) = \dfrac{g_{m_1}}{g_{ds_2} + g_{ds_4}}$

2) $G_m = g_{m_1}$

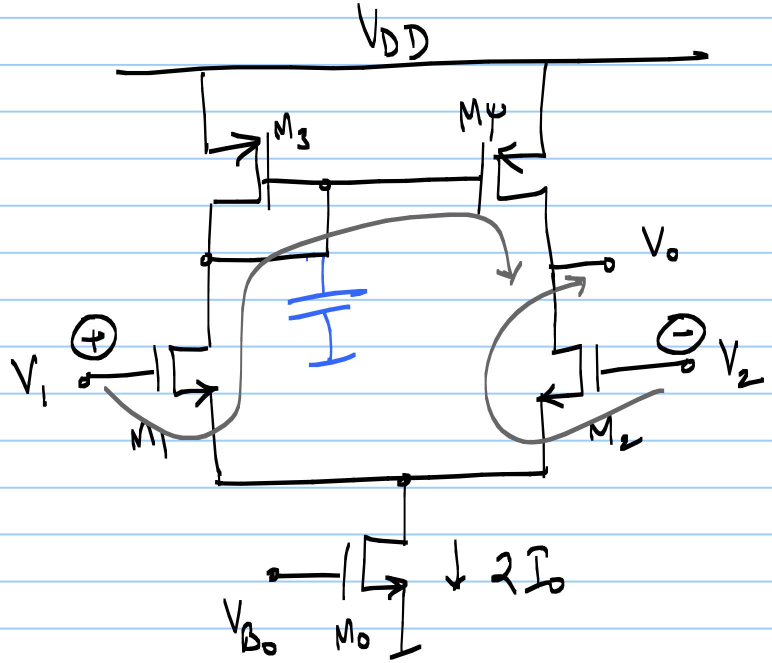
3) $\omega_u = \dfrac{G_m}{C_L} = \dfrac{g_{m_1}}{C_L} \longleftrightarrow \omega_d = \dfrac{1}{R_o C_L} = \dfrac{G_o}{C_L}$ (LHP)

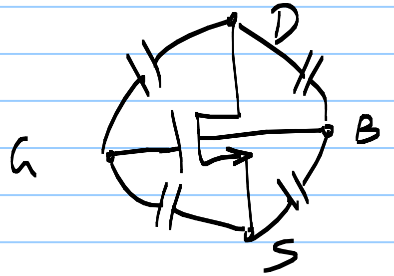
(UGF) (dominant pole)

4) Non-dominant poles & zeroes:

LHP $\omega_{nd} = \dfrac{g_{m_3}}{C_p}$

LHP $Z_{ND} = \dfrac{2 g_{m_3}}{C_p}$





Sat.

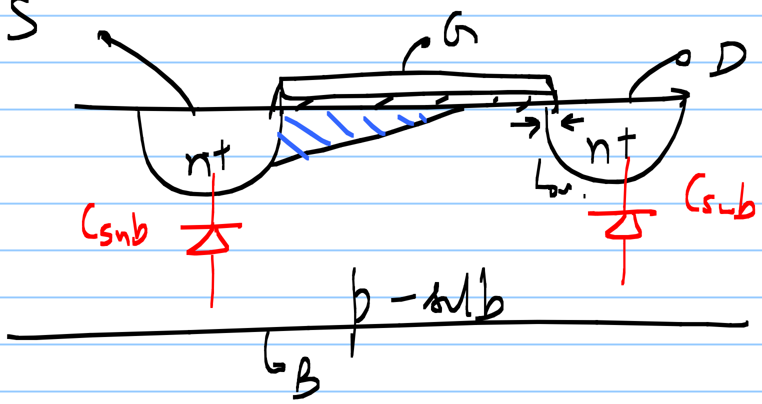
largest

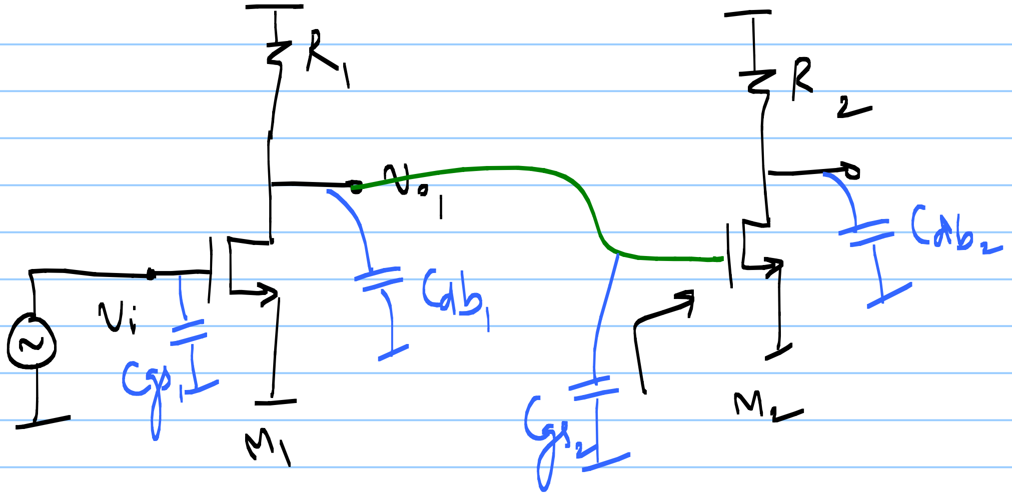
$C_{gs} = \frac{2}{3} W L C_{ox}$

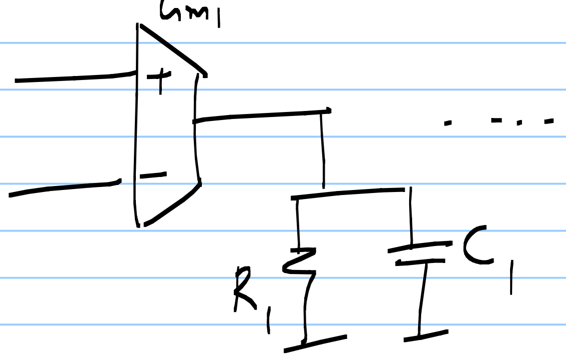
$C_{gd} = W L_{ov} C_{ox}$

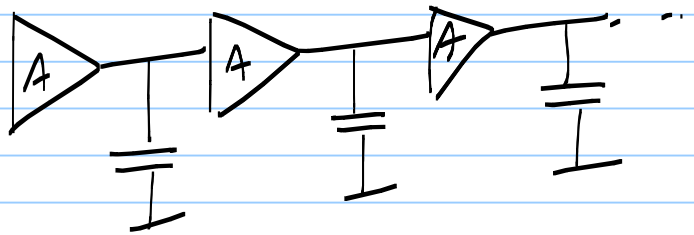
≈ small

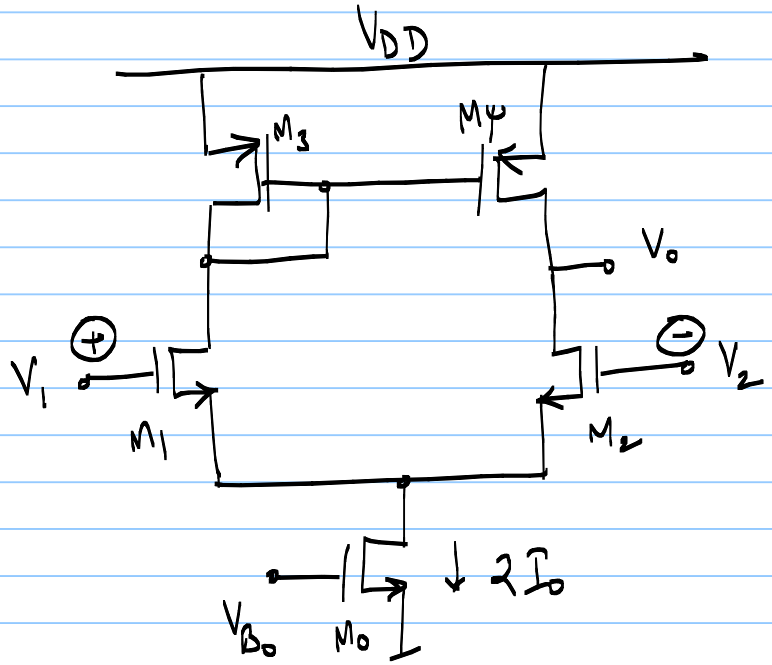
C_{db} = jn. cap, C_{sb} = jn cap




R₁
R₂
V₀₁
Cdb₂
Vi
Cgs₁
Cdb₁
M₁
Cgs₂
M₂

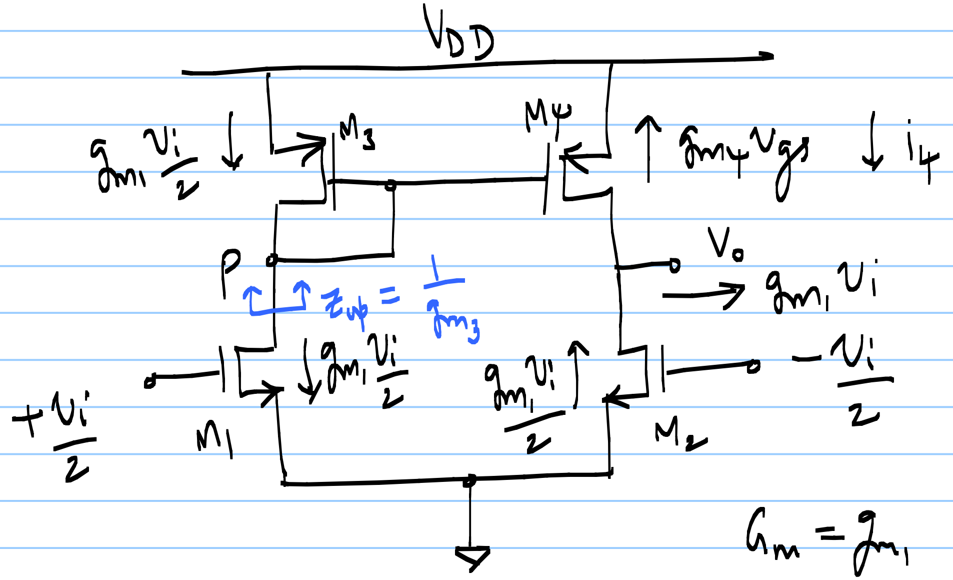

Gm₁
+
−
R₁
C₁
. . . .


A
A
A
. . .



$M_1 \equiv M_2$

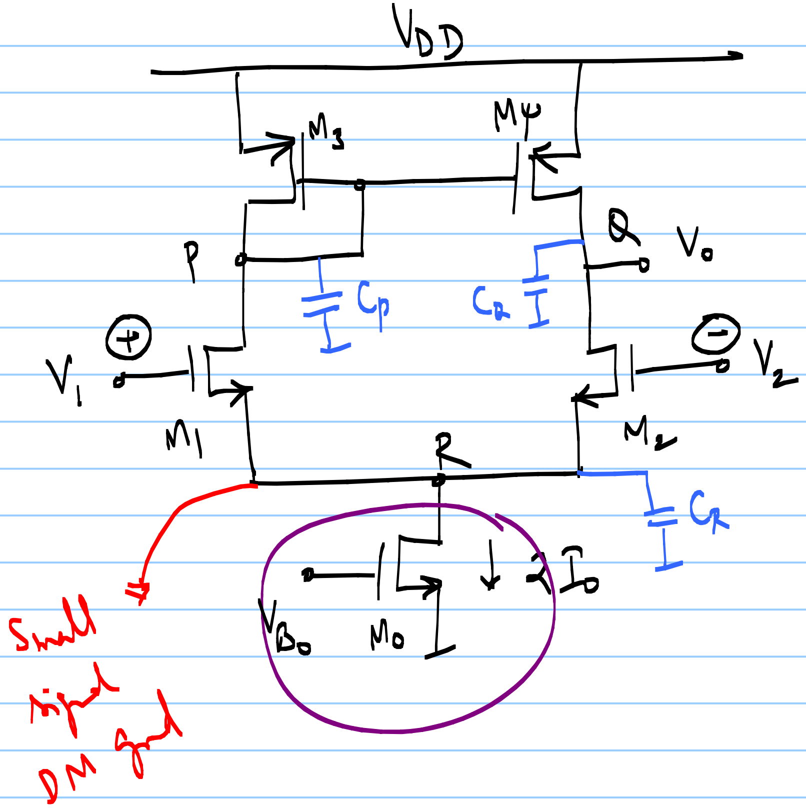
$M_3 \equiv M_4$



$G_m = g_{m1}$

$$v_p = -\frac{g_{m1} v_i/2}{g_{m3}}$$

$$i_4 = \frac{g_{m1}}{g_{m3}} \times g_{m4} \cdot \frac{v_i}{2} = g_{m1} v_i/2$$

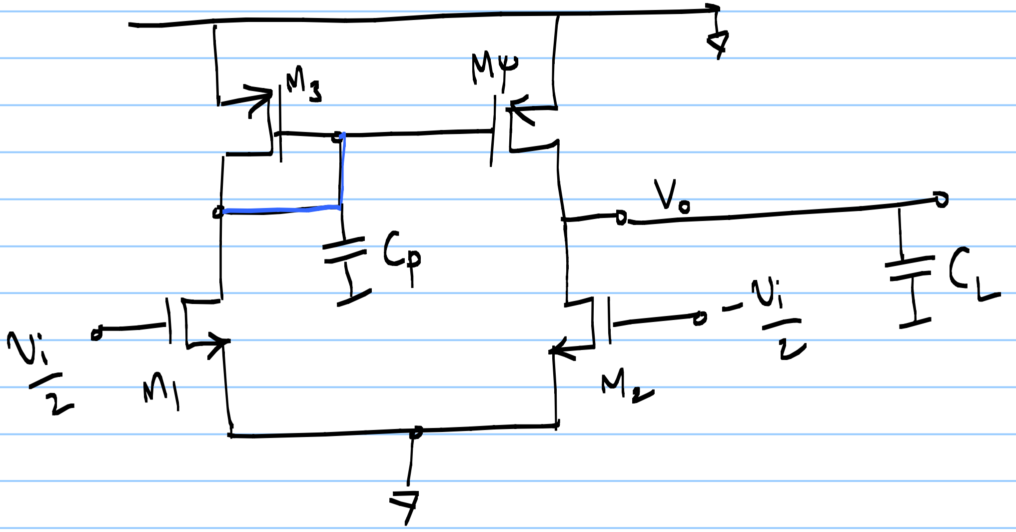


$$C_P = C_{db_3} + C_{gs_3} + C_{db_1} + C_{gs_4}$$

$$C_Q = C_{db_4} + C_{db_2} + C_L' = C_L$$

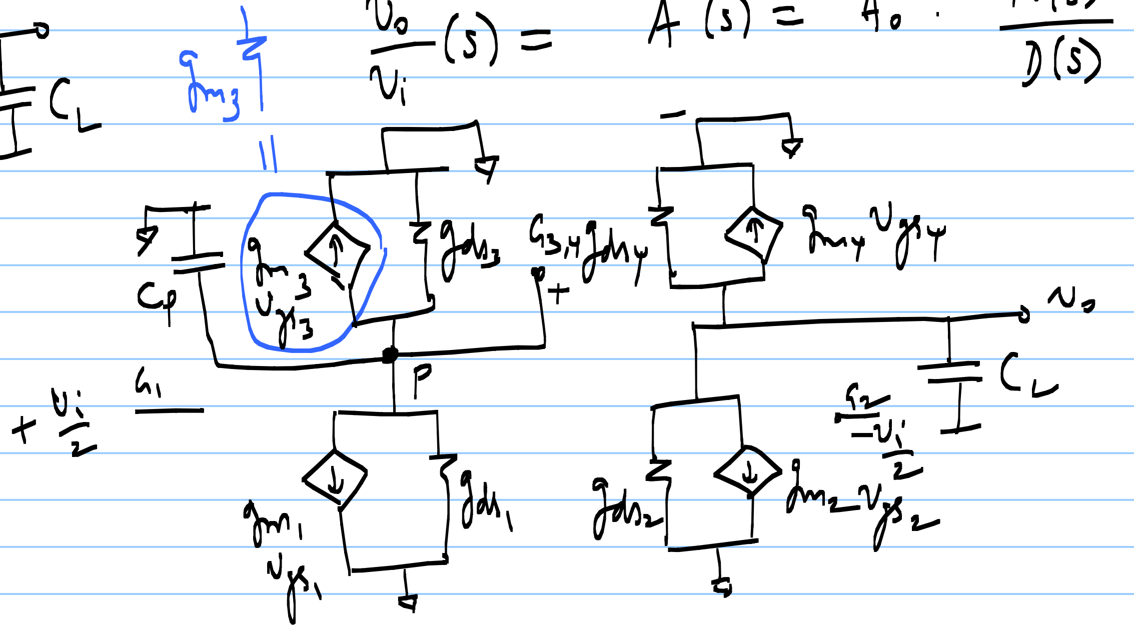
$$C_R = C_{db_0} + C_{sb_1} + C_{sb_2}$$ ← does not matter for DM analysis

→ matters for CM gain & CMRR



$$\frac{v_o}{v_i}(s) = ?$$

$$\frac{v_o}{v_i}(s) = A(s) = A_0 \cdot \frac{N(s)}{D(s)}$$



current pulled out of output node

$$i_o(s) = i_2(s) + i_4(s)$$

$$= g_{m_2} v_{gs_2} + g_{m_4} v_{gs_4}$$

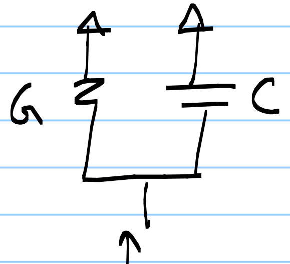
$$= -g_{m_2}\frac{v_i}{2} + g_{m_4}\cdot v_{gs_4}$$

$$i_1(s) = g_{m_1} v_{gs_1} = g_{m_1}\frac{v_i}{2}$$

$$v_p = -i_1(s)\cdot Z_p(s)$$

$$= -g_{m_1}\frac{v_i}{2}\cdot\frac{1}{g_{m_3}+g_{ds_3}+g_{ds_1}+sC_p}$$

$$g_{m_3} \gg g_{ds_1}, g_{ds_3}$$



$Z(s) =$

$$G = g_{m_3}+g_{ds_3}+g_{ds_1}$$

$$C = C_p$$

$$Y(s) = G + sC$$

$$Z(s) = \frac{1}{G+sC}$$

$$= \frac{1}{g_{m_3}+g_{ds_3}+g_{ds_1}+sC_p}$$

$$v_p(s) \simeq \frac{-g_{m_1}}{g_{m_3}+sC_p}\cdot\left(v_i/2\right) = v_{gs_4}$$

$$i_4(s) = g_{m_4}\cdot v_p(s) = \frac{-g_{m_1}g_{m_4}}{g_{m_3}+sC_p}\cdot\frac{v_i}{2} = \frac{-g_{m_1}}{1+sC_p/g_{m_3}}\cdot\frac{v_i}{2}$$

$$i_o(s) = -g_{m_2}\frac{v_i}{2} - \frac{g_{m_1}}{1+\frac{sC_p}{g_{m_3}}}\cdot\frac{v_i}{2} = -v_i(s)\cdot\frac{g_{m_1}}{2}\left[1+\frac{1}{1+\frac{sC_p}{g_{m_3}}}\right]$$

$$= -g_{m_1}\frac{v_i}{2}\left[\frac{2+sC_p/g_{m_3}}{1+sC_p/g_{m_3}}\right] = -g_{m_1}v_i\left[\frac{1+sC_p/2g_{m_3}}{1+sC_p/g_{m_3}}\right]$$

$$v_o(s) = -i_o(s) \times Z_o(s)$$

$$= g_{m_1} v_i \cdot \left[\frac{1+\frac{sC_p}{2g_{m_3}}}{1+\frac{sC_p}{g_{m_3}}}\right] \times \left[\frac{1}{g_{ds_2}+g_{d_4}+sC_L}\right]$$

$$A(s) = \frac{v_o}{v_i}(s) = \underbrace{\left[\frac{g_{m_1}}{g_{ds_2}+g_{ds_4}}\right]}_{A_o} \left[\frac{\overset{N(s)}{\left(1+\frac{sC_p}{2g_{m_3}}\right)}}{\underbrace{\left(1+\frac{sC_p}{g_{m_3}}\right)\left(1+\frac{sC_L}{g_{ds_2}+g_{ds_4}}\right)}_{D(s)}}\right]$$

$$A(s) = \frac{V_o}{V_i}(s) = \underbrace{\left[\frac{g_{m1}}{g_{ds2}+g_{ds4}}\right]}_{A_o}\left[\frac{\left(1+\frac{sC_p}{2g_{m3}}\right)}{\left(1+\frac{sC_p}{g_{m3}}\right)\left(1+\frac{sC_L}{g_{ds2}+g_{ds4}}\right)}\right]$$

$$Z_{ND} = \frac{2g_{m3}}{C_p}$$

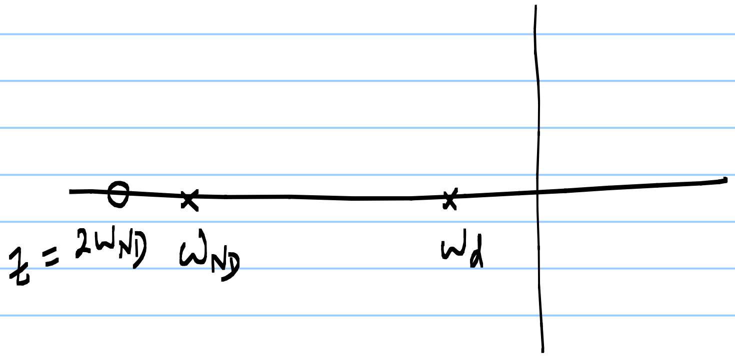
LHP zero

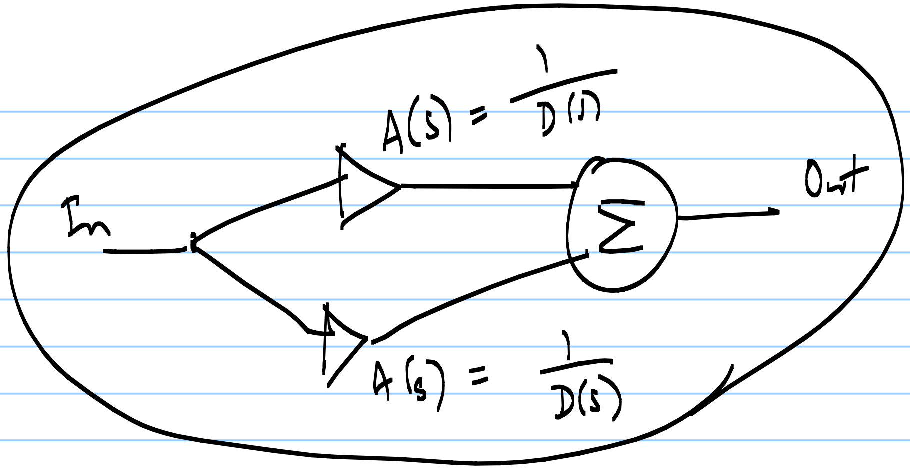
$$\omega_{ND} = \frac{g_{m3}}{C_p}$$

LHP pole

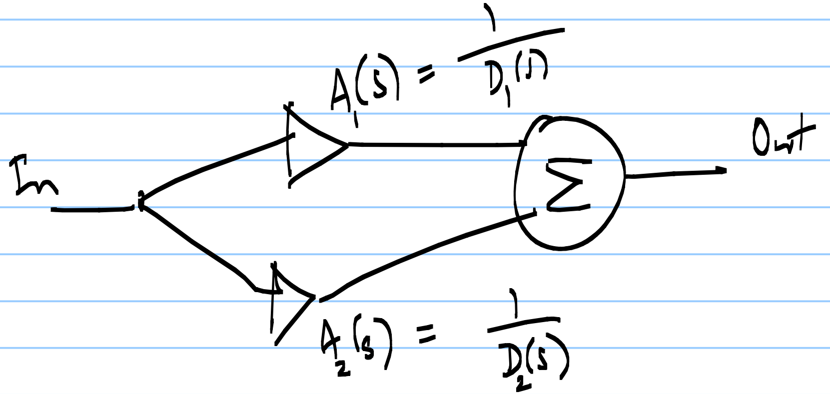
$$\omega_d = \frac{g_{ds2}+g_{ds4}}{C_L} = \frac{G_o}{C_L}$$

LHP pole



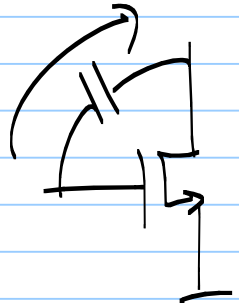


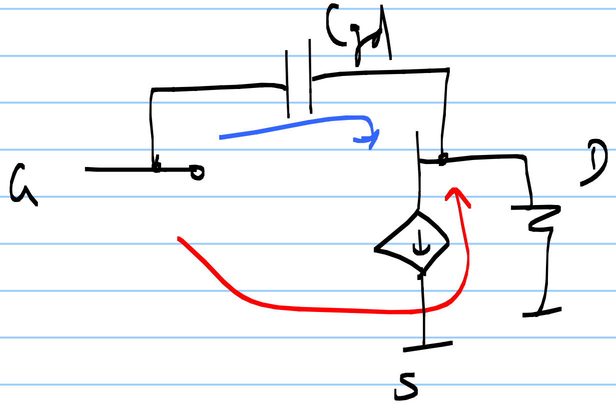
$$2A(s) = \frac{2}{D(s)}$$



$$A_1(s) + A_2(s)$$

$$= \frac{1}{D_1(s)} + \frac{1}{D_2(s)} = \frac{\overset{\text{Zero}}{\downarrow}N(s)}{D(s)}$$




Cgd
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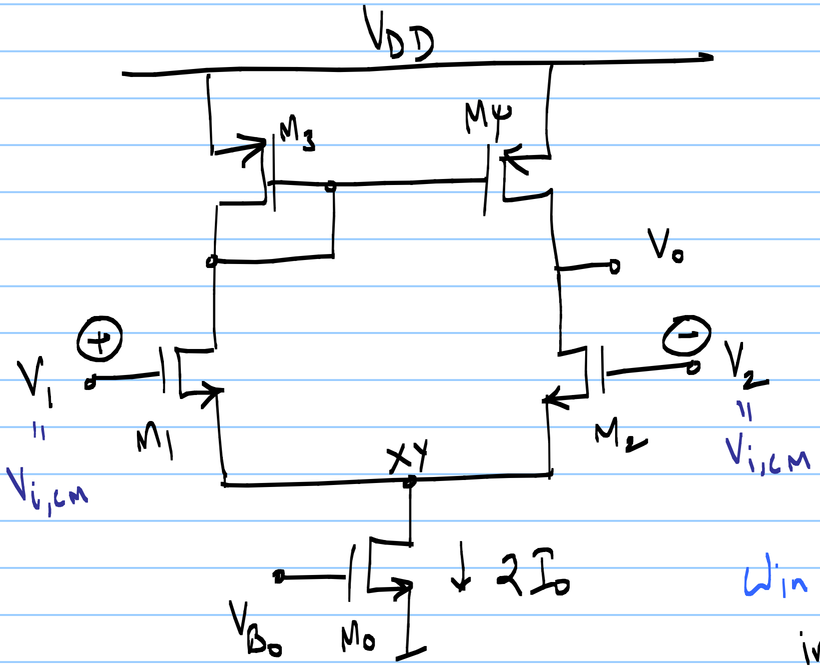
13/10/23

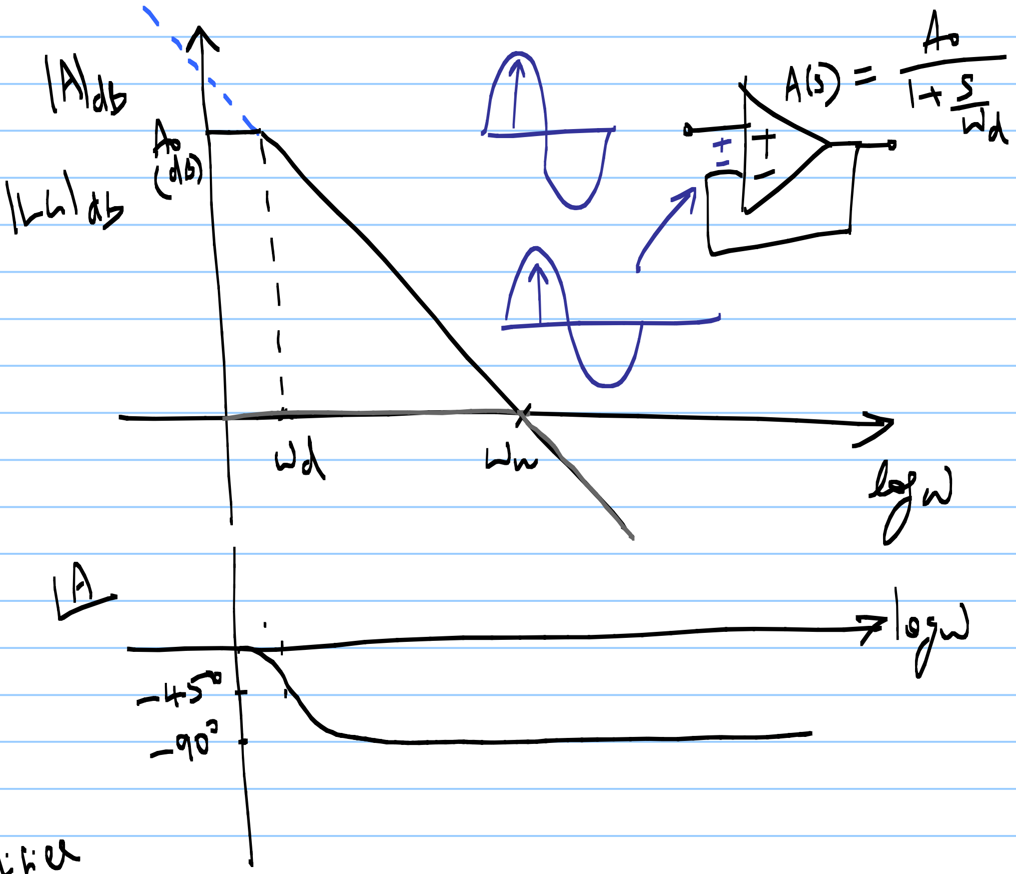
Lec 12

One-stage opamp datasheet



$\omega_{in} << \omega_u$

in closed loop amplifier



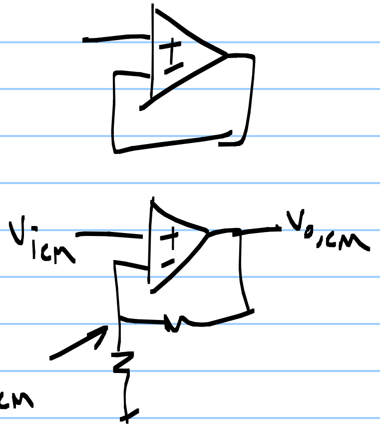
$$A(s) = \frac{A_0}{1+\frac{s}{\omega_d}}$$

5) Input CM range :

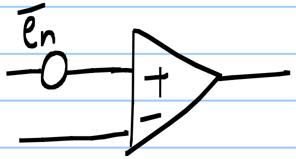
$$ICMR = \left\{ \underbrace{V_{GS_1}\Big|_{I_0} + V_{Dsat_0}\Big|_{2I_0}}_{ICMR_{min.}} \;,\; \underbrace{V_{DD} - V_{SG_3}\Big|_{I_0} + V_{T_1}}_{ICMR_{max}} \right\}$$

6) Output CM range

$$OCMR = \underbrace{\left\{ \underset{(min)}{V_{i,CM} - V_{T_2}} \;,\; \underset{(max)}{V_{DD} - V_{SG_4} + V_{T_4}} \right\}}_{}$$



7) <u>Noise</u> $\overline{e_n}$



$\overline{i_n} = 0$ @ low freq.

$$\frac{\overline{e_n^2}}{\Delta f} = ?$$

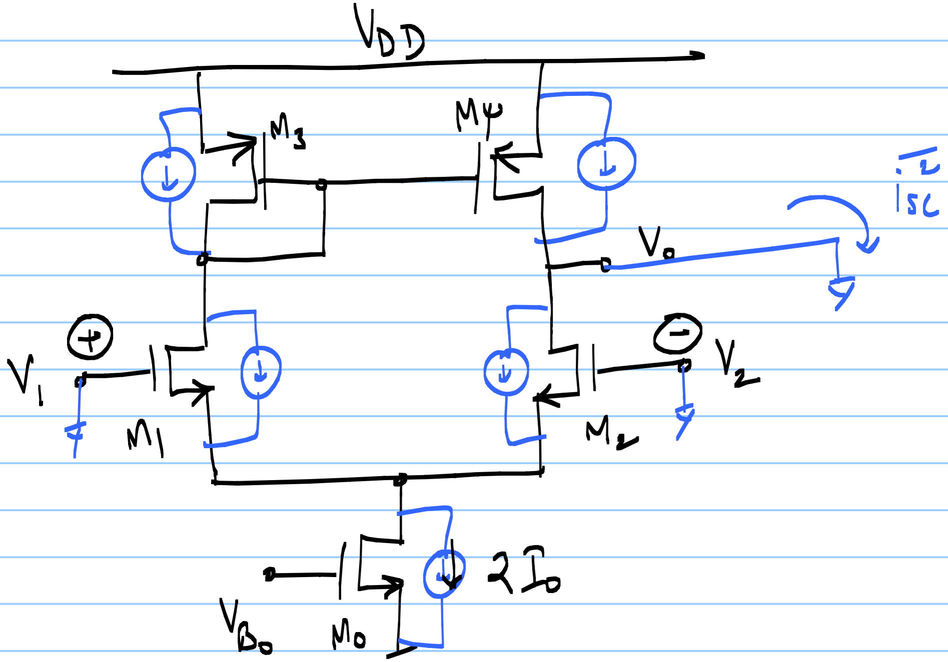
HW

$$\frac{8kT}{3} \cdot g_{m_1} \cdot \frac{1}{g_{m_1}^2}$$

$$\frac{\overline{e_n^2}}{\Delta f} = \underbrace{\frac{16kT}{3\,g_{m_1}}}_{\text{noise from } M_1 \,\&\, M_2} + \underbrace{\frac{16kT\,g_{m_3}}{3\,g_{m_1}^2}}_{\text{Noise from } M_3 \,\&\, M_4}$$

low noise : ↑ g_{m_1}

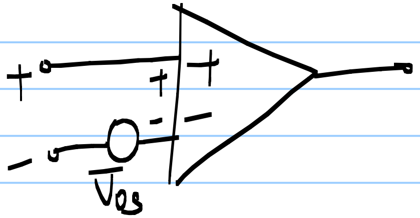
↓ g_{m_3}



$$\overline{e_n^2} = \frac{\overline{i_{sc}^2}}{g_{m1}^2}$$

HW

8) <u>Offset</u> $\overline{V_{os}}$



$\overline{i_{os}} = 0$

Assume mismatch in $V_{T_1} - V_{T_2}$

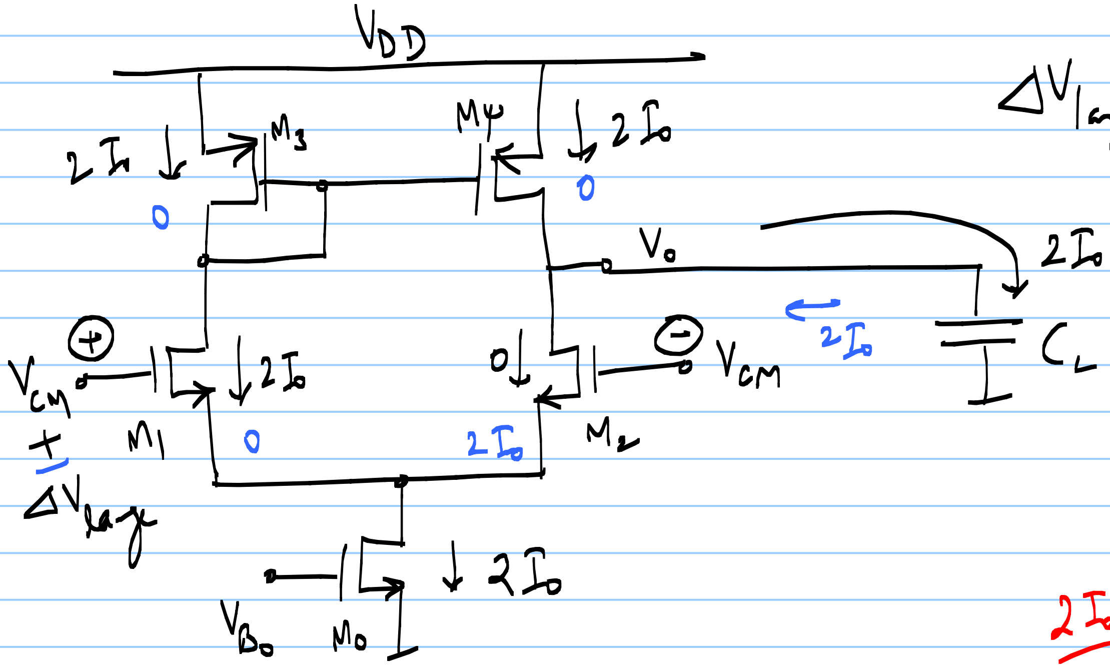
& $V_{T_3} - V_{T_4}$

$$\sigma_{os}^2 = \sigma_{V_{T_{1,2}}}^2 + \left(\frac{g_{m_3}}{g_{m_1}}\right)^2 \cdot \sigma_{V_{T_{3,4}}}^2$$

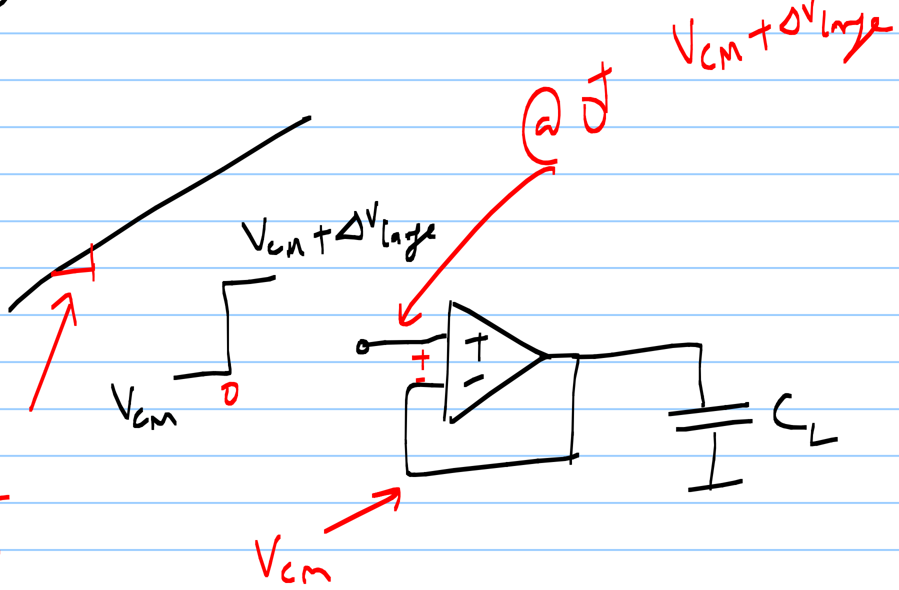
low offset : $\uparrow g_{m_1}$

$\downarrow g_{m_3}$

9) "Slew Rate" $= \dfrac{\pm 2 I_0}{C_L}$



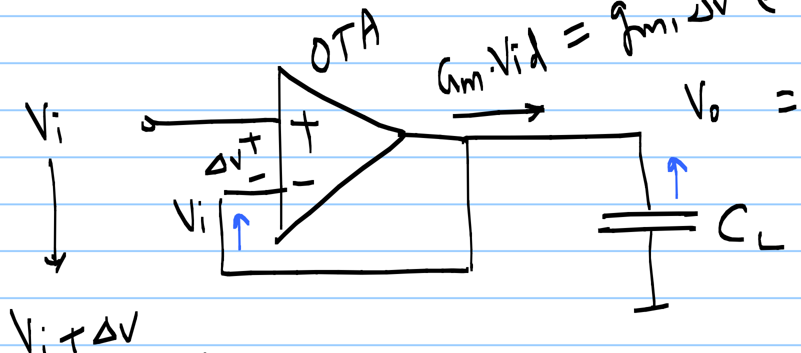
$$\Delta V_{large} = V_{GS_1} - V_{GS_2}$$



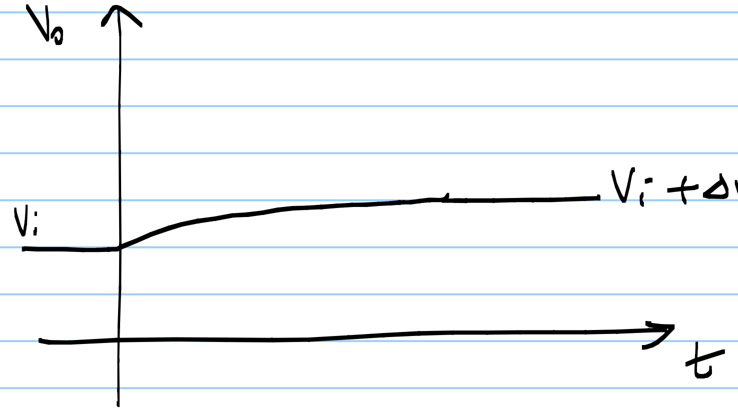
17/10/23

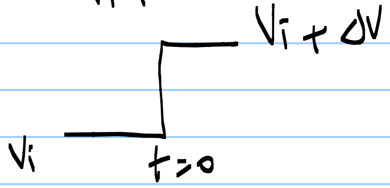
Lec 13

$G_m \cdot V_{id} = g_{m1} \Delta V$ @ $t = 0^+$



$V_o = V_i$

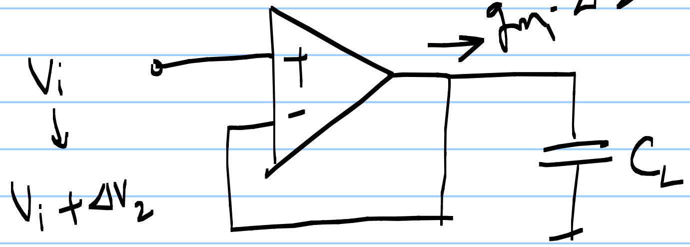


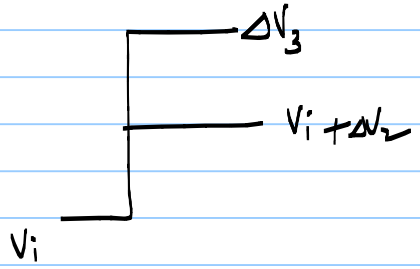


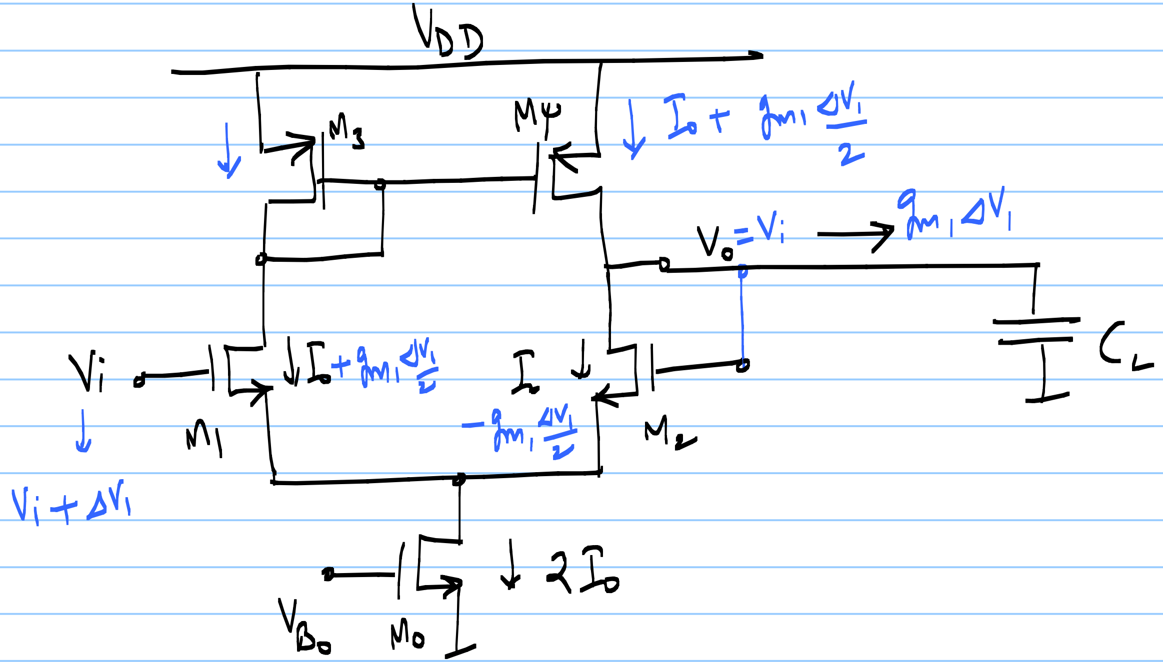
@ $t = 0^+$: $\left.\begin{array}{l} V_+ = V_i + \Delta V \\ V_- = V_i \end{array}\right)$ $V_{id}(t = 0^+) = \Delta V$

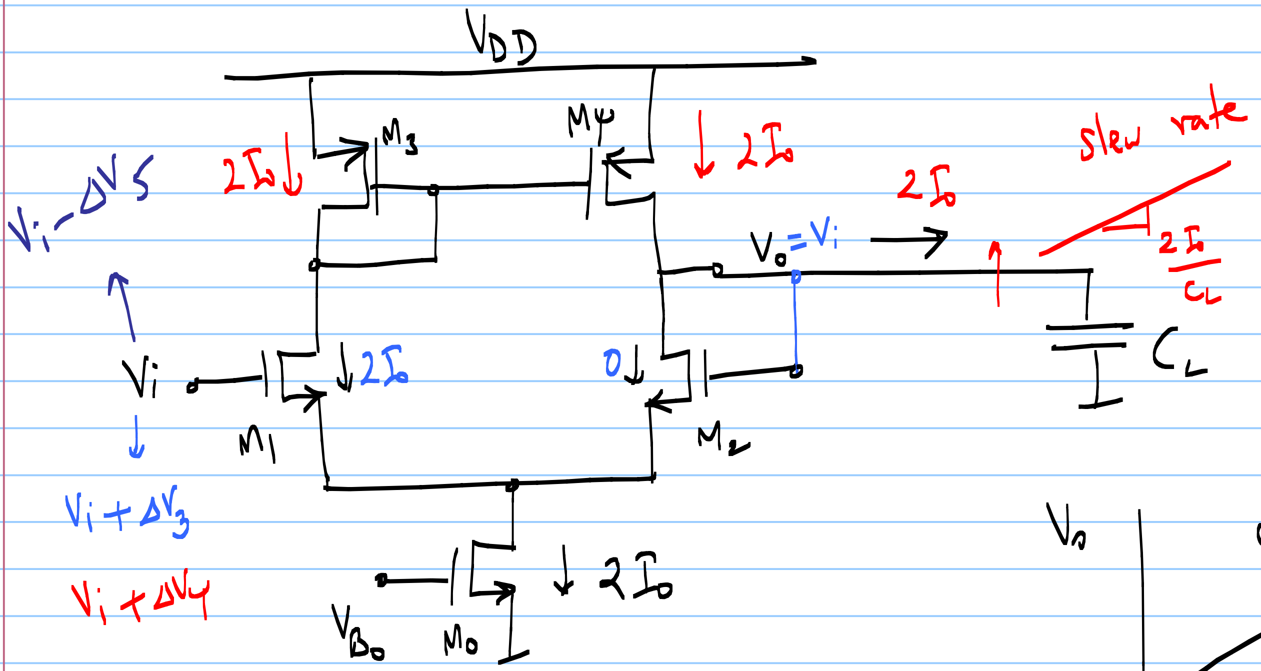
@ $t=0^+$

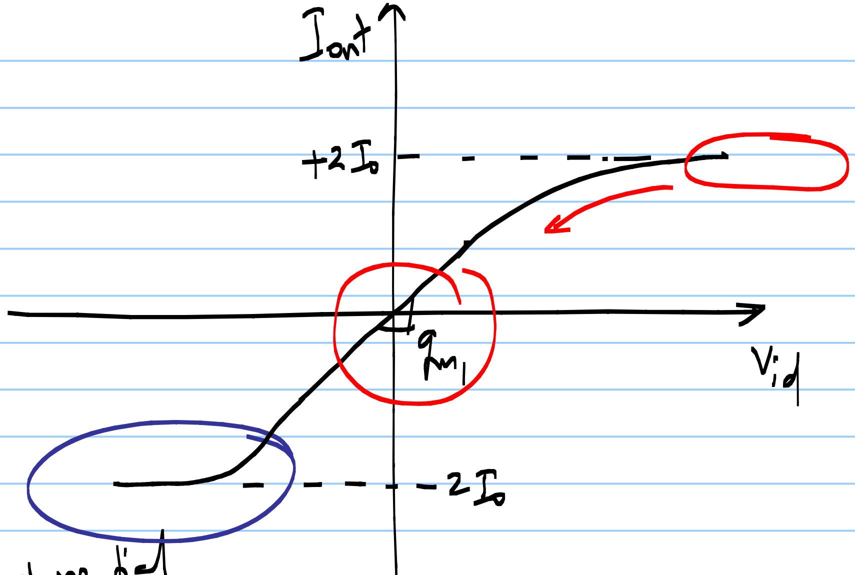
$\rightarrow g_m \cdot \Delta V_2 \sim g_{m_1} \cdot \Delta V_3$ etc.

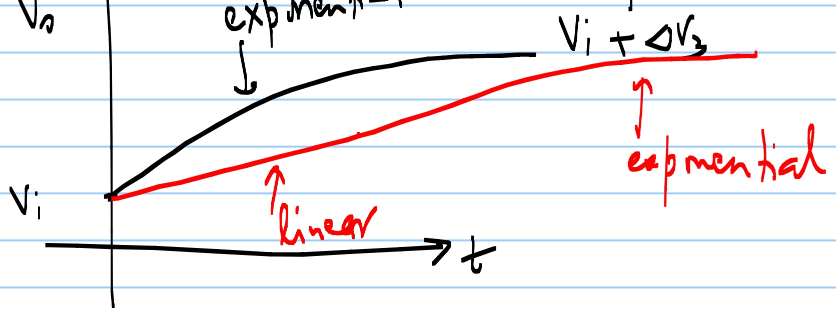






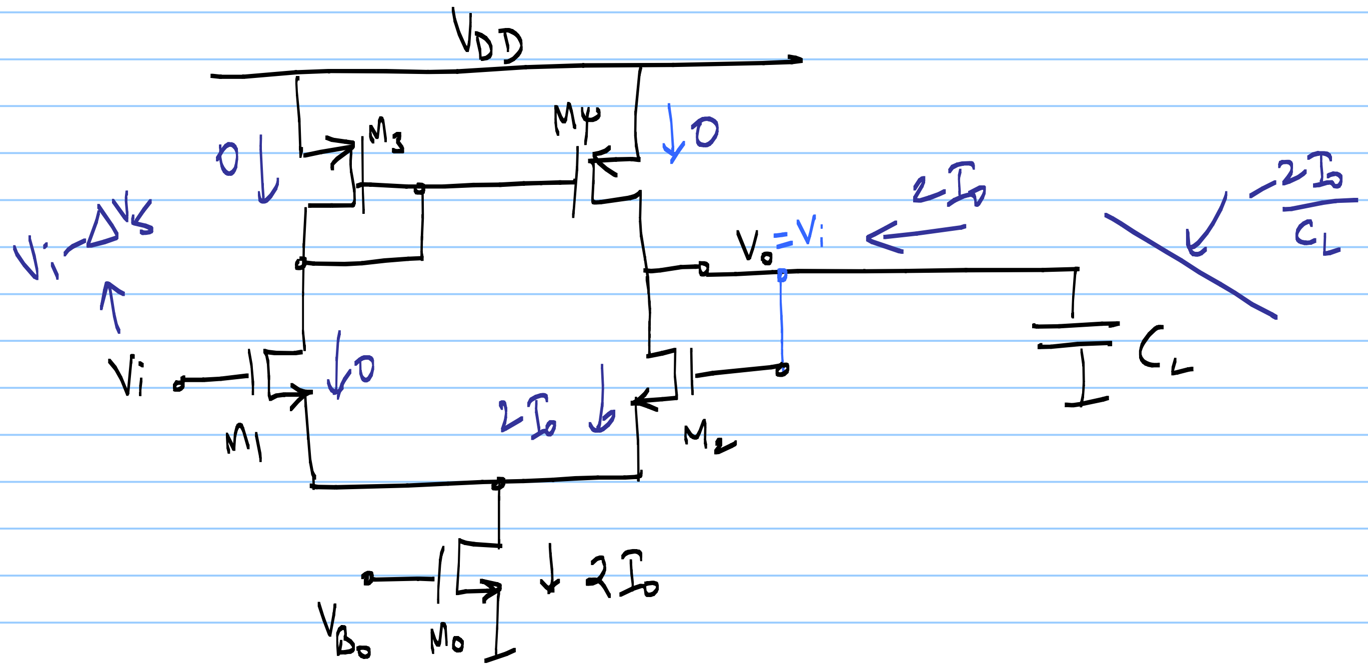




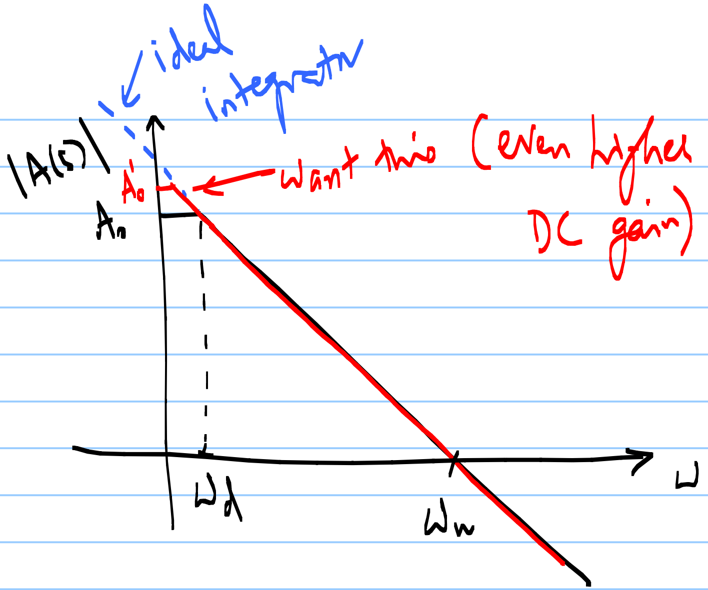


$\Delta V_4 > \Delta V_3$

$\Delta V_5 > \Delta V_4$

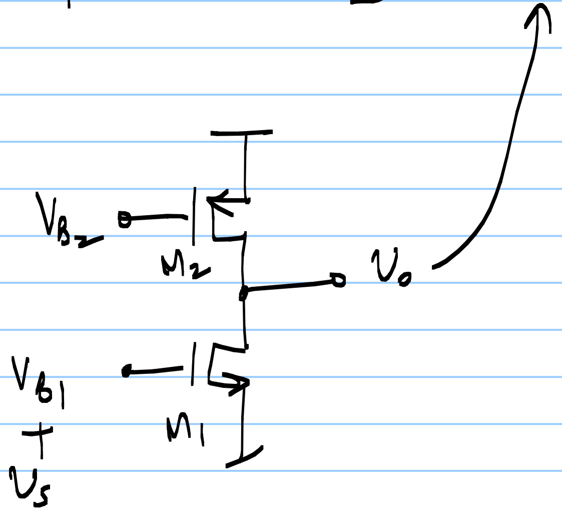


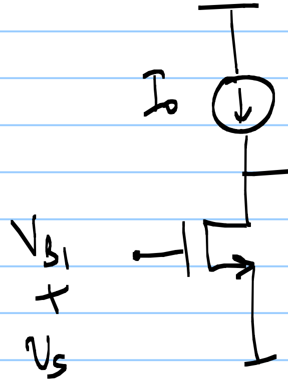
VDD
M3
M4
0
0
Vi-ΔVs
Vi
M1
0
2I0
M2
Vo=Vi
2I0
-2I0/CL
CL
VB0
M0
2I0



DC gain of one-stage opamp $= g_{m_1}(r_{ds_2} \| r_{ds_4})$

$$\approx \frac{1}{2}(g_m r_{ds})$$

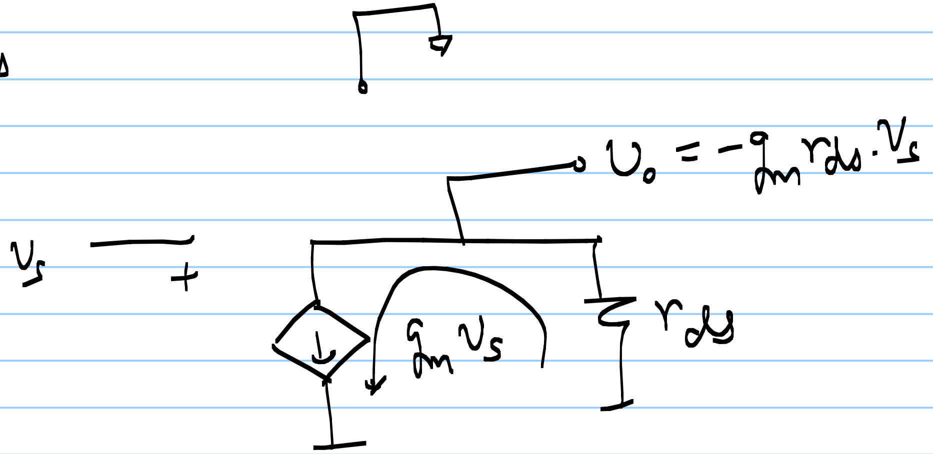


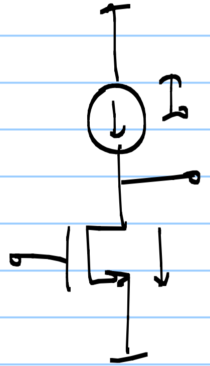


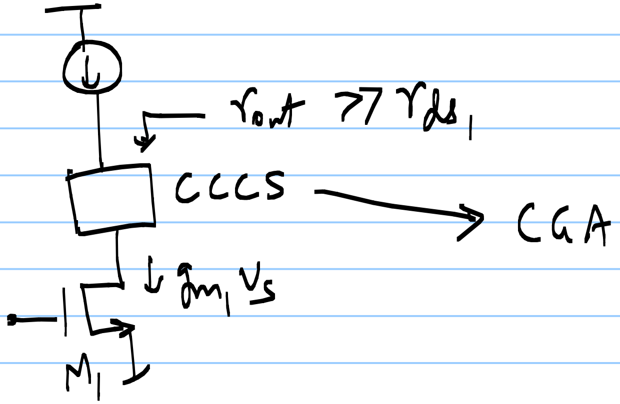
$\frac{v_o}{v_s} = -g_m r_{ds}$

→ VCCS → $G_m = g_{m1}$

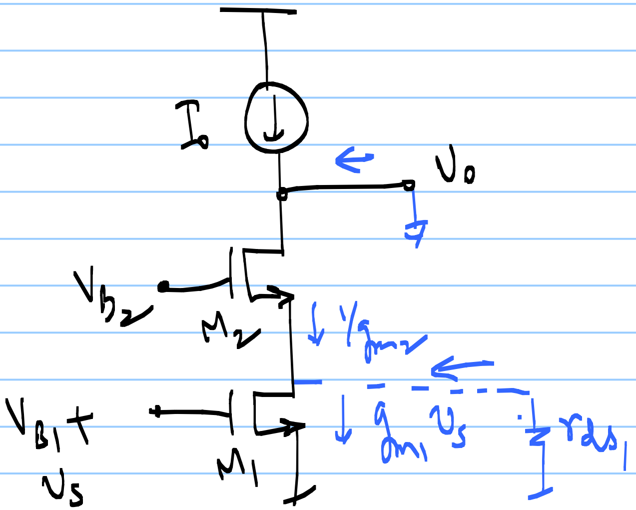
→ $r_{out} = r_{ds}$







Cascode Amplifier



$\equiv$

better VCCS

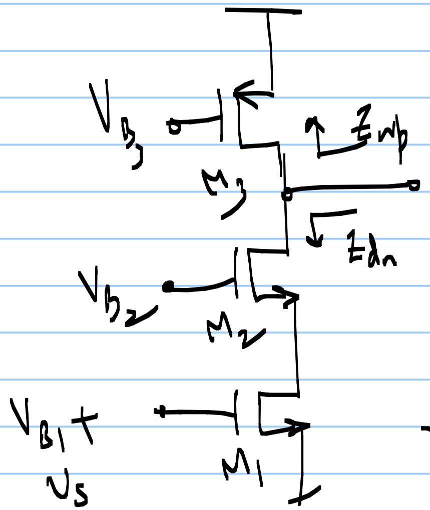
$G_m = -g_{m1}$

$r_{out} \approx (g_{m2} r_{ds2}) \cdot r_{ds1}$

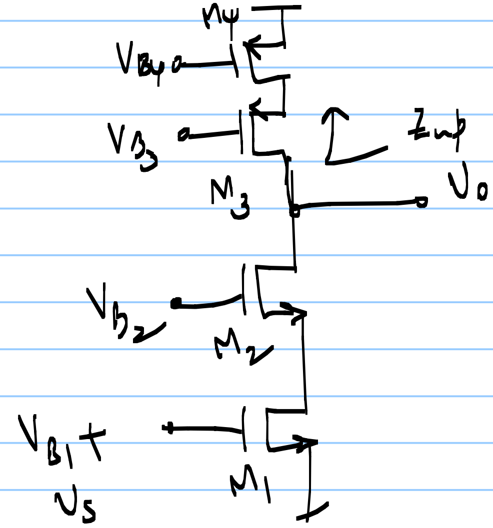
voltage gain $= G_m \cdot r_{out}$

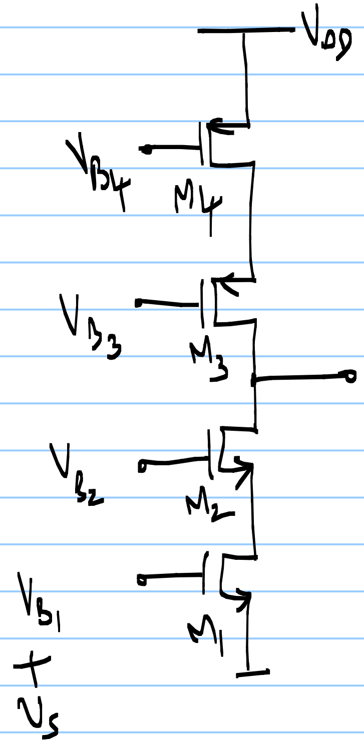
$= -g_{m1} g_{m2} r_{ds1} r_{ds2}$

same gain as 2-stage amplifier



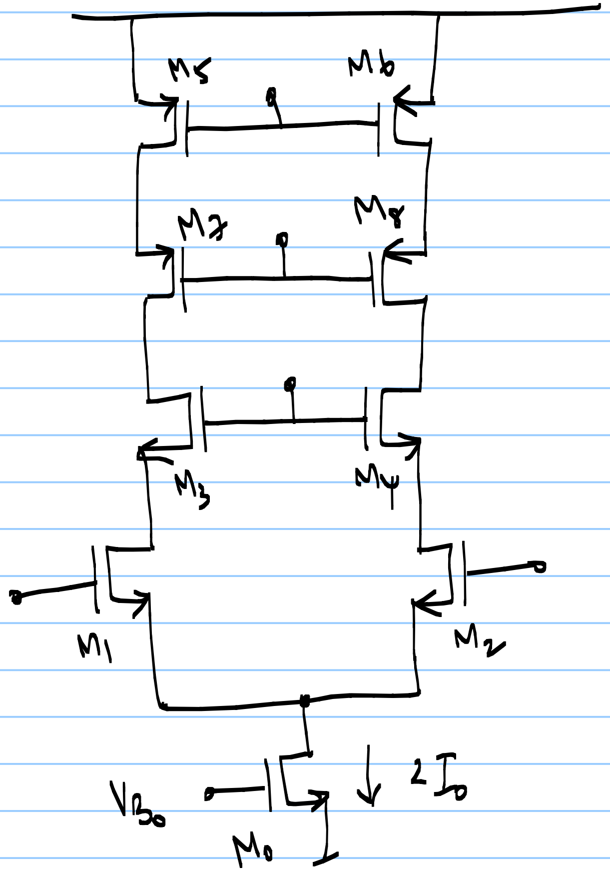
$$\frac{v_o}{v_s} = -(g_{m_1})\Big[\underbrace{(g_{m_2} r_{ds_2}) r_{ds_1}}_{(Z_{d_n})} \,\Big|\Big|\, \underbrace{r_{ds_3}}_{(Z_{up})}\Big] \approx -g_{m_1} r_{ds_3}$$



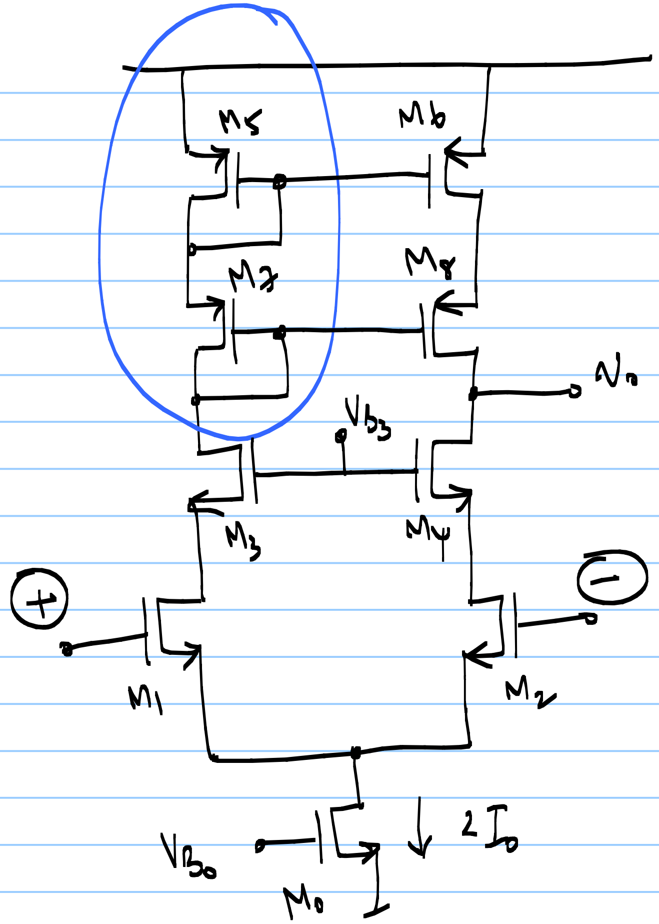


$$\frac{v_o}{v_s} = -\,g_{m_1}\left[(g_{m_3} r_{ds_3}) r_{ds_4} \,\|\, (g_{m_2} r_{ds_2}) \cdot r_{ds_1}\right]$$

$$\simeq \frac{1}{2}\left(g_m r_{ds}\right)^2$$

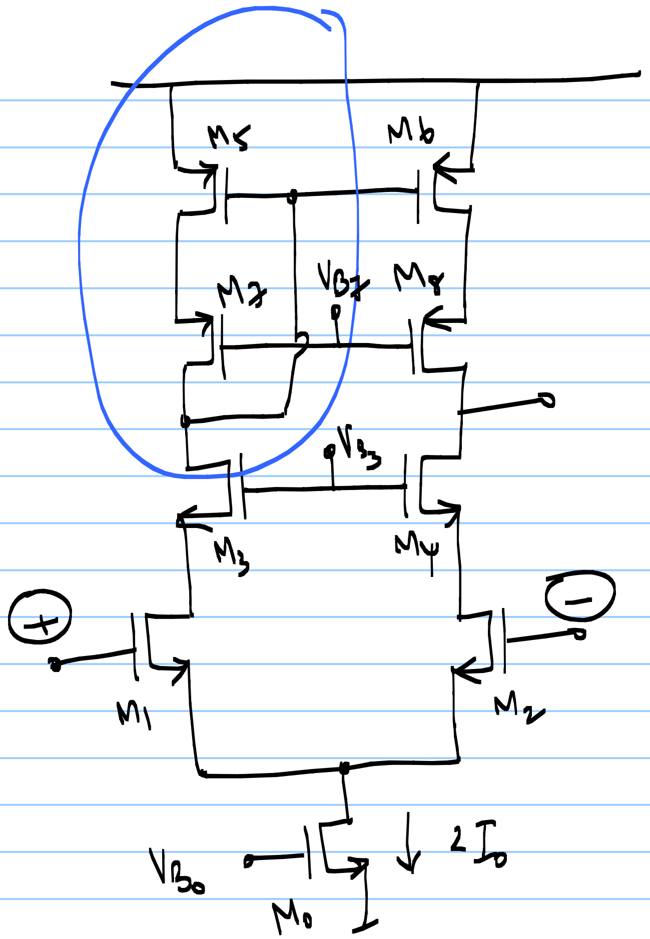


→

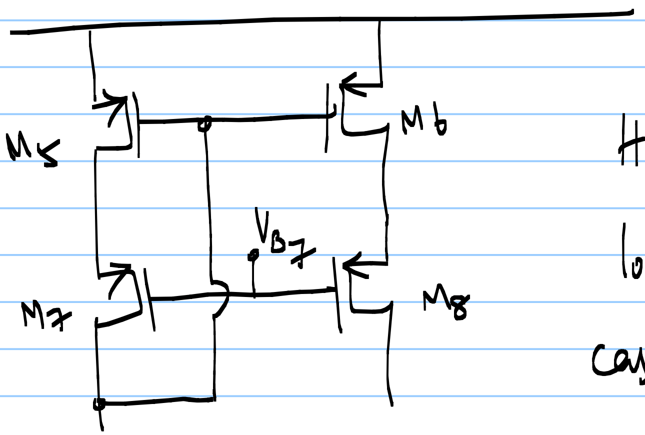


"Telescopic" Opamp

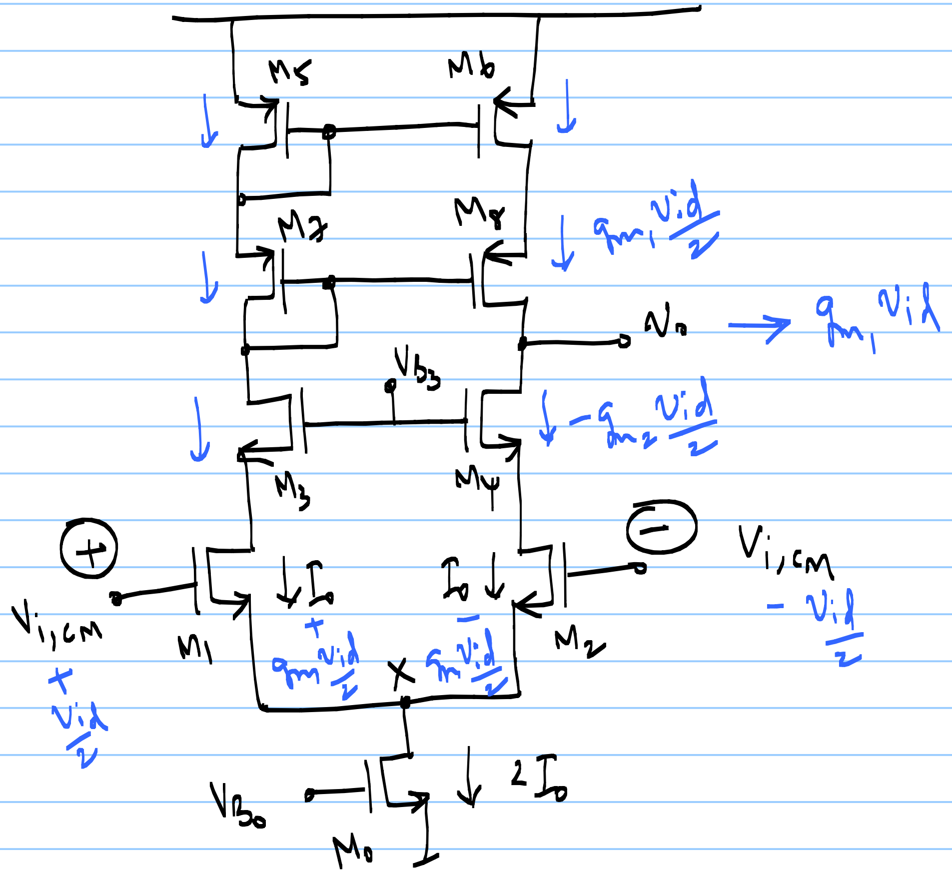
Also a "one-stage" opamp (technically)

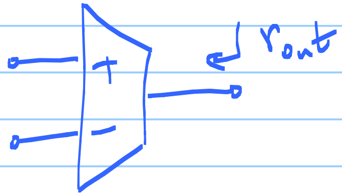


Telescopic opamp



High-swing or low-voltage cascode current mirror





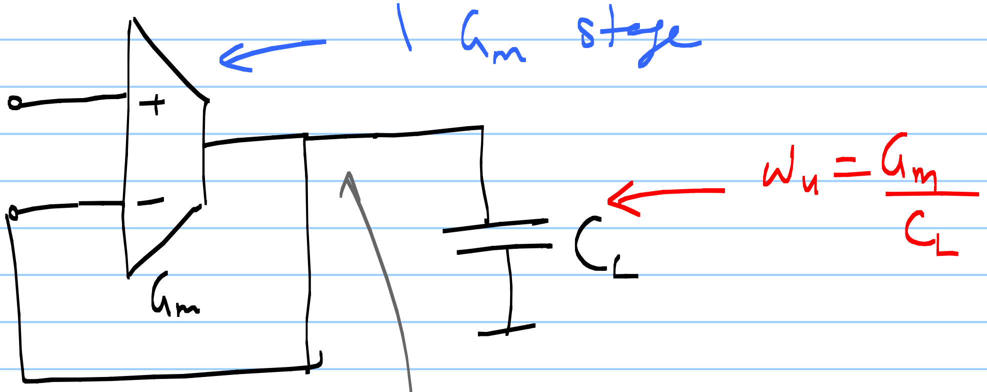
$G_M = g_{m1}$

$$r_{out} = \left[(g_{m4} r_{ds4}) r_{ds2} \;\Big\|\; (g_{m8} r_{ds8}) \cdot r_{ds1} \right]$$

$$A_0' = G_m \cdot r_{out} \gg A_0 \text{ for 5-T}$$

one-stage opamp

One-stage opamp ?



ω_d' @ output node

$\omega_d' = \frac{1}{r_{out} C_L}$ has decreased

increased

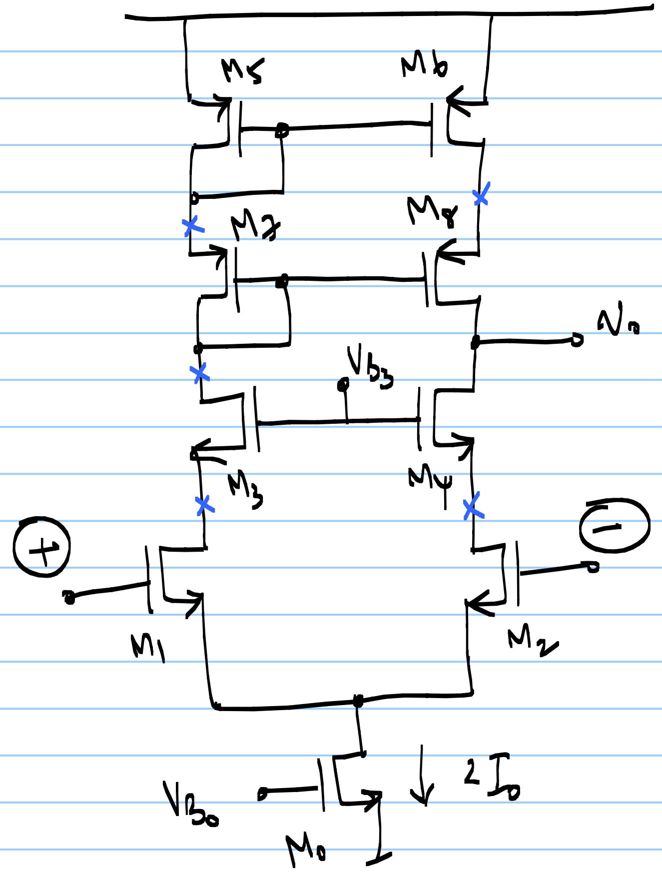
Telescopic Opamp Datasheet

1) DC gain $= g_{m_1} r_{out}$

$$r_{out} = \left[(g_{m_4} r_{ds_4}) r_{ds_2} \,\|\, (g_{m_8} r_{ds_8}) \cdot r_{ds_6} \right]$$

2) $G_m = g_{m_1}$

3) $\omega_u = \frac{G_m}{C_L} = \frac{g_{m_1}}{C_L}$ (UGF) $\longleftrightarrow$ $\omega_d = \frac{1}{\underset{\underset{r_{out}}{\|}}{R_o} C_L} = \frac{G_o}{C_L}$ (dominant pole) (LHP)

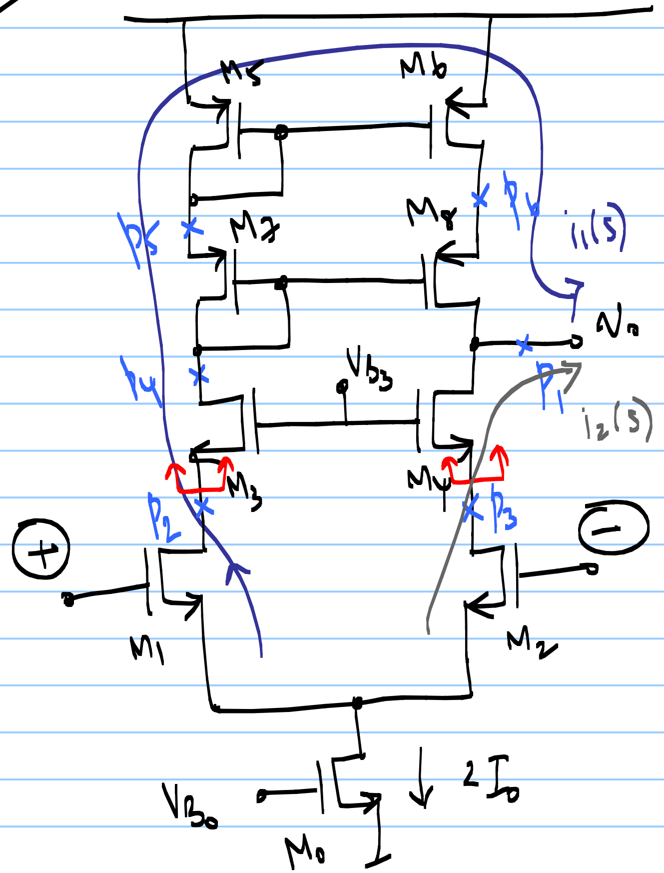


guess : 5 ND poles

how many zeroes ?

19/10/2023

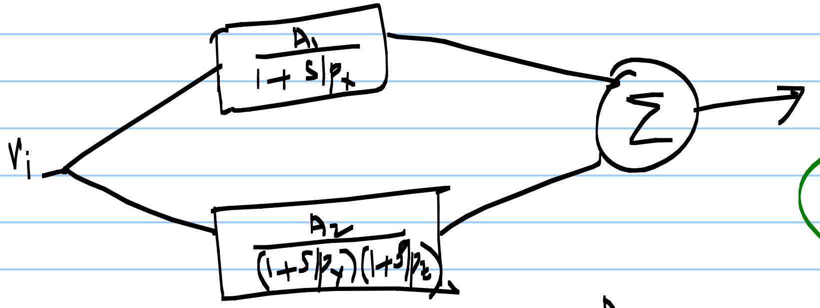
Lec 14



$$\frac{v_o}{v_{id}}(s) = \frac{N(s)}{D(s)}$$

Is $p_2 = p_3$?

$p_2 \neq p_3$ most likely



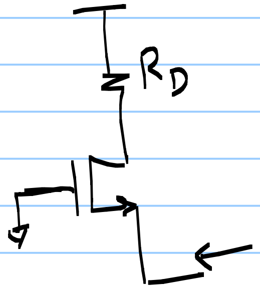
only 2 poles

$p_x = p_y$ or $p_x = p_z$

or neither

3 poles

$$\frac{v_o}{v_i}(s) = \frac{A_1}{1+\frac{s}{p_x}} + \frac{A_2}{\left(1+\frac{s}{p_y}\right)\left(1+\frac{s}{p_z}\right)}$$



$Z_{in} = f(g_m, r_{ds}, R_D)$ ← HW

$Z_{in} \simeq \frac{1}{g_m}$ if $R_D << r_{ds}$

$\boxed{V_{out}(s)} = i_{out}(s) \cdot Z_{out}(s)$

$= \underbrace{\left[i_1(s) + i_2(s) \right]} Z_{out}(s)$

$i_1(s) \rightarrow p_2, p_4, p_5, p_6, p_1$

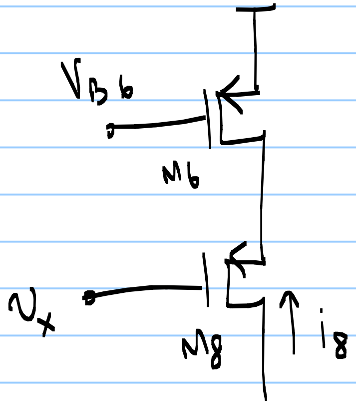
$i_2(s) \rightarrow p_3, p_1$

$\frac{V_o}{V_{id}}(s) = \frac{N(s)}{D(s)}$ — 4 zeroes

6 poles

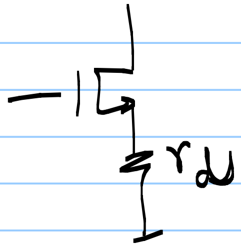
p_1 – Dominant pole

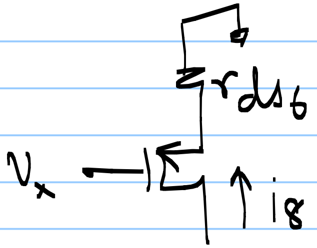
p_2 – p_6 – 5 ND poles



$$\frac{i_8}{v_x} = \frac{g_m}{1 + g_m r_{ds_6}} \simeq \frac{1}{\frac{1}{g_m} + r_{ds_6}}$$

— very small





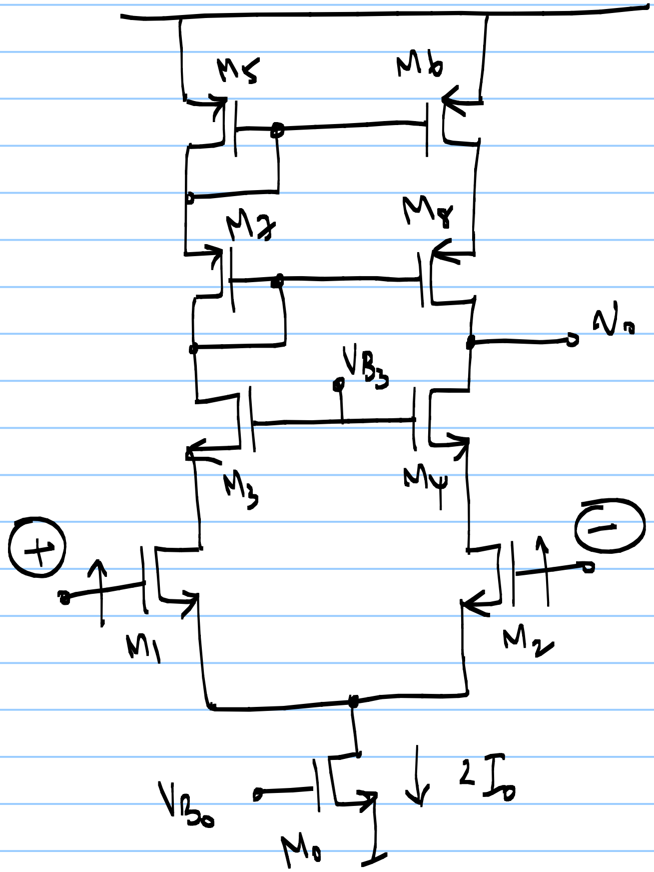
4) ND poles & Zeroes

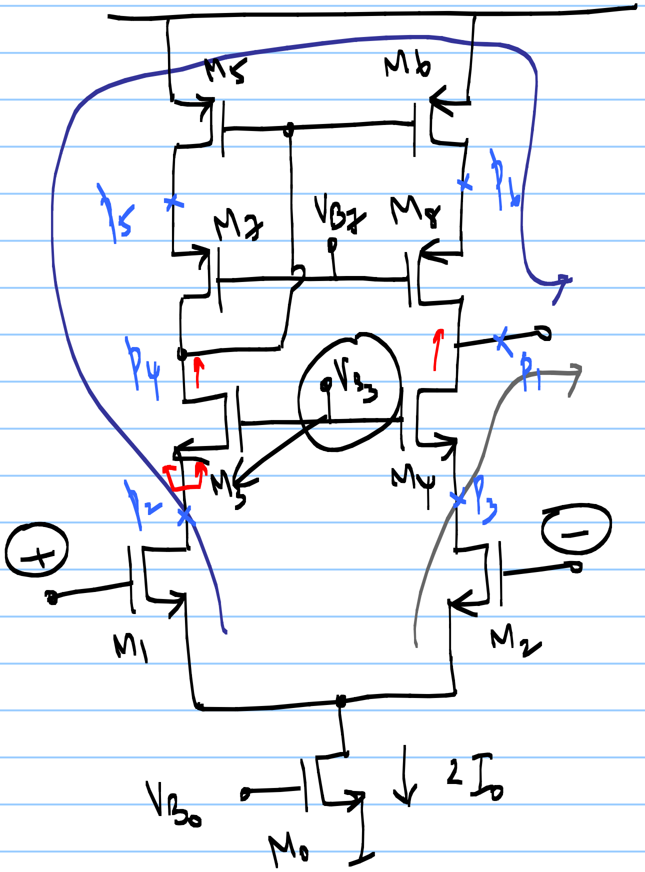
↓ 5 ND poles

↘ 4 ND zeroes

5) $$\mathrm{ICMR} = \left\{ V_{GS_1}\Big|_{I_0} + V_{DSat_0}\Big|_{2I_0} ,\; V_{T_1} + V_{B_3} - V_{GS_3}\Big|_{I_0} \right\}$$

6) $$\mathrm{OCMR} = \left\{ V_{B_3} - V_{T_4} ,\; V_{DD} - V_{SG_5}\Big|_{I_0} - V_{SG_7}\Big|_{I_0} + V_{T_8} \right\}$$





High swing Telescopic opamp

1) DC gain = same

2) G_m = same

3) W_n = same , W_d = same

4) ND poles = 5 LHP poles

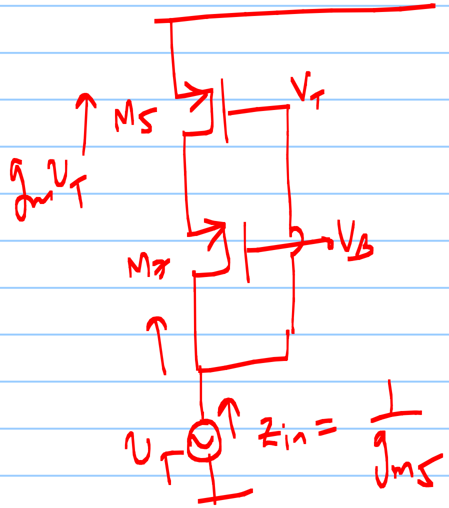
ND zeroes = 4

5) ICMR = same

6) OCMR = $\{V_{B_3} - V_{T4} , V_{B7} + V_{T8}\}$

$V_{B7} > V_{DD} - V_{SG_5} - V_{SG_7}$

(H.S.) (L.S.)



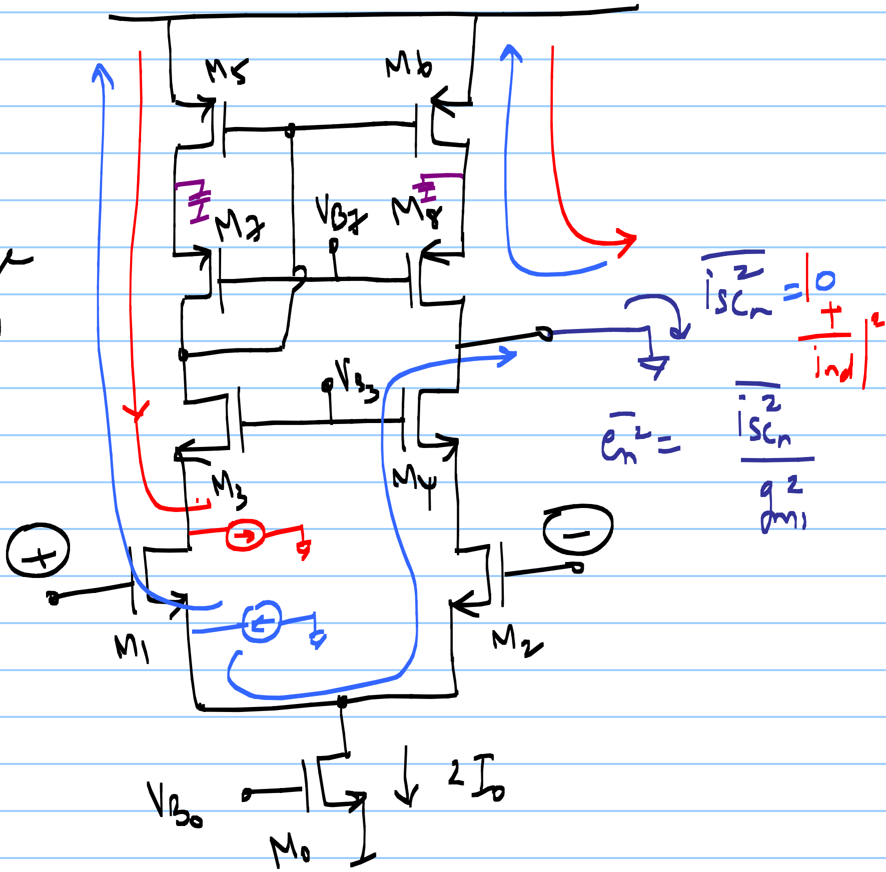
$p_2 \neq p_3$

7) Noise $\frac{\overline{e_n^2}}{\Delta f} = \frac{16kT}{3\, g_{m_1}} + \frac{16kT}{3\, g_{m_1}^2} \cdot g_{m5}$

↳ same as 1-stage opamp

- No noise from cascode devices M_3, M_4, M_7, M_8 @ low frequencies

- Noise of Telescopic opamp is worse than one-stage opamp @ high frequencies

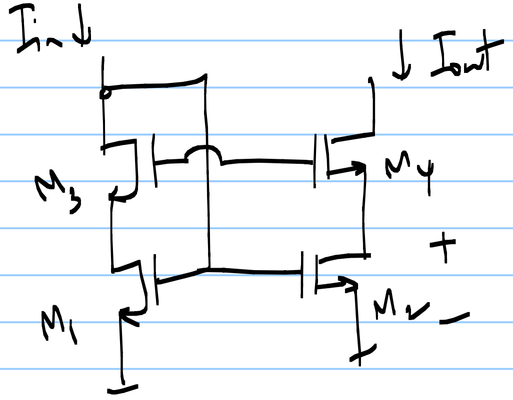


$\overline{i_{sc_n}^2} = |0 + i_{nd}|^2$

$\overline{e_n^2} = \frac{\overline{i_{sc_n}^2}}{g_{m_1}^2}$

8) **Mismatch** : $\sigma_{os}^2 = \sigma_{V_{T_{1,2}}}^2 + \left(\frac{g_{m5}}{g_{m1}}\right)^2 \sigma_{V_{T_{5-6}}}^2$

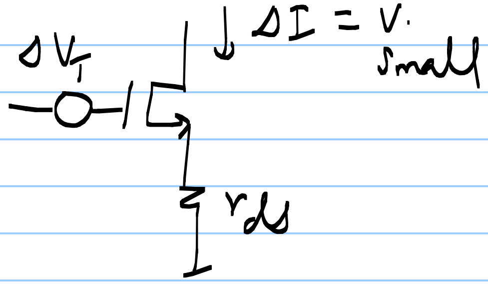
$M_{3,4}$ & M_{7-8} V_T -mismatches will not contribute to σ_{os}^2



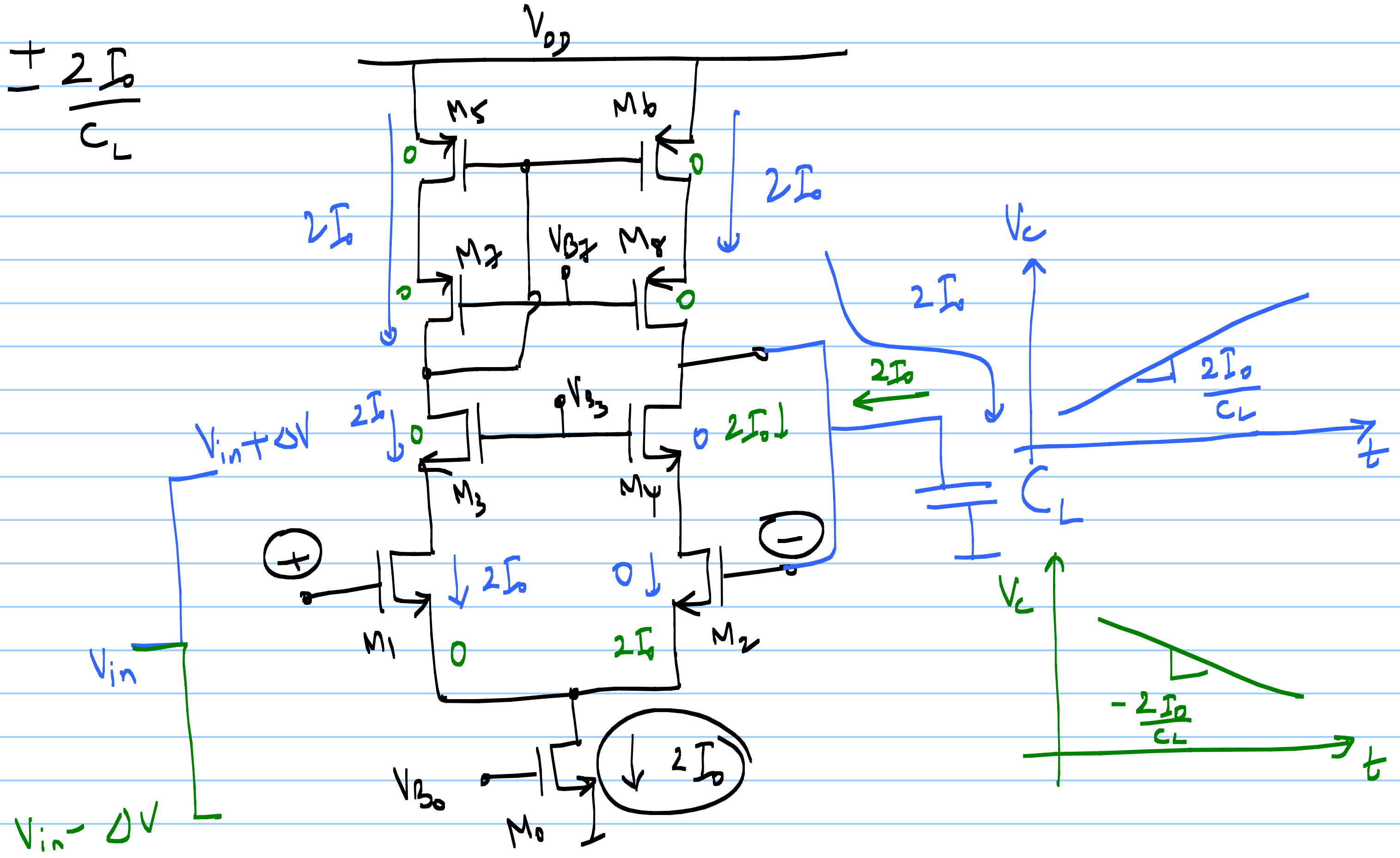
$V_{T_3} = V_T$, $V_{T_4} = V_T + \Delta V_T$

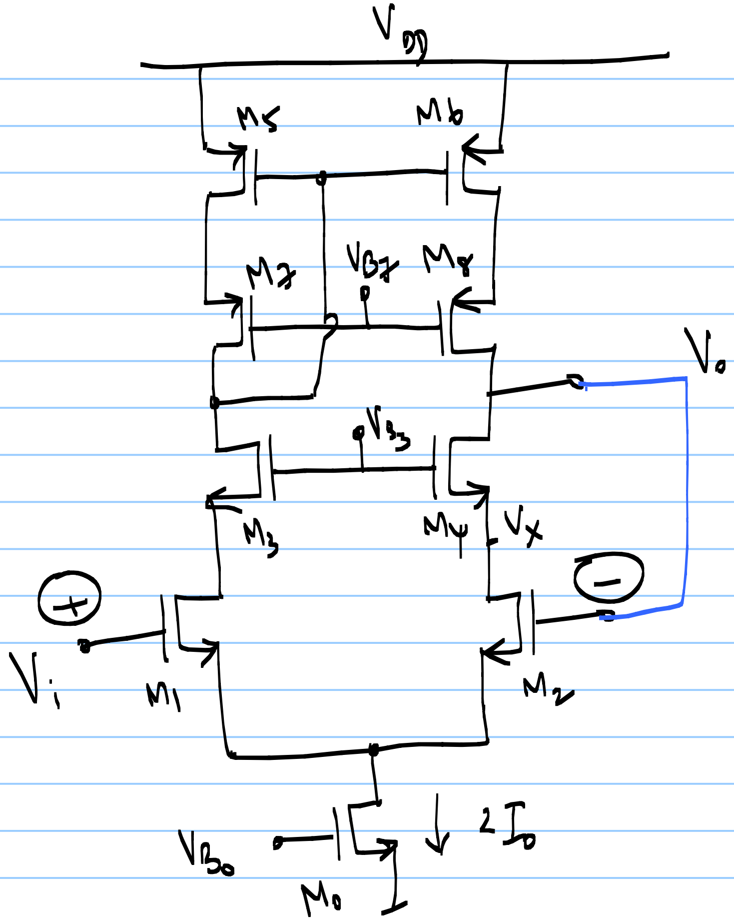
$\Delta I_{out} = ?$

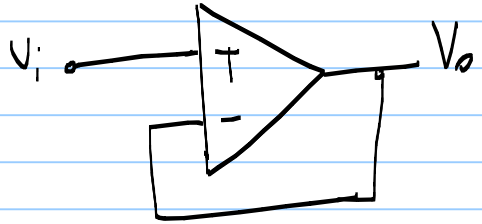
$V_{T_1} = V_{T_2} = V_T$



9) Slew Rate $= \pm \dfrac{2I_0}{C_L}$







$V_{o_{min}}$ = larger of → $\{V_{GS_2} + V_{Dsat_0}\}$

and

$\{V_{B_3} - V_{T4}\}$ ✓

$V_{o_{max}}$ = smaller of → $\{V_{B_3} - V_{GS_4} + V_{T_2}\}$ ✓

and

$\{V_{B7} + V_{T8}\}$

$V_{0,min} = V_{B_3} - V_{T4}$

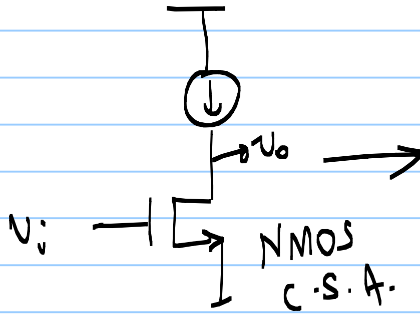
$V_{0,max} = V_{B_3} - V_{GS4} + V_{T_2} = V_{B_3} - V_{T4} - V_{Dsat_4} + V_{T2}$

$V_{0_{max}} - V_{0_{min}} = V_{T4} - V_{Dsat_4}$ very small !

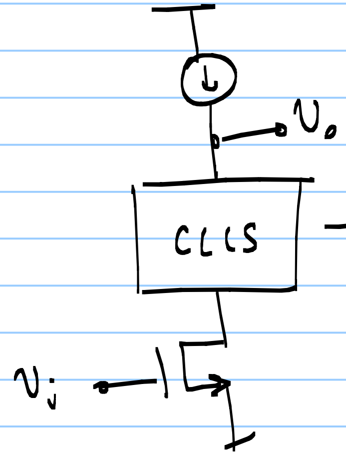
26/10/23

Lec 15

NMOS



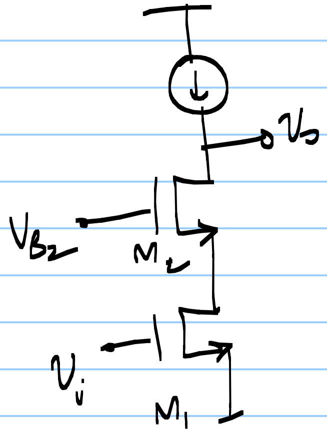
→



→ PMOS CGA

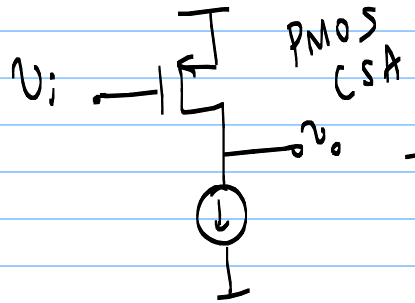
→ NMOS CGA

→

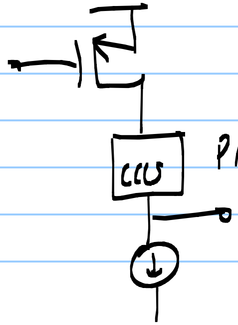


→ Telescopic opamp (NMOS)

PMOS

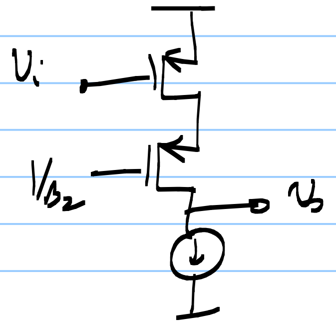


→



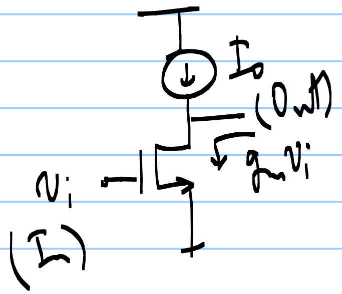
PMOS C.C.A

→

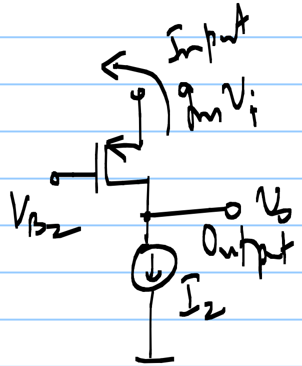


→ Telescopic opamp with PMOS input pair

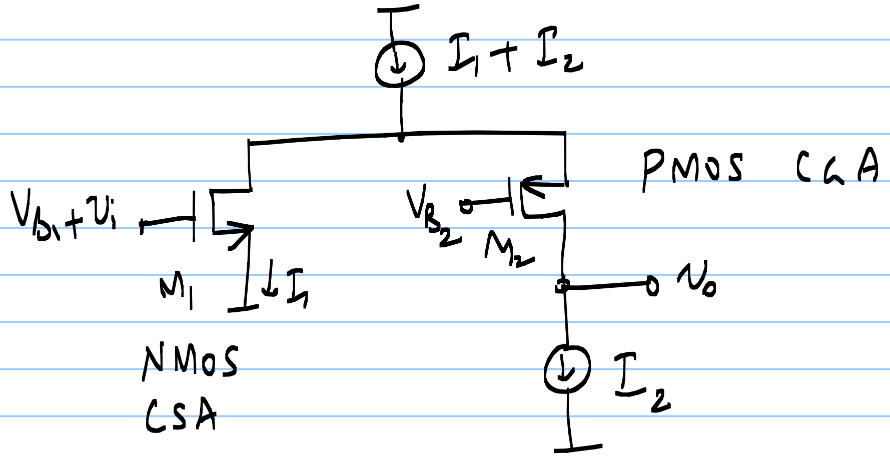
NMOS CSA + PMOS C.G.A ?



+

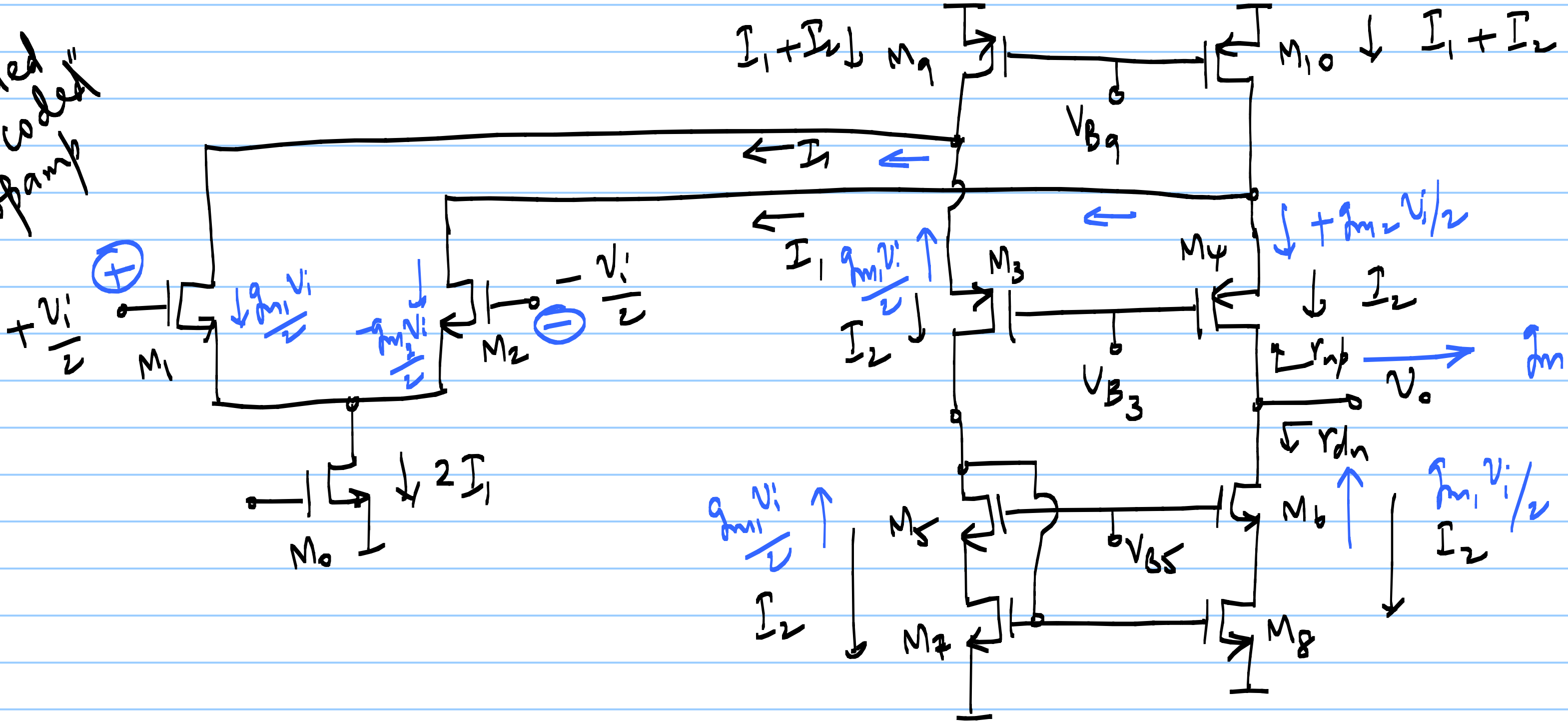


⇒

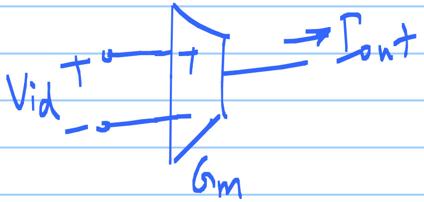


"Folded Cascode" Amplifier

"Folded Cascoded" opamp



$g_{m1}v_i$



$$G_m = \frac{\partial I_{out}}{\partial V_{id}}$$

$$= g_{m1}$$

for small signals

1) DC gain = $g_{m_1} r_{out}$ ✓ $\qquad r_{out} = \left(r_{dn} \,||\, r_{up}\right)$ = slightly smaller than that of Telescopic opamp

$r_{dn} = g_{m_6} r_{ds_6} \cdot r_{ds_8}$

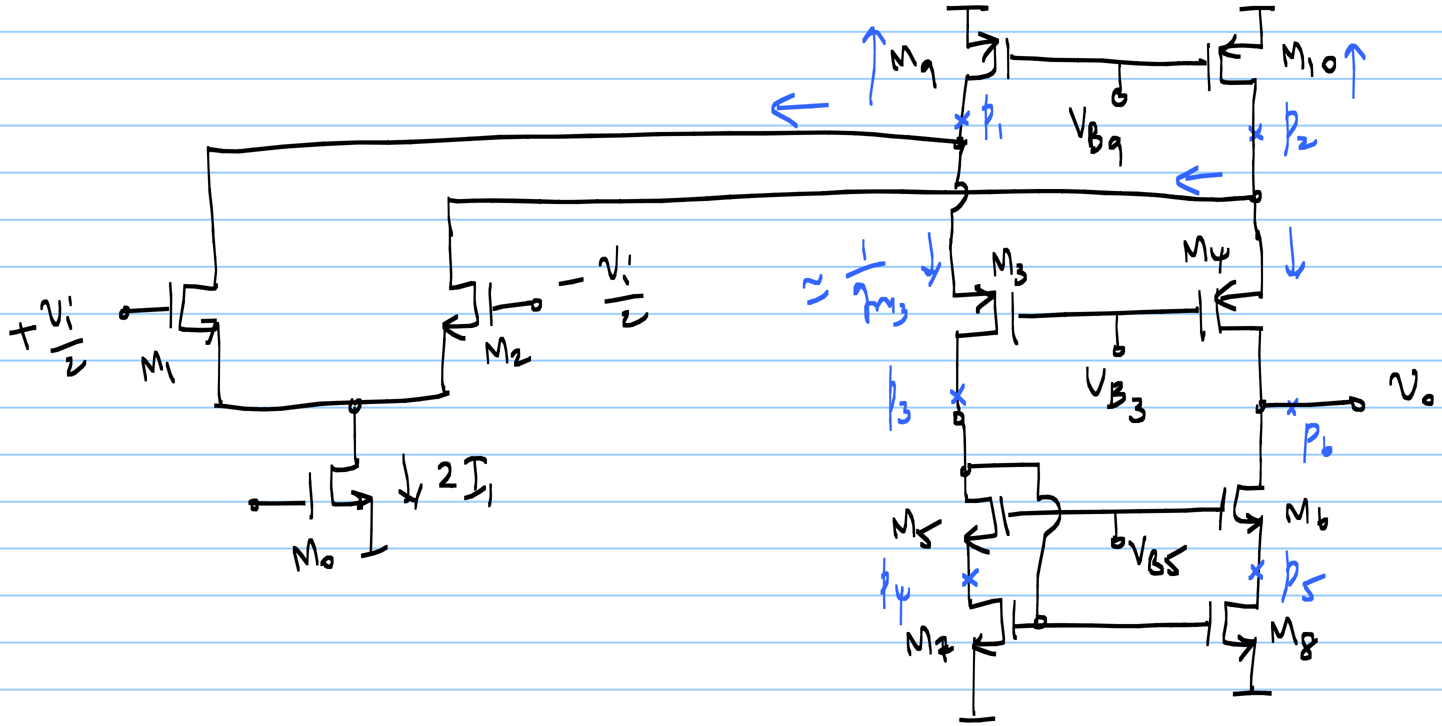
$r_{up} = g_{m_4} r_{ds_4} \cdot \left(r_{ds_2} \,||\, r_{ds_{10}}\right)$

2) $G_m = g_{m_1}$

3) $\omega_u = \frac{G_m}{C_L} = \frac{g_{m_1}}{C_L}$; $\qquad \omega_d$ ✓ $= \frac{g_{out}}{C_L} = \frac{1}{r_{out} C_L} = p_6$

dominant pole

4) ND poles & zeroes : $\underbrace{5 \text{ ND poles}}_{p_1 - p_5}$ & 4 zeroes


M9
M10
p1
p2
VB9
M3
M4
≈ 1/gm3
p3
VB3
vo
p6
M5
M6
VB5
p4
p5
M7
M8
+ vi/2
M1
M2
- vi/2
2I1
M0

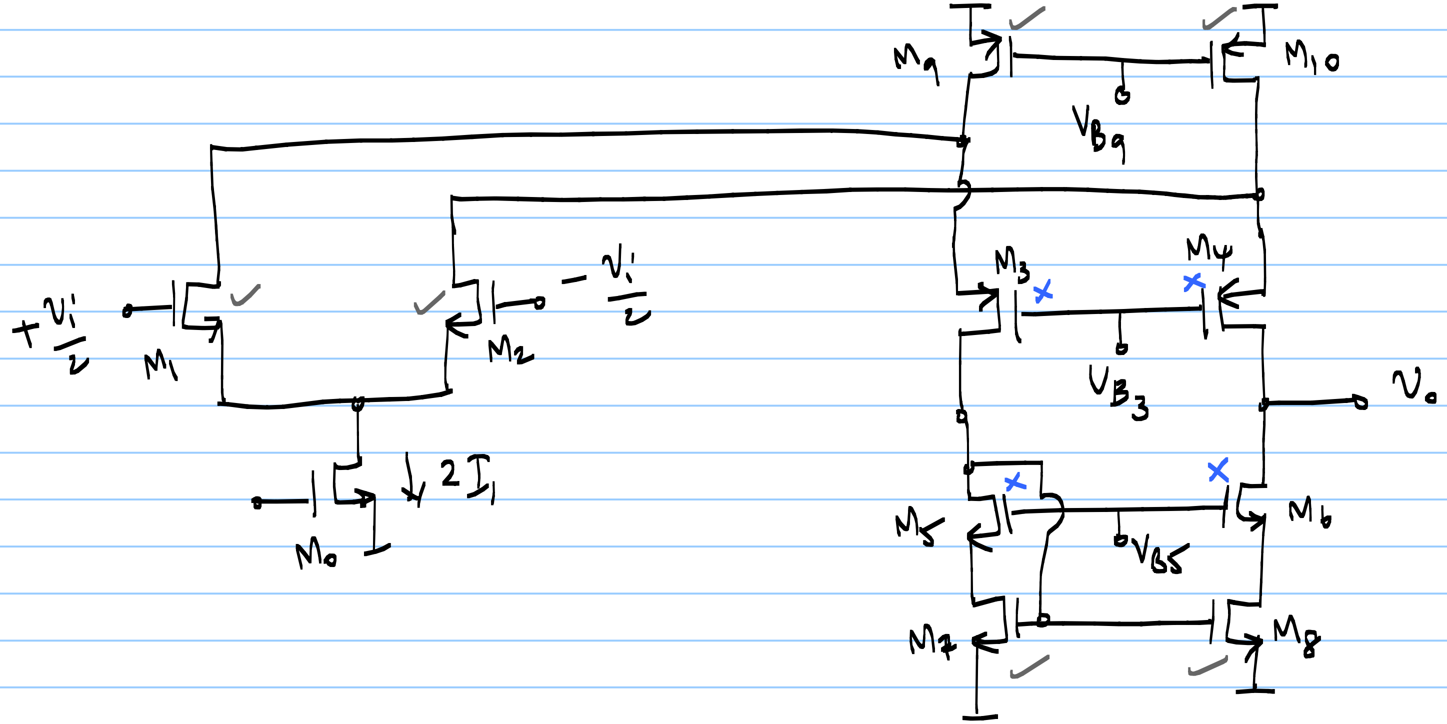
5) $$ICMR = \left\{ V_{Dsat_0}\Big|_{2I_1} + V_{GS_1}\Big|_{I_1} \; , \; V_{B_3} + V_{SG_3}\Big|_{I_2} + V_{T_1} \right\}$$

6) $$OCMR = \left\{ V_{B5} - V_{T6} \; , \; V_{B_3} + V_{T4} \right\}$$

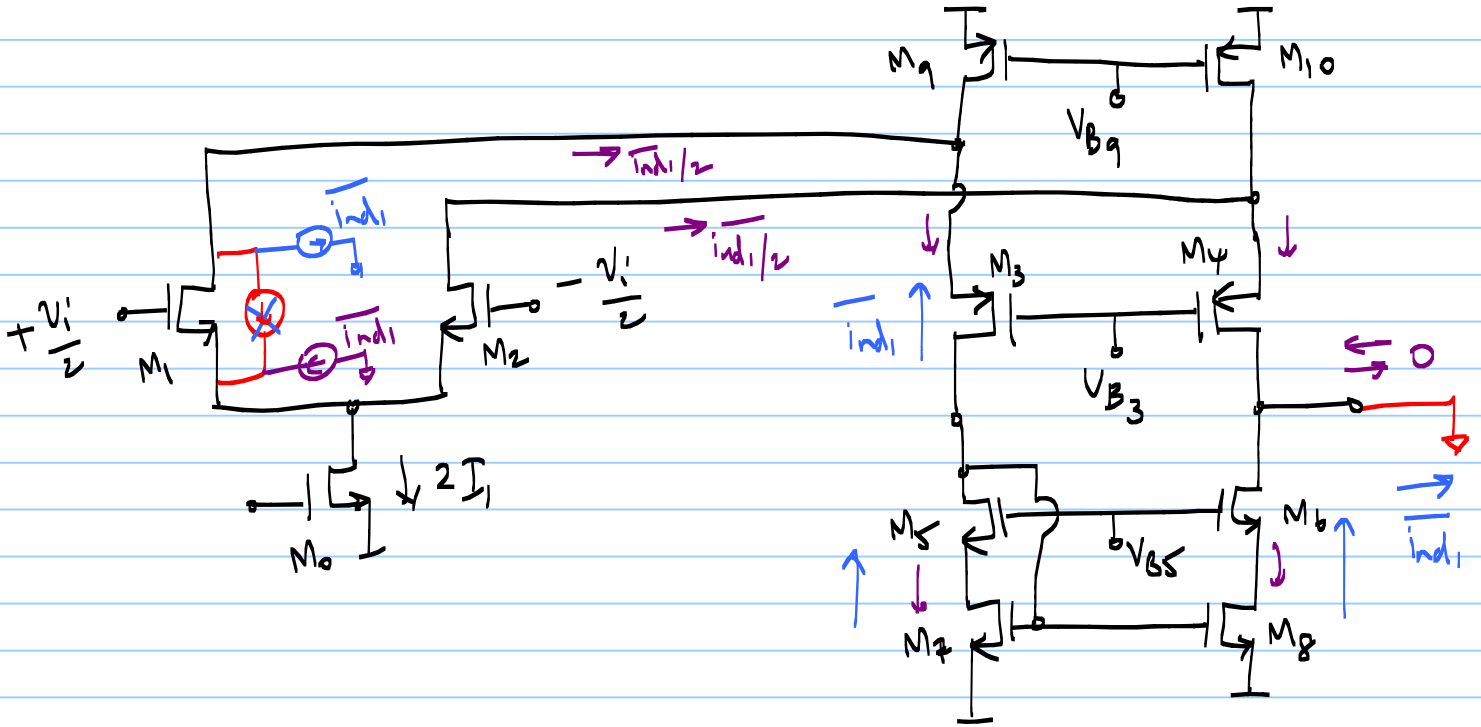
7) Noise $$\frac{\overline{e_n^2}}{\Delta f} = \frac{1}{g_{m_1}^2}\left[\overline{i_{nd_1}^2} + \overline{i_{nd_2}^2} + \overline{i_{nd_7}^2} + \overline{i_{nd_8}^2} + \overline{i_{nd_9}^2} + \overline{i_{nd_{10}}^2} \right]$$

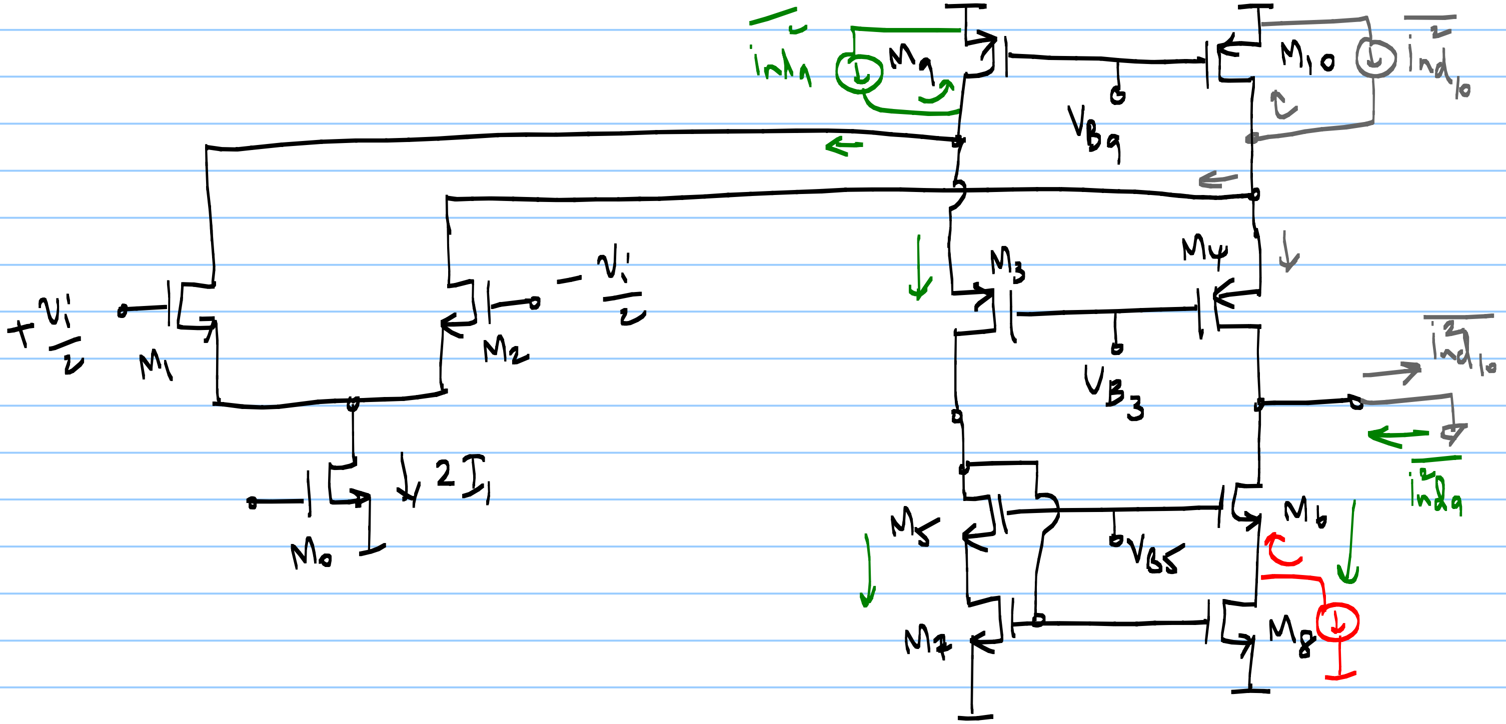
$$= \frac{16kT}{3g_{m_1}} + \frac{16kT\, g_{m_7}}{3g_{m_1}^2} + \frac{16kT\, g_{m_9}}{3g_{m_1}^2} \Bigg\}$$

slightly worse than that of telescopic or one-stage opamp



× noise of cascode devices do not contribute to $\overline{e_n}$ at low freq.


M9
M10
V_{B9}
$\overline{i_{nd1}}/2$
$\overline{i_{nd1}}/2$
$\overline{i_{nd1}}$
$\overline{i_{nd1}}$
$+\frac{v_i}{2}$
$-\frac{v_i}{2}$
M1
M2
M3
M4
V_{B3}
$\overline{i_{nd1}}$
0
M5
M6
V_{B5}
$\overline{i_{nd1}}$
M7
M8
$2I_1$
M0



$+\frac{v_i}{2}$
M_1
M_2
$-\frac{v_i}{2}$
M_0
$2I_1$
$\overline{i_{nd_9}^2}$
M_9
M_{10}
$\overline{i_{nd_{10}}^2}$
V_{B9}
M_3
M_4
V_{B3}
$\overline{i_{nd_{10}}^2}$
$\overline{i_{nd_9}^2}$
M_5
M_6
V_{B5}
M_7
M_8

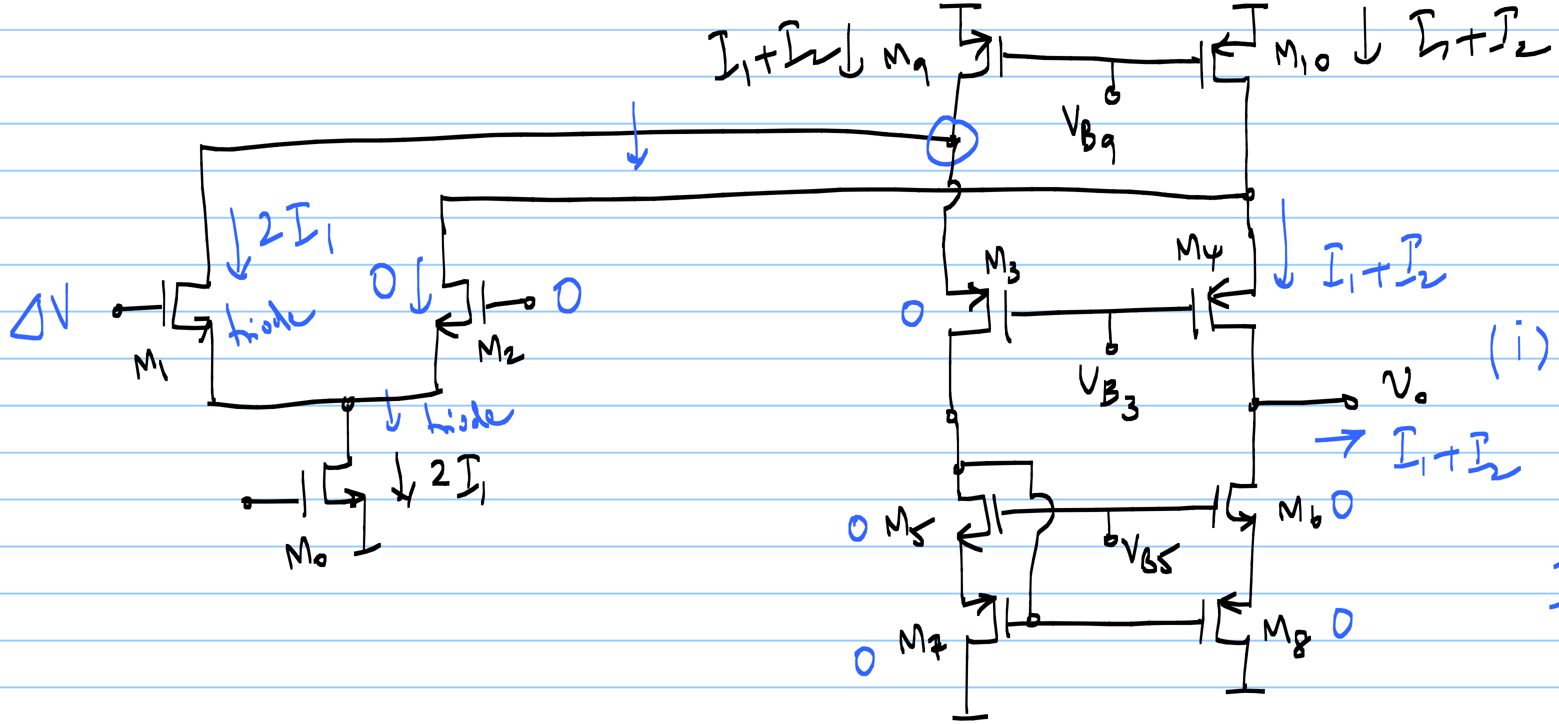
8) <u>Mismatch</u> $\sigma_{os}^2 = \sigma_{V_{T_{1,2}}}^2 + \left(\frac{g_{m9}}{g_{m1}}\right)^2 \cdot \sigma_{V_{T_{9,10}}}^2 + \left(\frac{g_{m7}}{g_{m1}}\right)^2 \cdot \sigma_{V_{T_{7,8}}}^2$

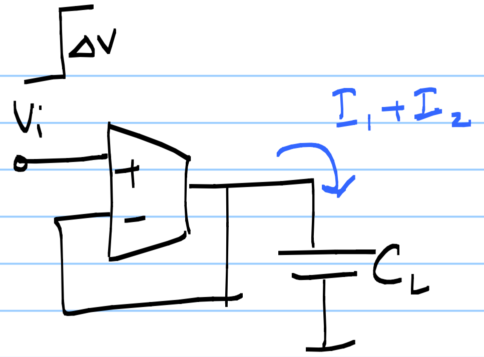
9) <u>Slew Rate</u> – 2 cases

$$\frac{I_1 + I_2}{C_L} \quad \text{or} \quad \frac{2I_1}{C_L}$$

HW – ΔV

(–ve SR)

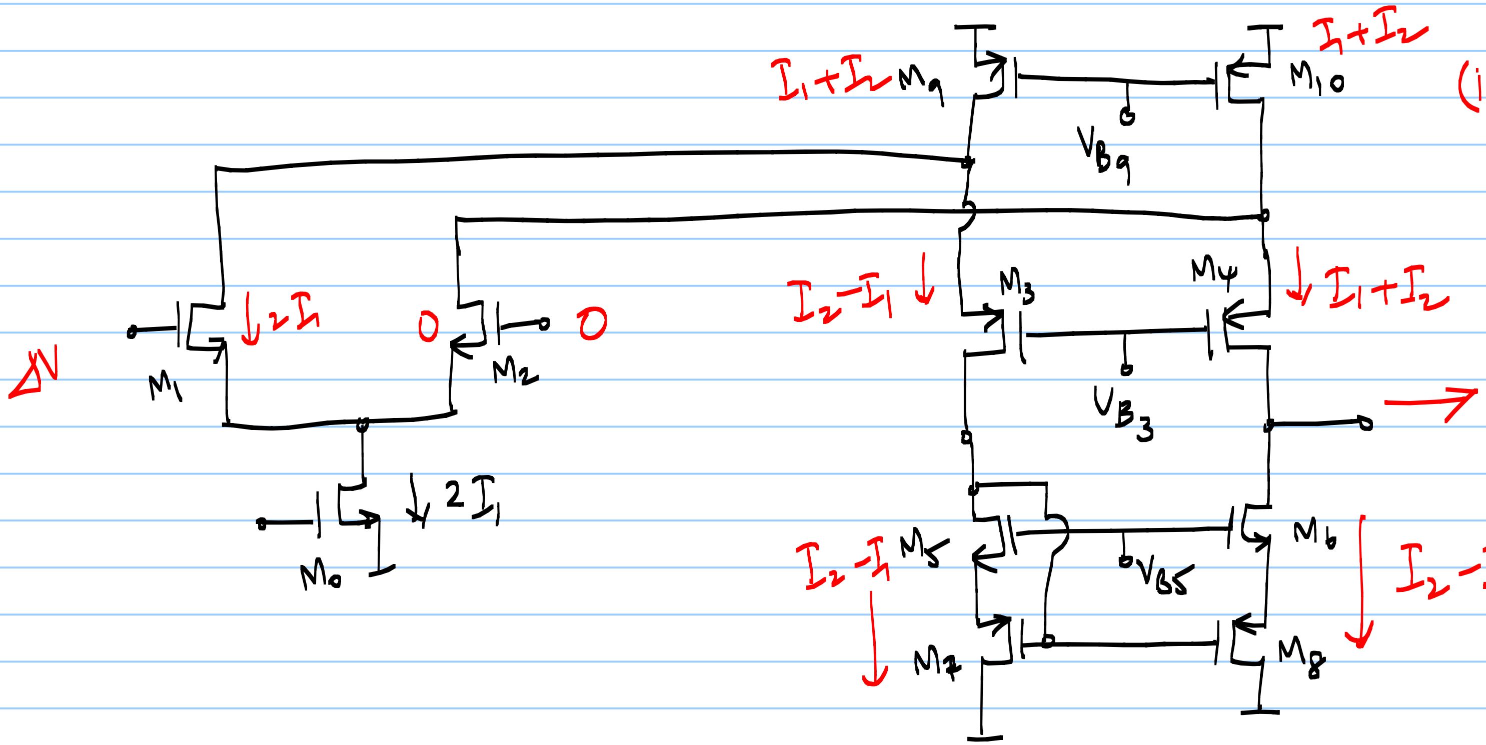




(i) $2I_1 > I_1 + I_2$

$I_1 > I_2$

$I_{out} = I_1 + I_2$



(ii) $2I_1 < I_1 + I_2$

$I_1 < I_2$

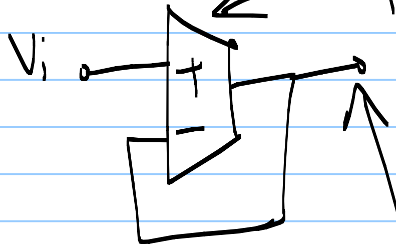
$I_b = I_1 + I_2 - 2I_1$

$= I_2 - I_1$

$I_{out} = (I_1 + I_2) - (I_2 - I_1)$

$= 2I_1$

HW



folded cascode

swing limits in unity gain f/b.

⇒ Confirm that the problem in Telescopic opamp does not exit here

HW PMOS input pair versions of all opamps

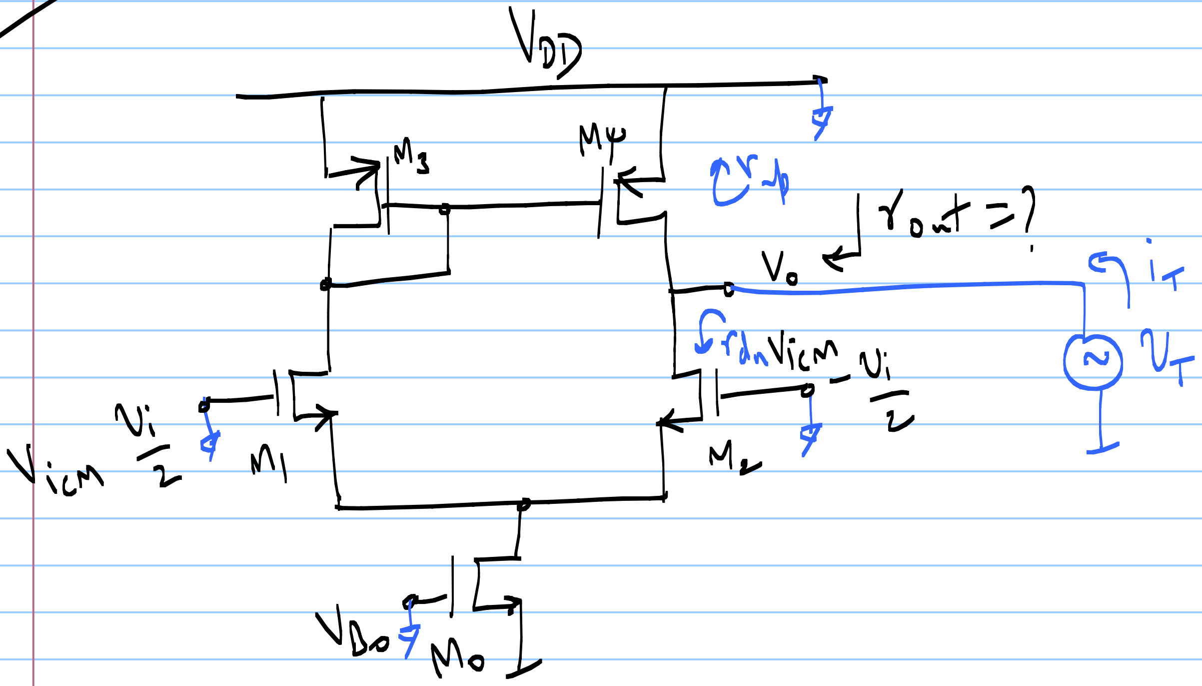
↳ Only difference is in ICMR & OCMR

Quiz 1 → Saturday Nov 4th, 2023

9- 10am

27/10/23

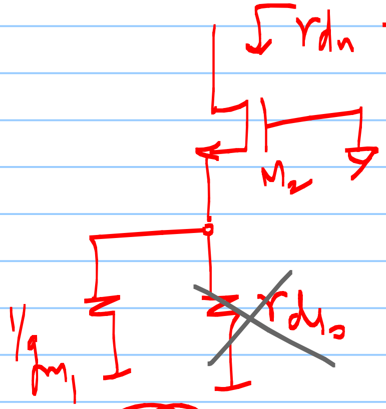
Lec 16



$$r_{out} = r_{up} \| r_{dn}$$

$$\approx r_{ds_4} \| r_{ds_2}$$

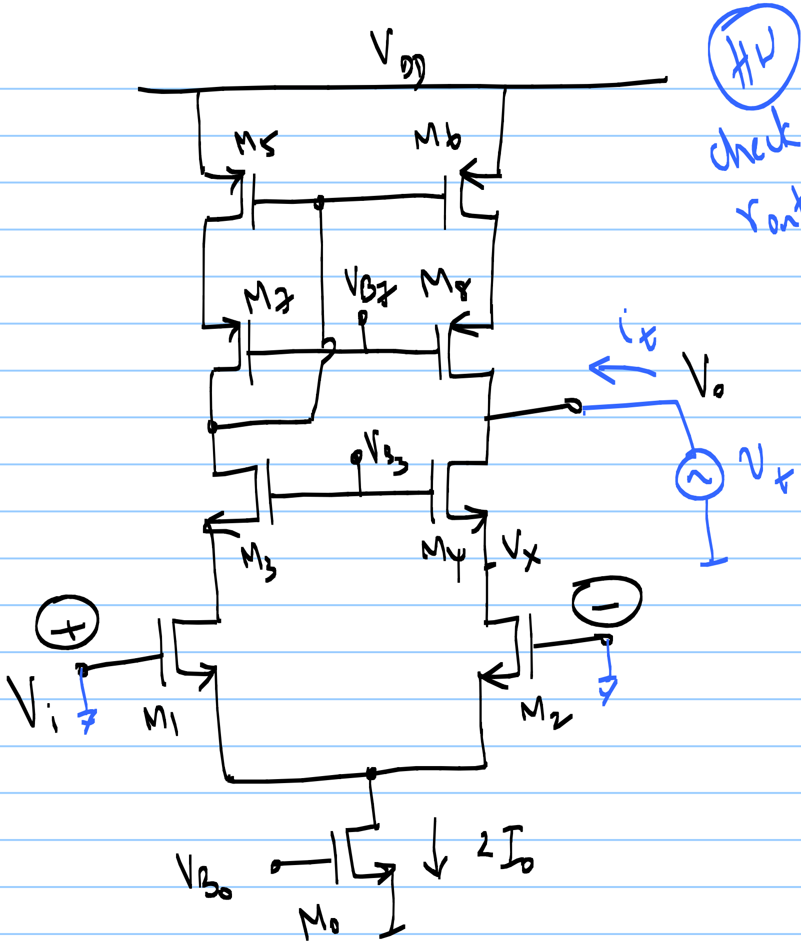
?



$$r_{dn} = r_{ds_2} + \frac{1}{g_{m_1}} + g_{m_2} r_{ds_2} \cdot \frac{1}{g_{m_1}}$$

$$\approx 2 r_{ds_2}$$

HW Cross check

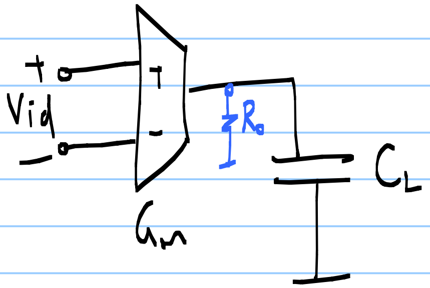


HW

check r_{out} for telescopic & folded cascode as well.

$r_{out} = \frac{v_t}{i_t}$

One Stage Opamp
→ 5T OTA
→ Telescopic
→ Folded Cascode



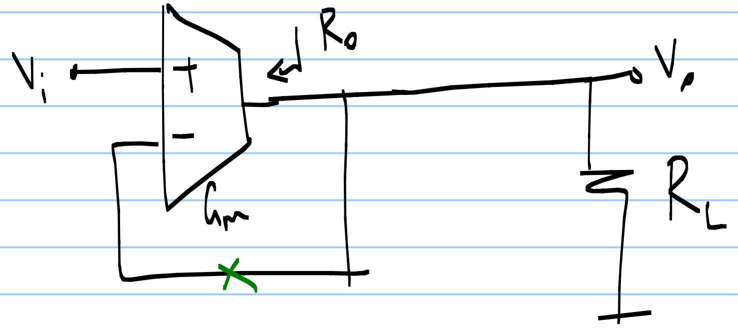
$$\omega_u = \frac{G_m}{C_L}$$

1) We need higher gain? Not necessarily the issue

↳ one-stage opamps have gain

5-T OTA $\simeq g_m r_{ds}$

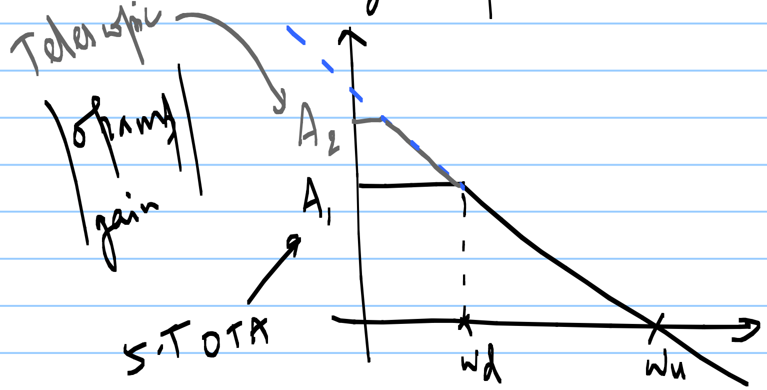
Telescopic/Folded Cascode $\simeq (g_m r_{ds})^2$ ✓

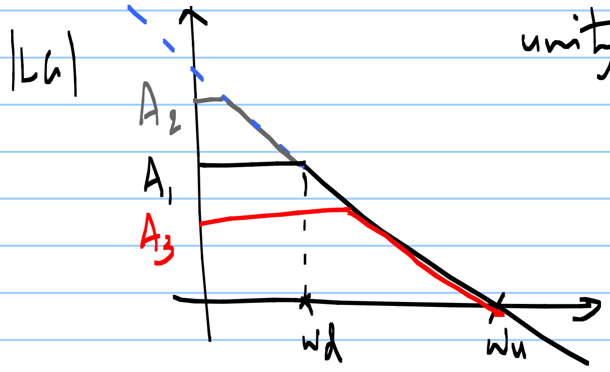


$$|LG| = G_m(\underset{\text{high}}{R_o}\,||\,\underset{\text{low}}{R_L})$$

$$\approx G_m R_L \quad (\text{too low})$$

* One stage opamp cannot drive resistive loads

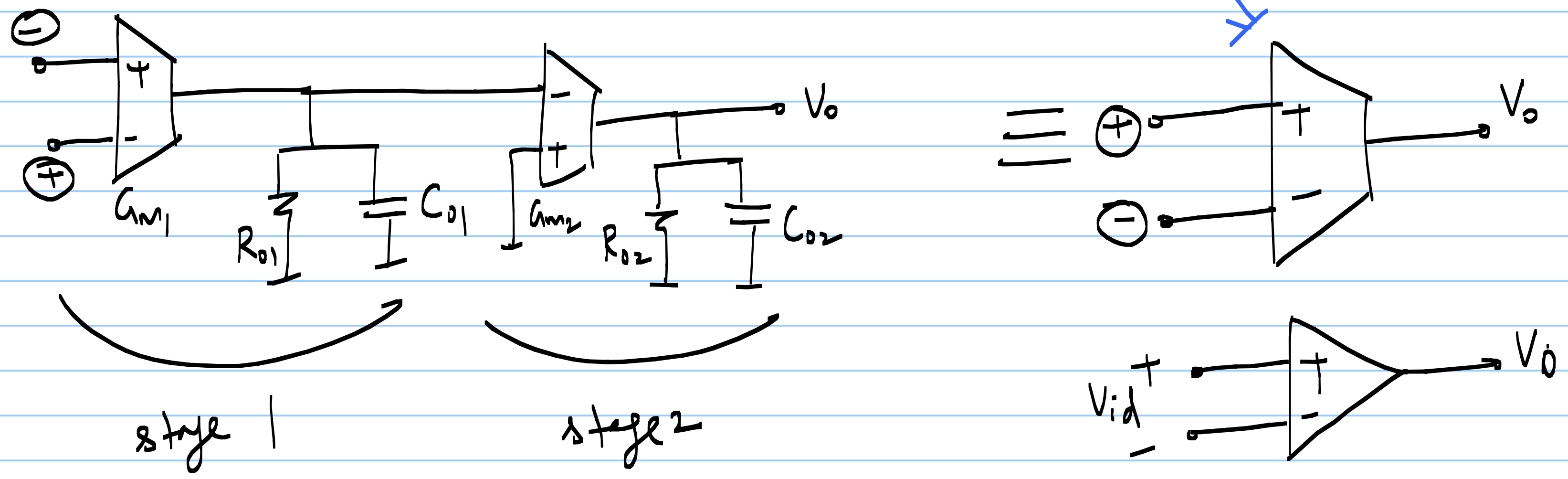


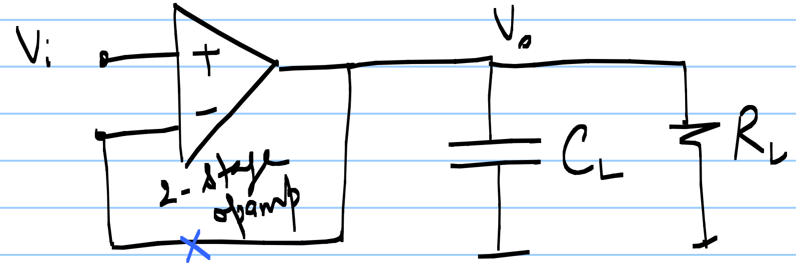


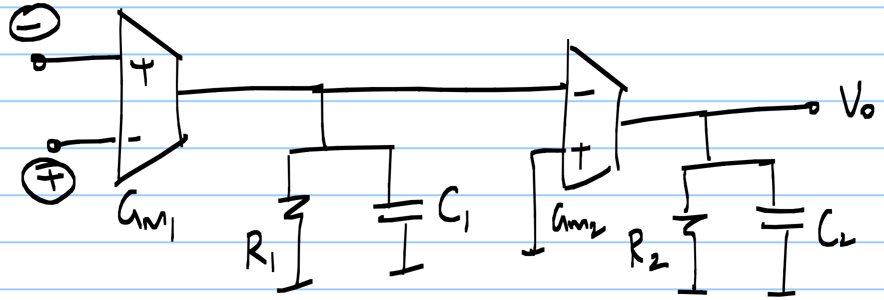
unity gain CL amp

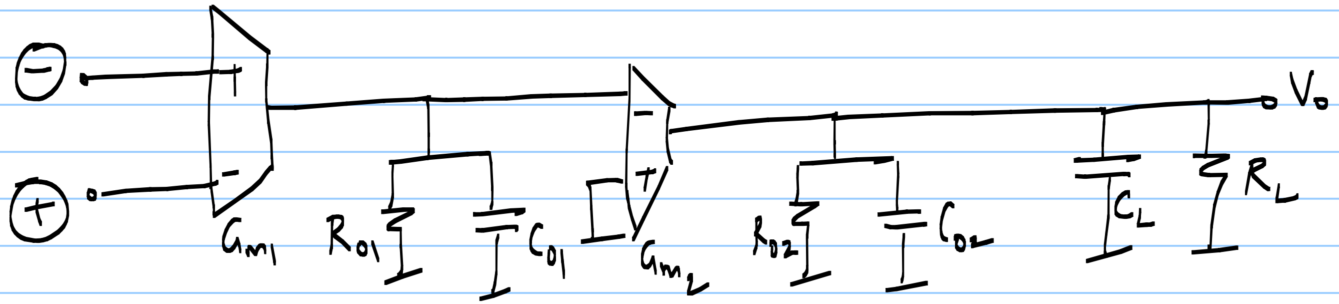
A_3

Two Stage Opamp







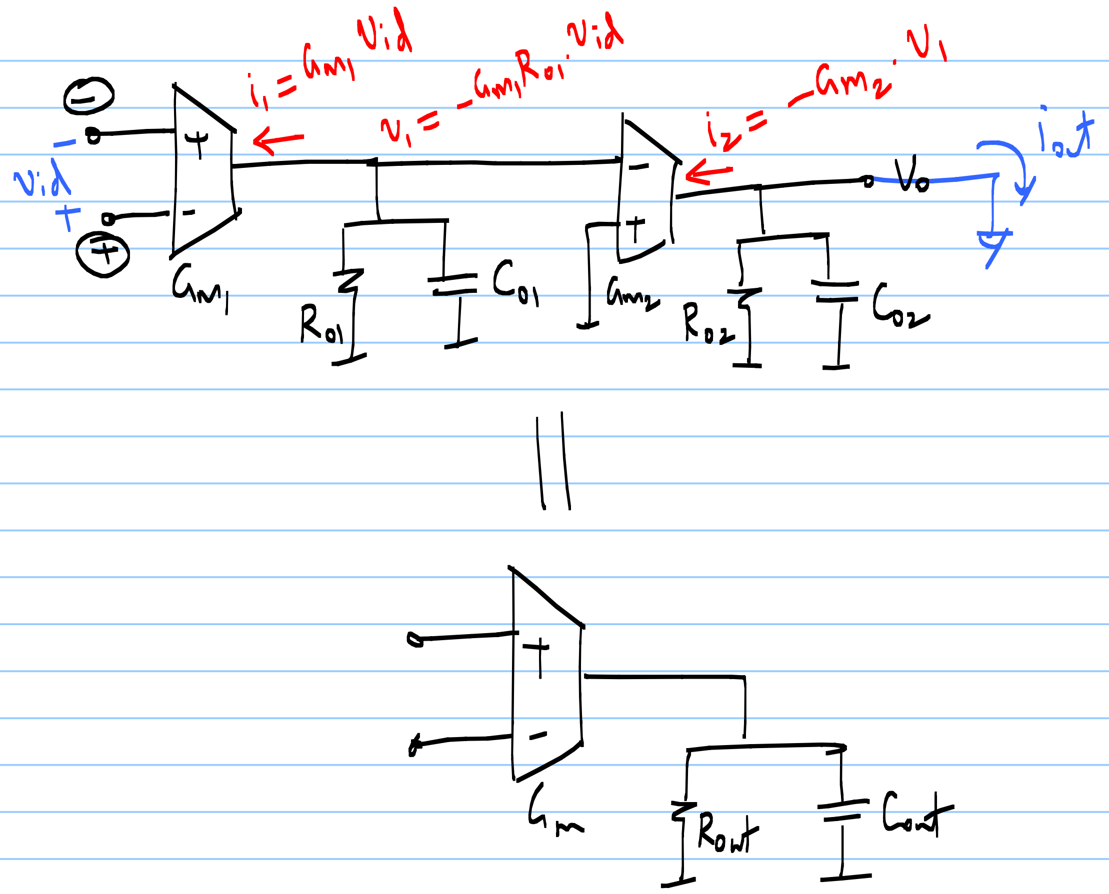


$$R_2 = R_{o2} || R_L$$

$$C_2 = C_{o2} || C_L$$

$$R_1 = R_{o1}$$

$$C_1 = C_{o1}$$

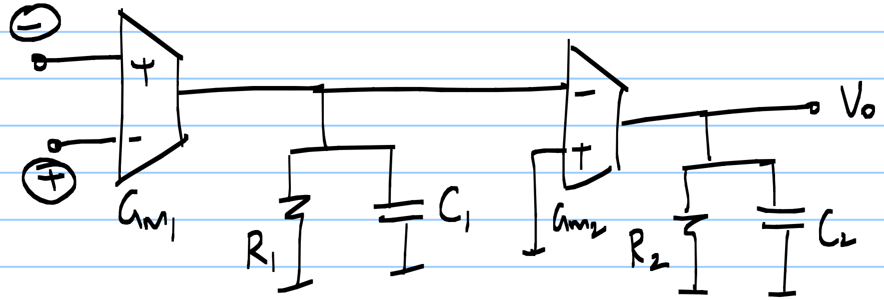


DC gain $= (G_{m1} R_{o1}) \cdot (G_{m2} R_{o2})$

can be close to $(g_m r_{ds})^2$

$R_{out} = R_{o2}$ (large)

$G_m = \frac{i_{out}}{v_{id}} = (G_{m1} R_{o1}) G_{m2}$

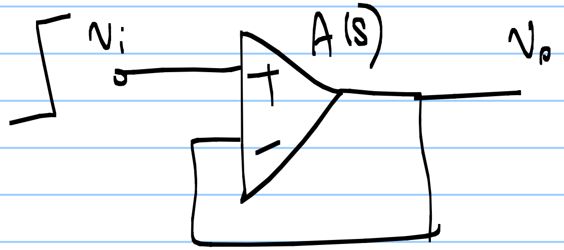


pole p_1 pole p_2

2-pole system

↳ technically "unconditionally" stable

↳ If p_1 & p_2 are close to each other, ringing



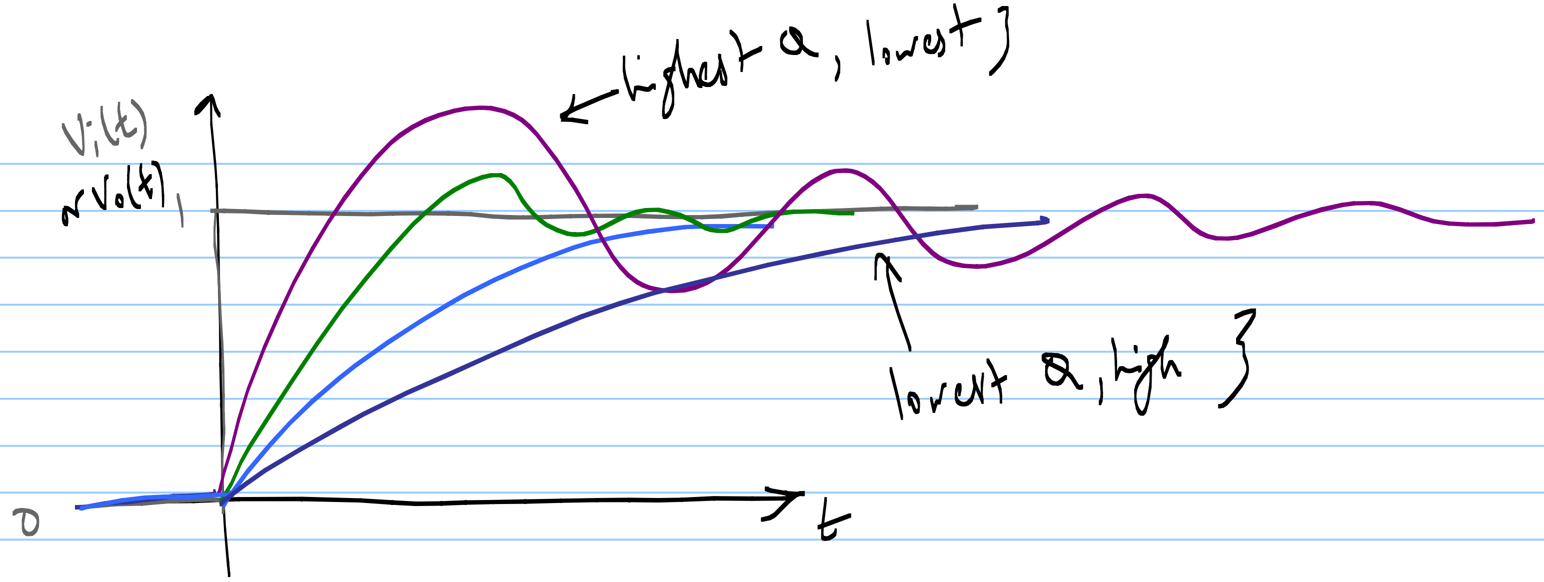
A = DC gain

$$A(s) = \frac{A}{(1+s/p_1)(1+s/p_2)}$$

$$CLG = \frac{A(s)}{1+A(s)} = \frac{N(s)}{D(s)}$$

$$D(s) = A + (1+s/p_1)(1+s/p_2)$$

$$= (1+A) + s\left(\frac{1}{p_1}+\frac{1}{p_2}\right) + \frac{s^2}{p_1 p_2}$$



$$D'(s) = \frac{D(s)}{1+A} = 1 + \frac{s}{1+A}\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \frac{s^2}{(1+A)p_1p_2}$$

$$= 1 + 2\zeta\left(\frac{s}{\omega_n}\right) + \left(\frac{s}{\omega_n}\right)^2 \quad \text{or} \quad 1 + \frac{s}{Q\omega_n} + \frac{s^2}{\omega_n^2}$$

* try $p_1 = p_2 = p$ $\{$ identical stages 1 & 2 $\}$

$$D'(s) = 1 + \frac{s}{\frac{1}{2}p(1+A)} + \frac{s^2}{p^2(1+A)}$$

$$\omega_n = p\sqrt{(1+A)}$$

$$Q = \frac{\sqrt{1+A}}{2}$$

A = high (DC gain)

$\Downarrow$

Q = high

$\Downarrow$

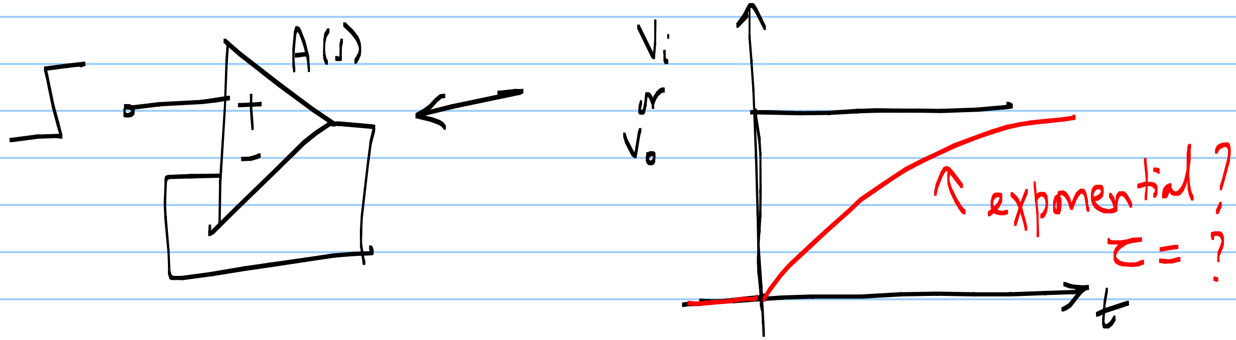
ringing in step response

$$CLG = \frac{A(s)}{1+A(s)} = \frac{A\Big/(1+s/p_1)(1+s/p_2)}{1+\dfrac{A}{(1+s/p_1)(1+s/p_2)}}$$

$$= \frac{A}{A+(1+s/p_1)(1+s/p_2)} = \frac{N(s)}{D(s)}$$

$$= \frac{A}{A+1+s\left(\dfrac{1}{p_1}+\dfrac{1}{p_2}\right)+\dfrac{s^2}{p_1p_2}} = \frac{\overbrace{A/(1+A)}^{N'(s)}}{\underbrace{1+\dfrac{s}{1+A}\left(\dfrac{1}{p_1}+\dfrac{1}{p_2}\right)+\dfrac{s^2}{(1+A)p_1p_2}}_{D'(s)}}$$

$$A(s) = \frac{A}{(1+s/p)}$$



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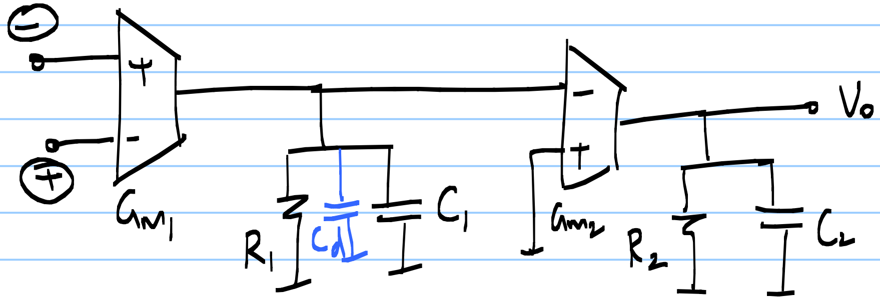
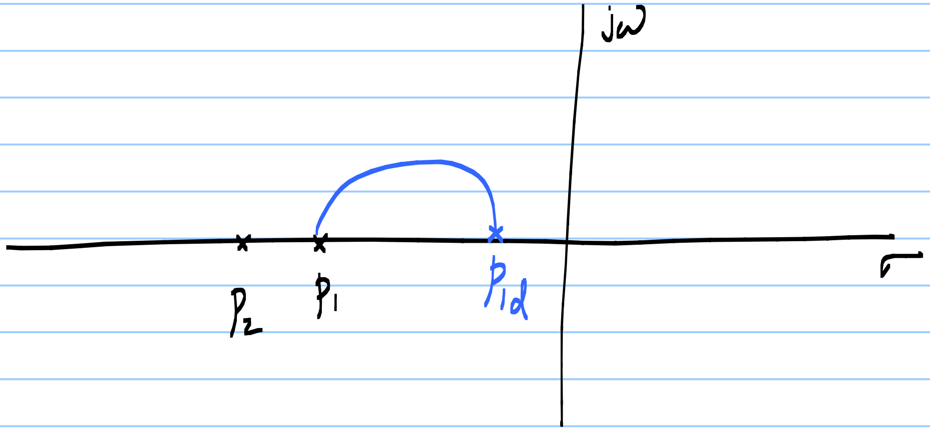
Lec 17





pole p_1 pole p_2

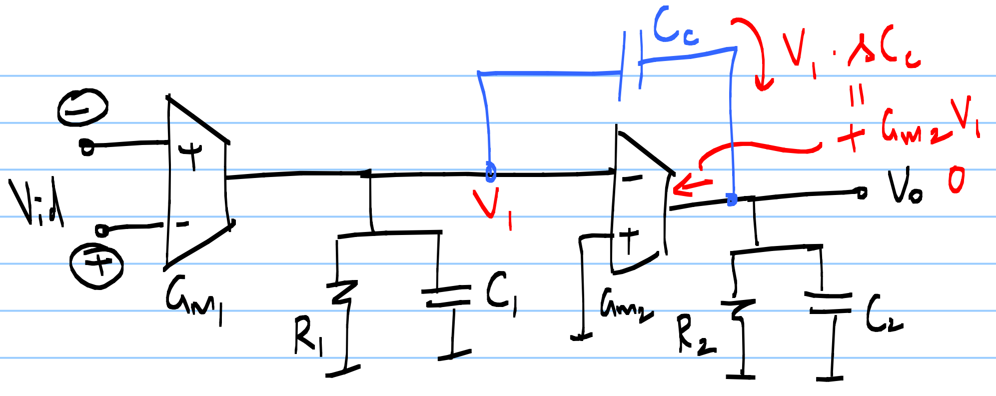
perform freq. compensation → 1) Dominant pole comp. → add cap C_d

$p_1 \rightarrow p_{1d}$

$p_{1d} << \omega_u$ (high A_0)

$p_2 > \omega_u$ (high PM)

need $C_d >> C_1$ because $p_{1d} << p_2$ is required



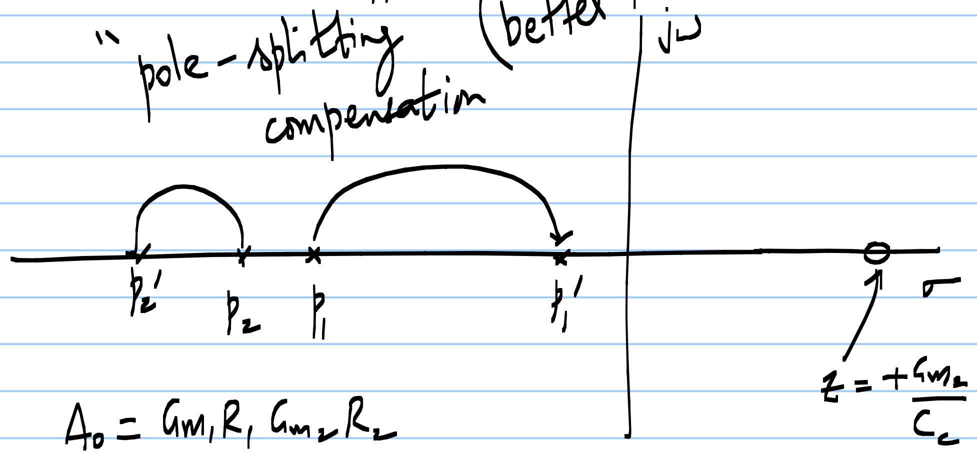
pole p_1 pole p_2

2) Miller Compensation

$$\frac{V_o}{V_{id}}(s) = \frac{A_0\,(1 - s/z)}{\left(1+\frac{s}{p_1'}\right)\left(1+\frac{s}{p_2'}\right)}$$

$$C_{2eff} \simeq C_2 + \frac{C_1 C_c}{C_1 + C_c}$$

"pole-splitting" compensation (better PM)



$A_0 = G_{m1}R_1G_{m2}R_2$

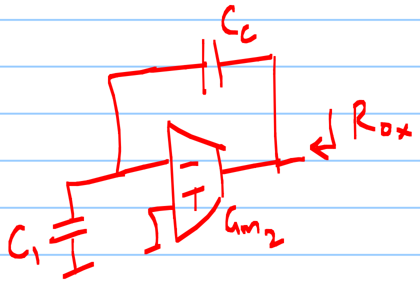
$$z = \frac{G_{m2}}{C_c}$$

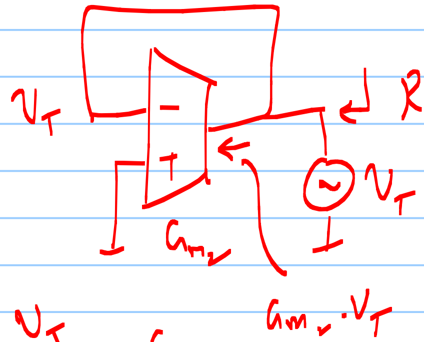
$$p_1' \simeq \frac{1}{R_1 \cdot C_{1eff}} \; ; \; C_{1eff} \approx C_1 + (G_{m2}R_2)C_c$$

$R_{2eff} = R_2 \,||\, R_{ox}$ ✓

$$p_2' = \frac{1}{R_{2eff} \cdot C_{2eff}} \; ; \; C_{2eff.} = C_2 + f(C_c, C_1)$$

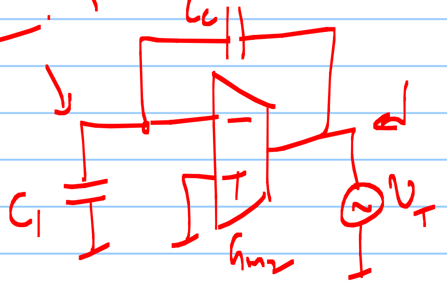
HW





$R_{out} = \frac{1}{G_{m_2}}$

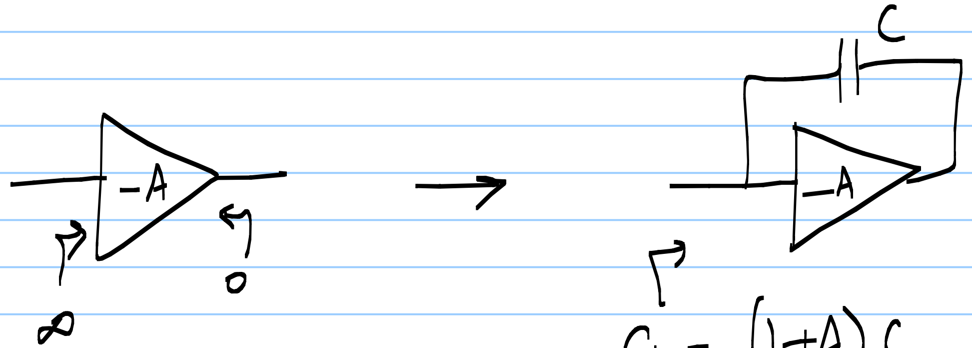
$\frac{C_c}{C_c + C_1} \cdot V_T$



$R_{out} = \left(\frac{C_c + C_1}{C_c}\right) \cdot \frac{1}{G_{m_2}}$

$i_T = V_T \cdot \frac{G_{m_2} \cdot C_c}{C_c + C_1}$

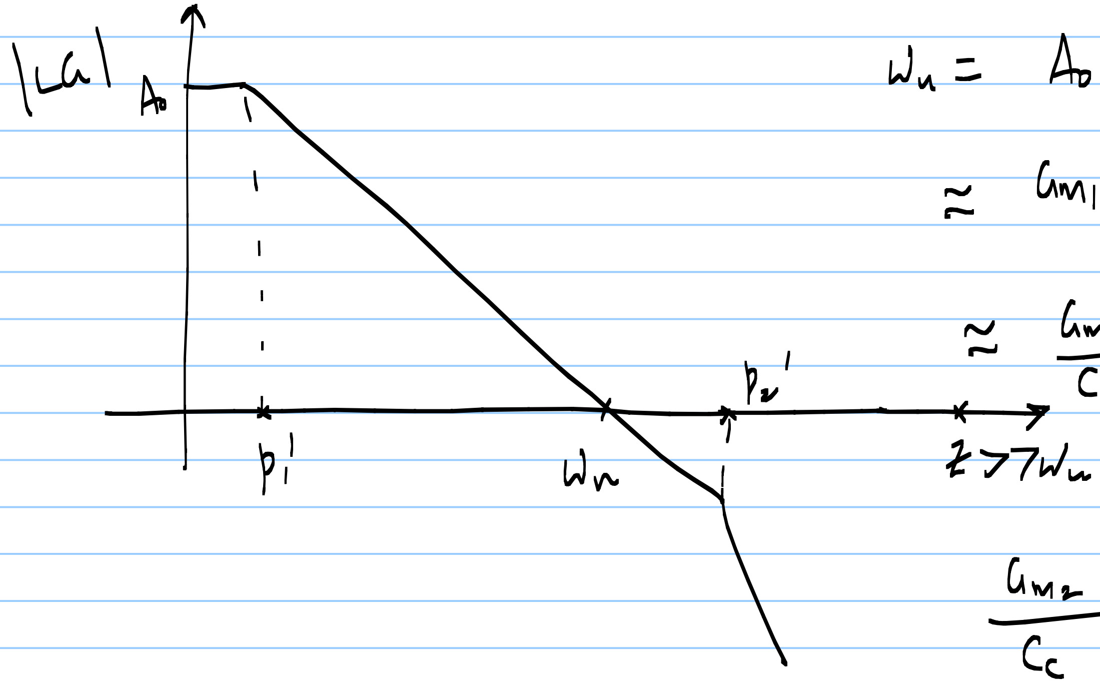
Miller Effect



$C_{in} = (1+A)\,C$

Miller Compensation :

ω_u after compensation $= \dfrac{G_{m1}}{C_c}$

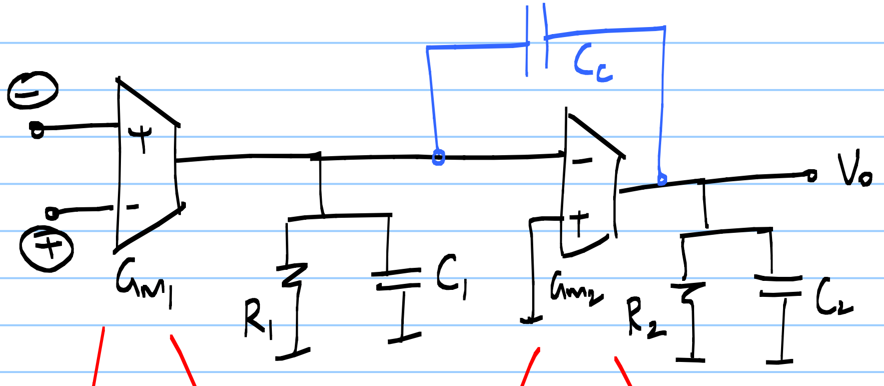


$$\omega_u = A_0 \cdot p_1'$$

$$\approx G_{m1} R_1 \, G_{m2} R_2 \cdot \frac{1}{R_1 \cdot G_{m2} R_2 C_c}$$

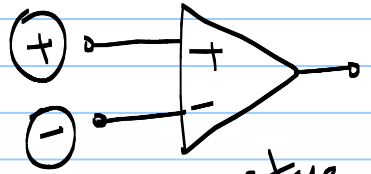
$$\approx \frac{G_{m1}}{C_c}$$

$$\frac{G_{m2}}{C_c} \gg \frac{G_{m1}}{C_c} \Rightarrow \boxed{G_{m2} \gg G_{m1}}$$



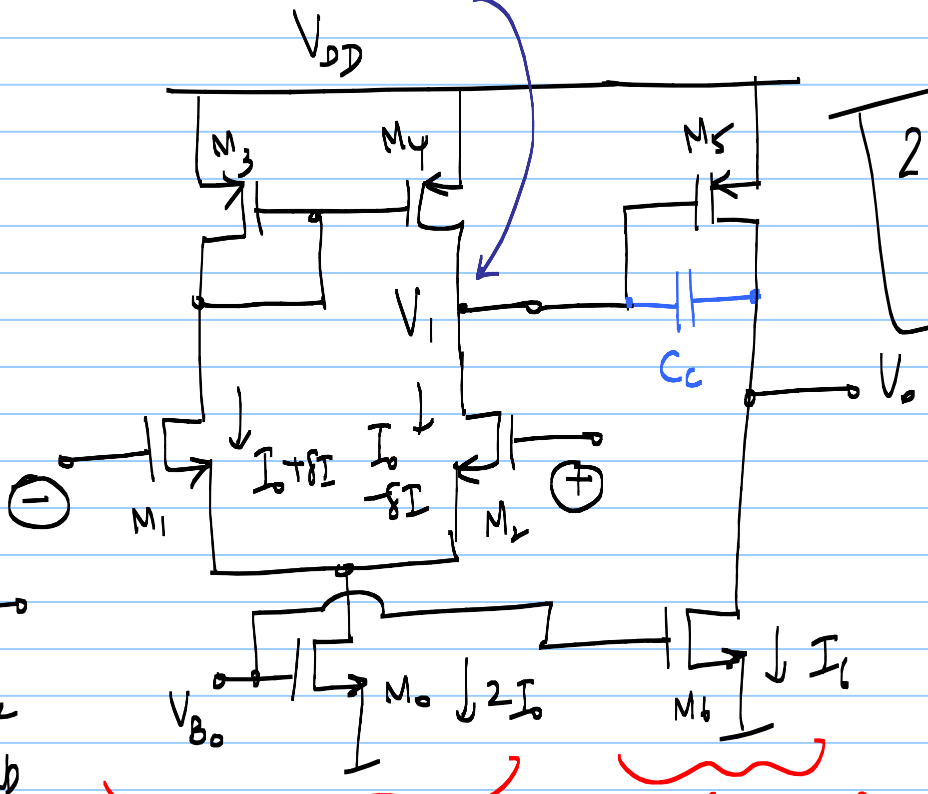
looks like 5-T OTA

CSA with active load



2-stage opamp

expected DC voltage = $V_{DD} - V_{SG_{3,4}}$



$G_{m_1} = g_{m_{1,2}}$

$G_{m_2} = g_{m_5} >> g_{m_{1,2}}$

2-stage opamp

$R_{o1} = r_{ds_4} || r_{ds_2}$ | $C_{o1} = C_{db_4} + C_{db_2} + C_{gs_5}$

$R_{o2} = r_{ds_5} || r_{ds_6}$ | $C_{o2} = C_{db_5} + C_{db_6}$

1) $G_{m_1} = g_{m_1}$; $G_{m_2} = g_{m_5}$

2) $Z \gg \omega_u \Rightarrow \frac{G_{m_2}}{C_c} \gg \frac{G_{m_1}}{C_c} \Rightarrow G_{m_2} \gg G_{m_1} \Rightarrow \boxed{g_{m_5} \gg g_{m_1}}$

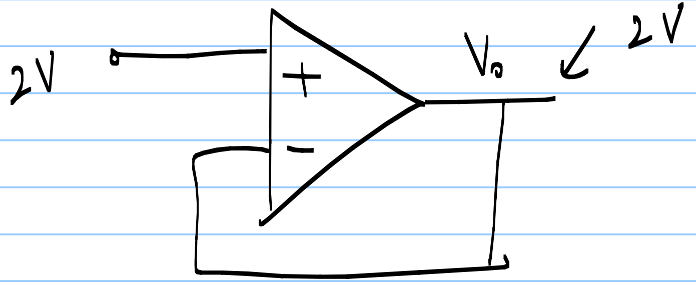
3) $V_{DD} - V_{SG_{3,4}}\Big|_{I_0} = V_{DD} - V_{SG_5}\Big|_{I_6}$

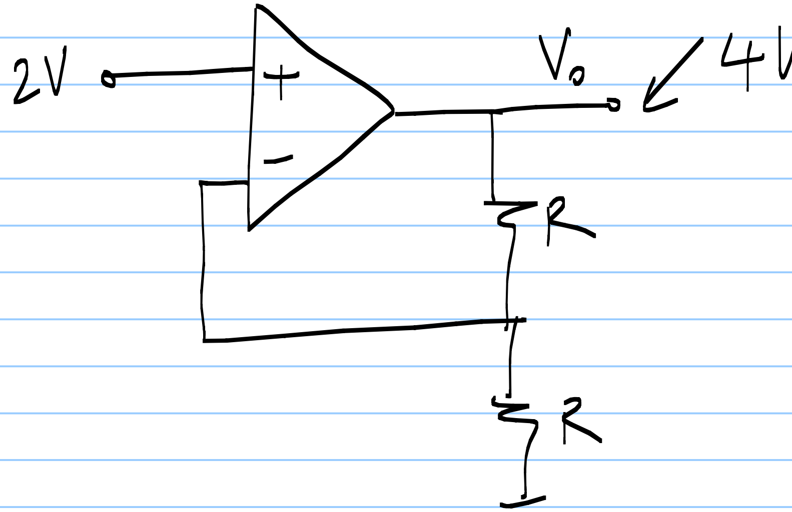
$V_{SG_{3,4}}\Big|_{I_0} = V_{SG_5}\Big|_{I_6} \Rightarrow V_{Dsat_{3,4}}\Big|_{I_0} = V_{Dsat_5}\Big|_{I_6}$

$$\sqrt{\frac{2\,I_o}{\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4}}} = \sqrt{\frac{2\,I_b}{\mu_p C_{ox}\left(\frac{W}{L}\right)_5}}$$

$$\Rightarrow \boxed{\frac{I_o}{(W/L)_{3,4}} = \frac{I_6}{(W/L)_5}}$$

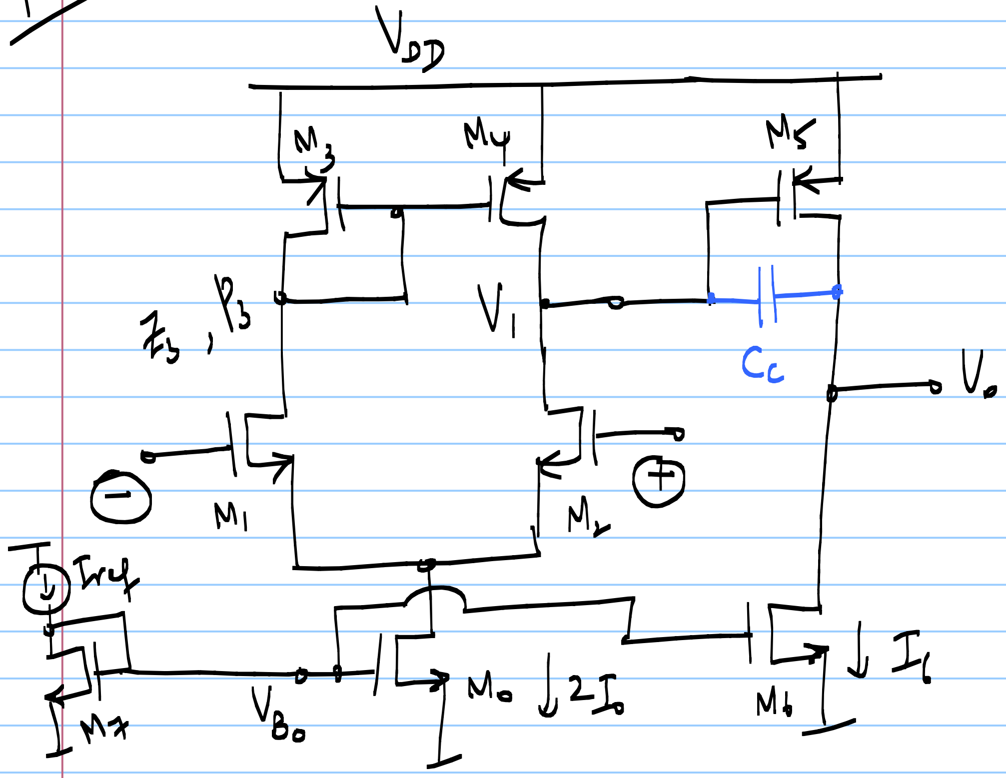
4) $L_{3,4} = L_5$


2V
+
−
V₀
2V


2V
+
−
V₀
4V
R
R

9/11/2023

Lec 18



$L_7 = L_0 = L_6$

$W_7 : W_0 : W_6 = I_{ref} : 2I_0 : I_6$

2-stage opamp datasheet

1) DC gain $= G_{m_1} R_1 \cdot G_{m_2} R_2$

$= g_{m_1} \cdot (r_{ds_2} || r_{ds_4}) \cdot g_{m_5} \cdot (r_{ds_5} || r_{ds_6})$

2) $G_m = g_{m_1} (r_{ds_2} || r_{ds_4}) \cdot g_{m_5}$

3) $\omega_u = \frac{g_{m_1}}{C_c}$; $\omega_d = \frac{\omega_u}{\text{DC gain}}$

4) ND poles & zeroes : poles: p_2' ; p_3

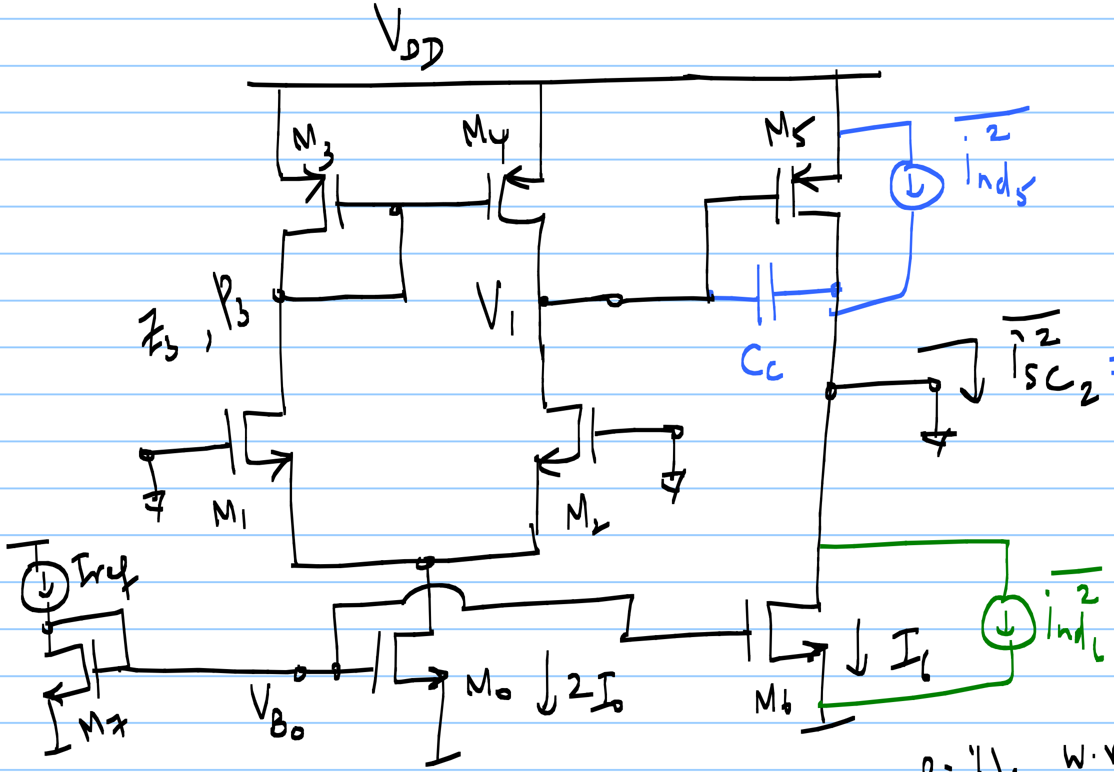
zeroes: z ; z_3

5) ICMR : $\left\{ V_{GS_1}\Big|_{I_0} + V_{Dsat_0}\Big|_{2I_0} ,\ V_{DD} - V_{SG_3}\Big|_{I_0} + V_{T_1} \right\}$

6) OCMR : $\left\{ V_{Dsat_6} ,\ V_{DD} - V_{Dsat_5} \right\}$

7) Noise :

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{16kT}{3g_{m_1}} + \frac{16kT \cdot g_{m_3}}{3g_{m_1}^2}$$



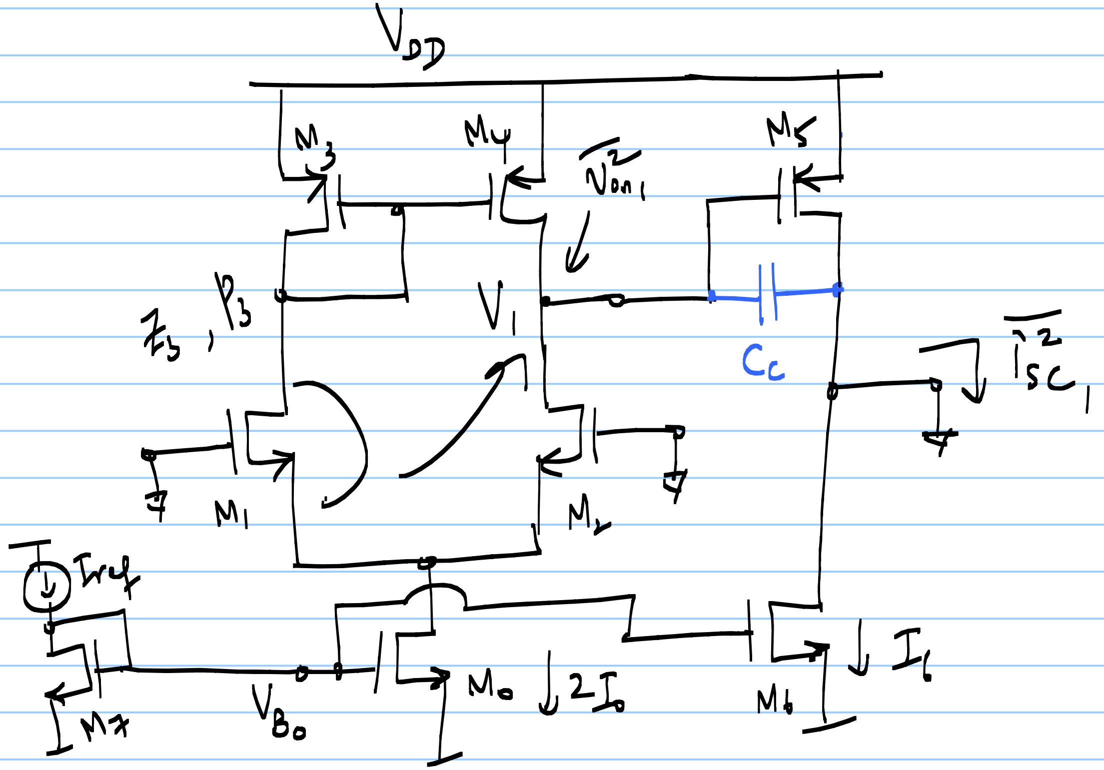
$$\overline{e_n^2} = \underset{\substack{\uparrow \\ \text{1st stage}}}{\overline{e_{n_1}^2}} + \underset{\substack{\uparrow \\ \text{2nd stage}}}{\overline{e_{n_2}^2}}$$

$$\overline{i_{sc_2}^2} = \overline{i_{nd_5}^2} + \overline{i_{nd_6}^2} + \overline{i_{nd_7}^2}$$

$$\overline{e_{n_2}^2} = \frac{\overline{i_{nd_5}^2} + \overline{i_{nd_6}^2} + \overline{i_{nd_7}^2}}{G_m^2}$$

$$= \frac{\overline{i_{nd_5}^2} + \overline{i_{nd_6}^2} + \overline{i_{nd_7}^2}}{\left[g_{m_1}(r_{ds_2} \| r_{ds_4}) \cdot g_{m_5}\right]^2}$$

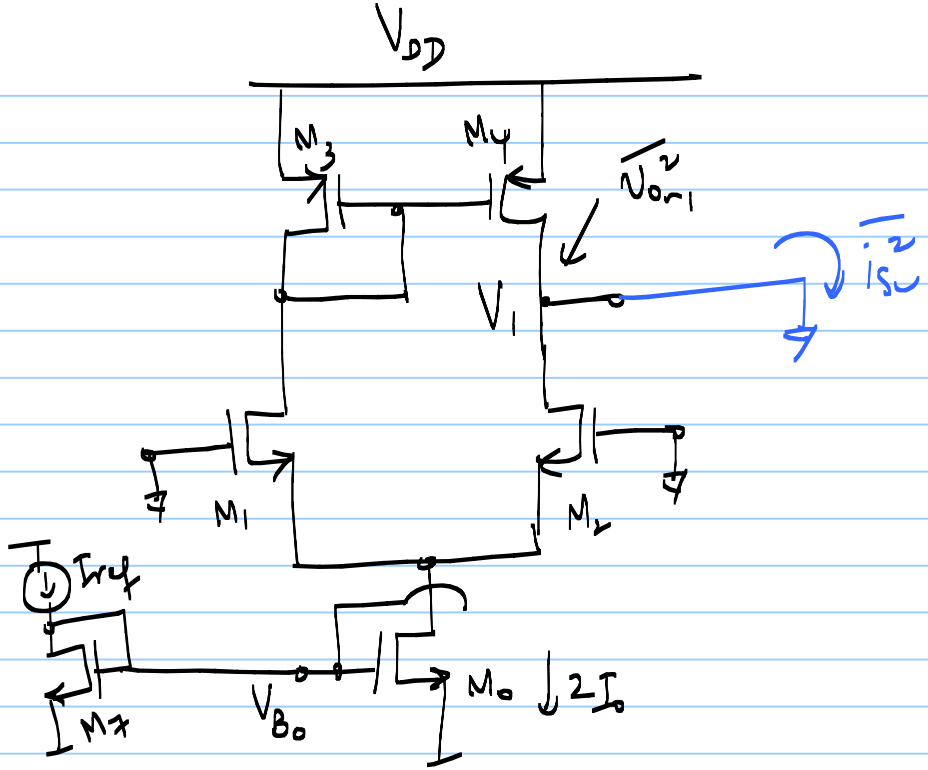
negligible w.r.t. $\overline{e_{n_1}^2}$ due to large gain of stage 1 ←



$$\overline{v_{on_1}^2} = \left[\frac{8kT}{3} g_{m_1} + \frac{8kT}{3} g_{m_2} + \frac{8kT}{3} g_{m_3} + \frac{8kT}{3} g_{m_4}\right] \cdot (r_{ds_2} || r_{ds_4})^2$$

$$\overline{i_{sc_1}^2} = \overline{v_{on_1}^2} \cdot g_{m_5}^2$$

$$\overline{e_{n_1}^2} = \frac{\overline{i_{sc_1}^2}}{G_m^2}$$



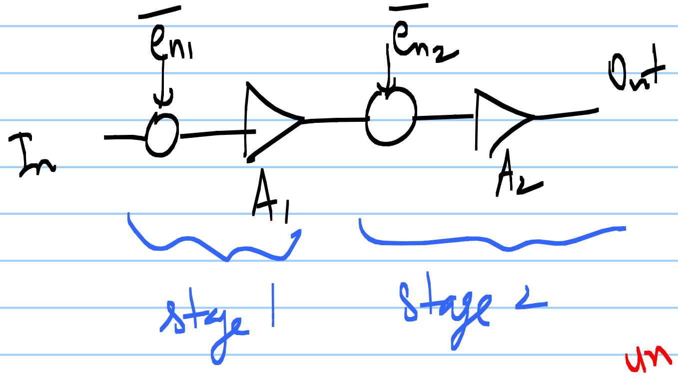
$$\overline{e_n^2} = \frac{\overline{i_{sc}^2}}{G_m^2}$$

$$\overline{e_n^2} = \frac{\overline{v_{on1}^2}}{A^2}$$

$$\frac{\overline{e_{n_1}^2}}{\Delta f} = \frac{\left[\frac{8kT}{3} g_{m_1} + \frac{8kT}{3} g_{m_2} + \frac{8kT}{3} g_{m_3} + \frac{8kT}{3} g_{m_4}\right] \cdot \left(r_{ds_2} \| r_{ds_4}\right)^2 \cdot g_{m_5}^2}{\left[g_{m_1}\left(r_{ds_2} \| r_{ds_4}\right) \cdot g_{m_5}\right]^2}$$

$$\frac{\overline{e_{n_1}^2}}{\Delta f} = \frac{16kT}{3 g_{m_1}} + \frac{16kT \, g_{m_3}}{3 \, g_{m_1}^2}$$

Cascaded stage noise



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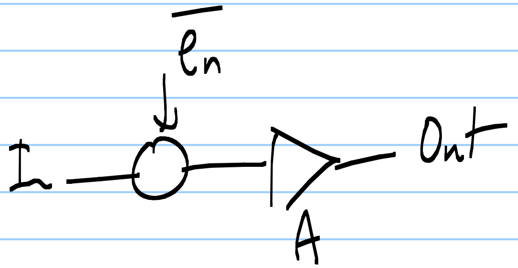
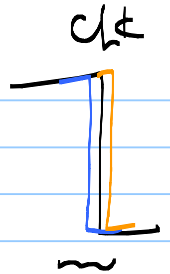




jitter = uncertainty in clk edge

$$A = A_1 A_2$$

un-correlated

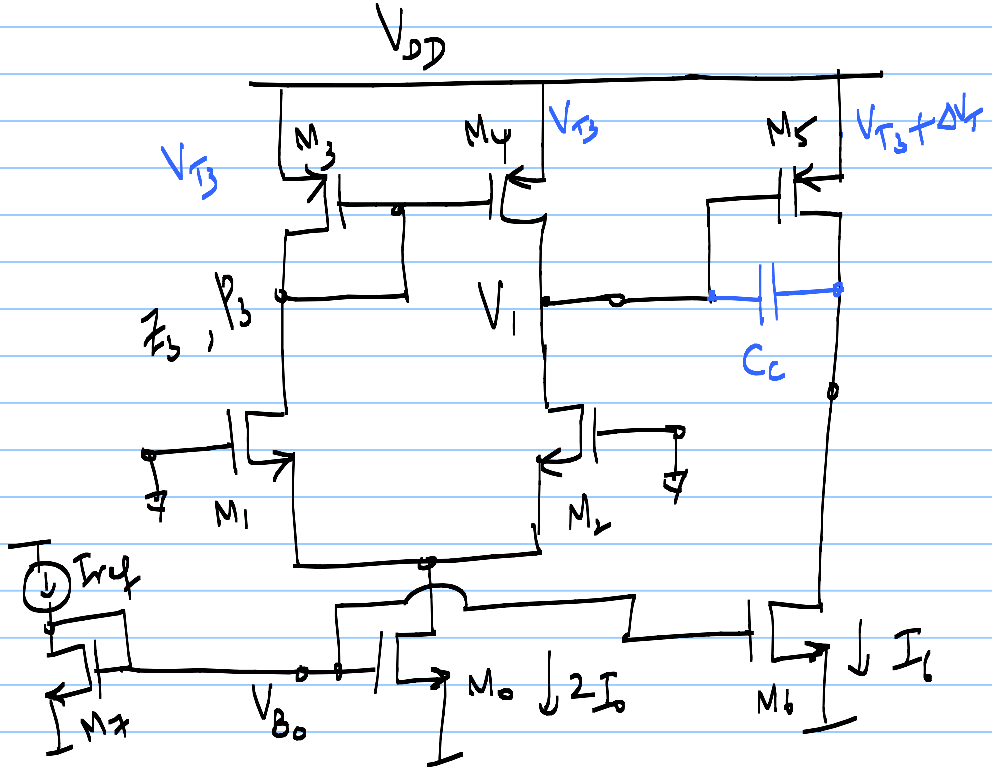
Noise @ Out squared voltage density

$$= \overline{e_{n1}^2}\cdot(A_1A_2)^2 + \overline{e_{n2}^2}\cdot A_2^2 \quad = \quad = \overline{e_n^2}\cdot A^2 = \overline{e_n^2}\,(A_1A_2)^2$$

$$\overline{e_n^2}\,(A_1A_2)^2 = \overline{e_{n1}^2}\,(A_1A_2)^2 + \overline{e_{n2}^2}\cdot A_2^2$$

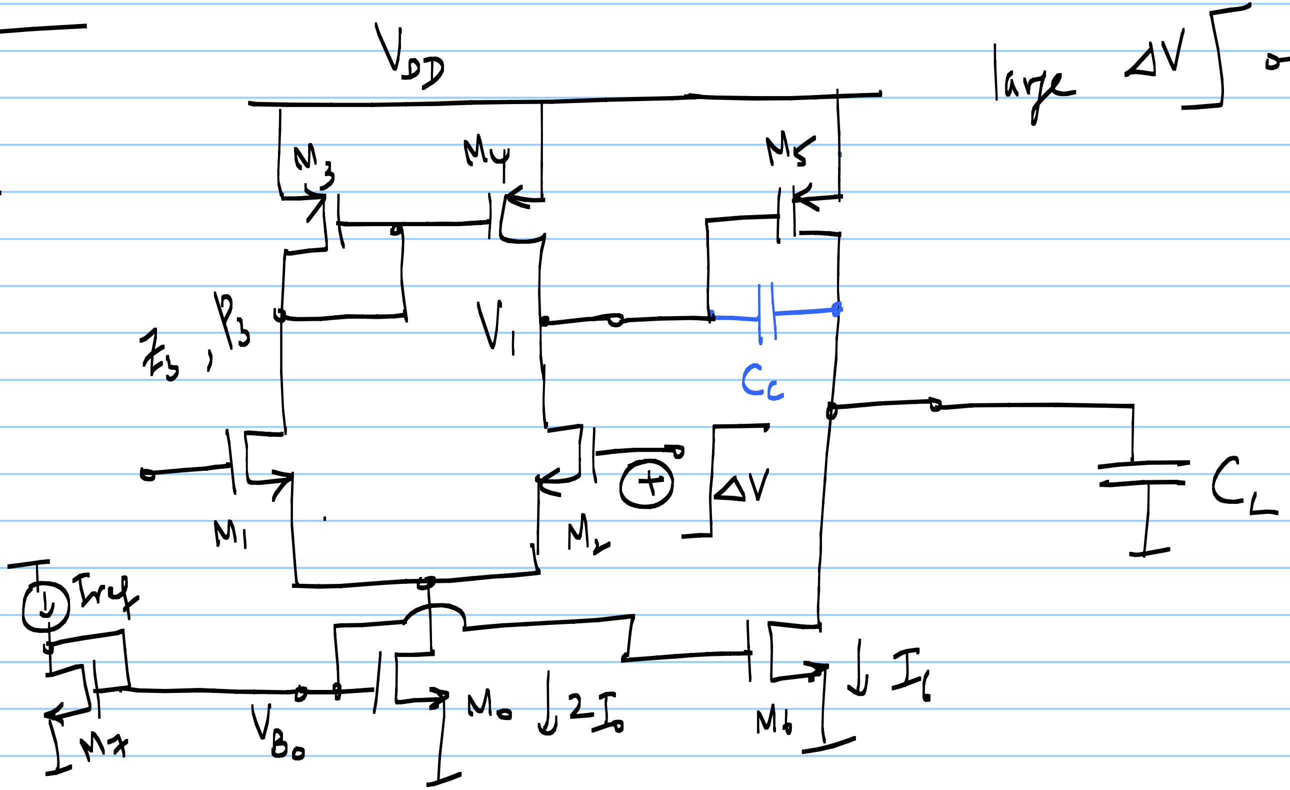
$$\overline{e_n^2} = \overline{e_{n1}^2} + \overline{e_{n2}^2}/A_1^2$$

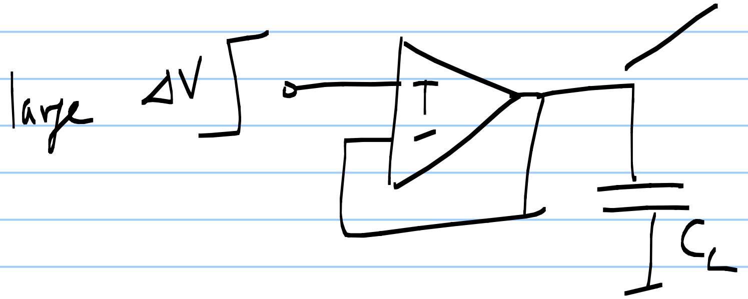
8) <u>Offset</u> $\sigma_{os}^2 \simeq \sigma_{V_{T1,2}}^2 + \sigma_{V_{T3,4}}^2 \left(\frac{g_{m3}^2}{g_{m1}^2}\right) + (\ldots)$



a) Slew rate

$$SR \simeq \pm \frac{2I_0}{C_c}$$





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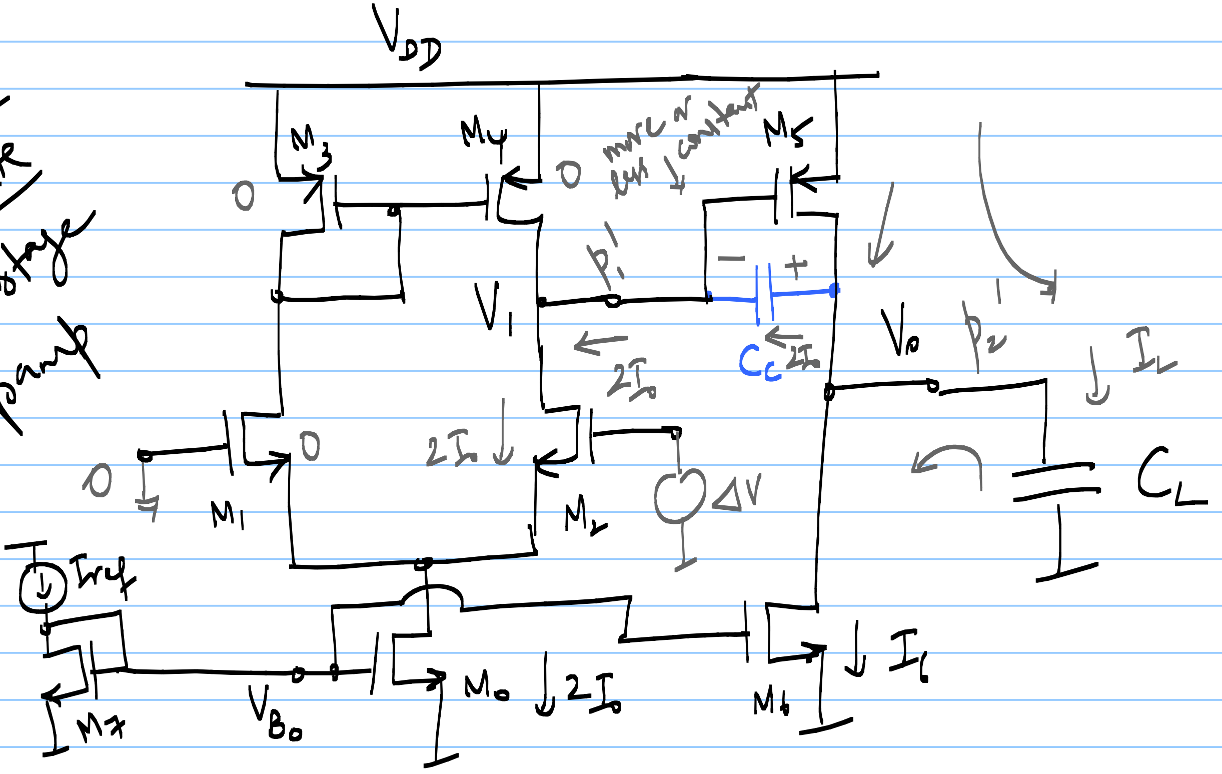
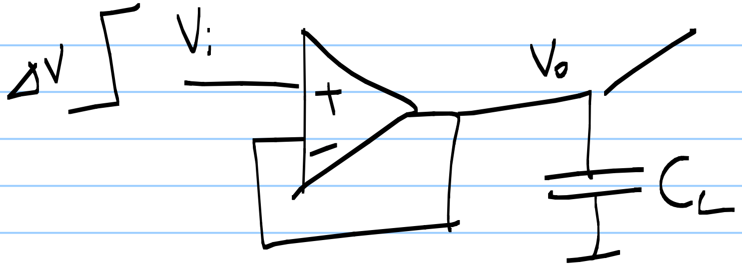
Lec 19

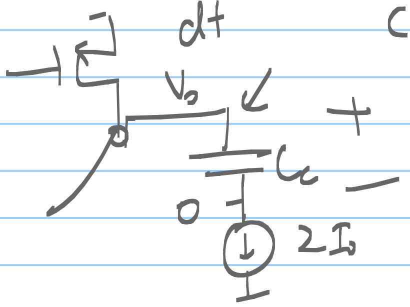
Slew Rate of 2-stage opamp

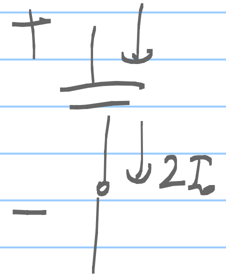




$$2I_0 = C_c \frac{dV_o}{dt}$$

$$\frac{dV_o}{dt} = \frac{2I_0}{C_c}$$





$$I_L = C_L \underbrace{\frac{dV_o}{dt}}_{\frac{2I_o}{C_c}} \leftarrow \text{from } M_5$$

* Assume C_L is not too large

* Assume $Z \gg \omega_u$

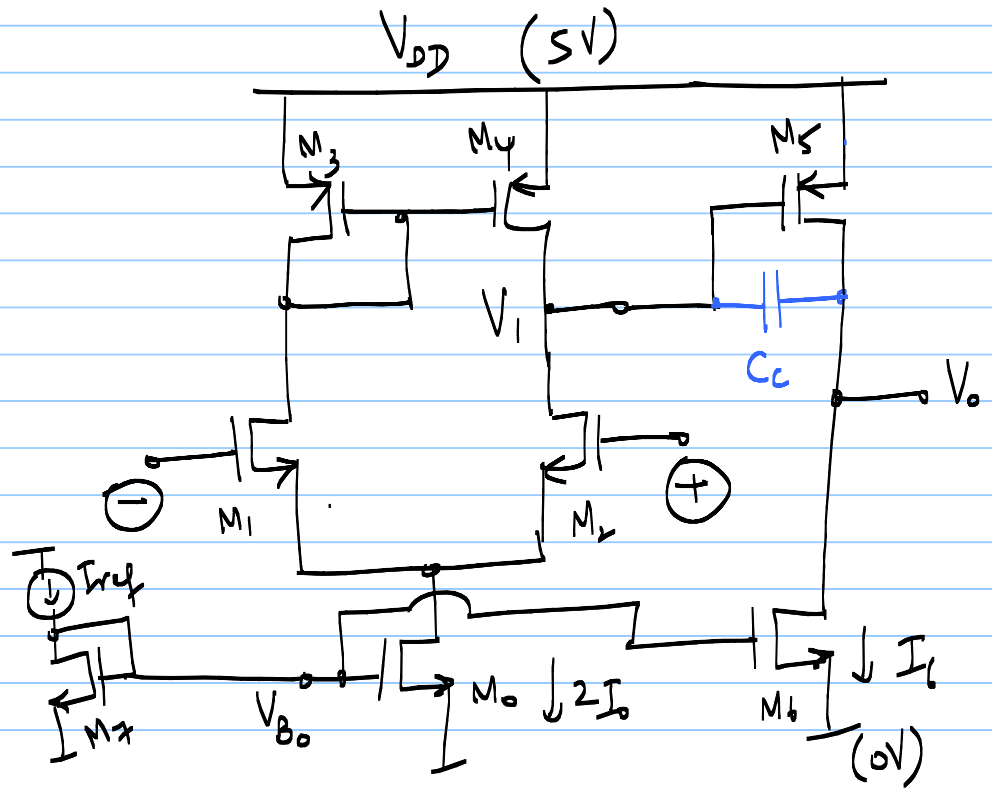
$$\Rightarrow \frac{g_{m_2}}{C_c} \gg \frac{g_{m_1}}{C_c} \Rightarrow g_{m_2} \gg g_{m_1} \Rightarrow \boxed{g_{m5} \gg g_{m_1}}$$

* $2I_o$ is small enough so that $SR \simeq \frac{2I_o}{C_c}$ is valid

* Cross check via simulations

$$\Rightarrow I_6 \gg I_o \,;\, \left(\frac{W}{L}\right)_5 \gg \left(\frac{W}{L}\right)_1$$

Design Example



Specs

1) $A_o \geqslant 80$ dB ($= 10^4$ V/V)
2) $V_{o_{p-p}} \geqslant 4$V
3) $f_u \geqslant 5$ MHz
4) SR $\geqslant$ 10 V/μs
5) Unity gain PM $\geqslant 60°$
6) $C_L = 10$ pF

MOSFET parameters : $\mu_n C_{ox} = 50\mu A/V^2$; $\mu_p C_{ox} = 25\mu A/V^2$

$V_{Tn} = V_{Tp} = 1V$; $V_{DD} = 5V$

$L_{min} = 2\mu m$; $C_{ox} = 1.5 fF/\mu m^2$

$(\lambda L_{min})_p = 0.1 \mu m/V$; $(\lambda L_{min})_n = 0.04 \mu m/V$

* $\left.\begin{array}{l} (\lambda L) = \text{constant} \\ r_{ds} = \dfrac{1}{\lambda I} \end{array}\right)$ $L \uparrow \Rightarrow \lambda \downarrow \Rightarrow r_{ds} \uparrow$

* Assume ICMR is large enough e.g. $> 1.5V \Rightarrow$ NMOS input pair

1) $f_u = \dfrac{g_{m_1}}{2\pi C_c}$

2) $SR = \dfrac{2I_0}{C_c}$

3) $A_0 = (g_{m_1} R_{o1}) \cdot (g_{m_5} R_{o2})$

4) $PM > 60^\circ \Rightarrow \tan 60^\circ = \sqrt{3} = \dfrac{p_2}{\omega_u}$

$p_2 = \sqrt{3}\ \omega_u$

5) $L_3 = L_4 = L_5$

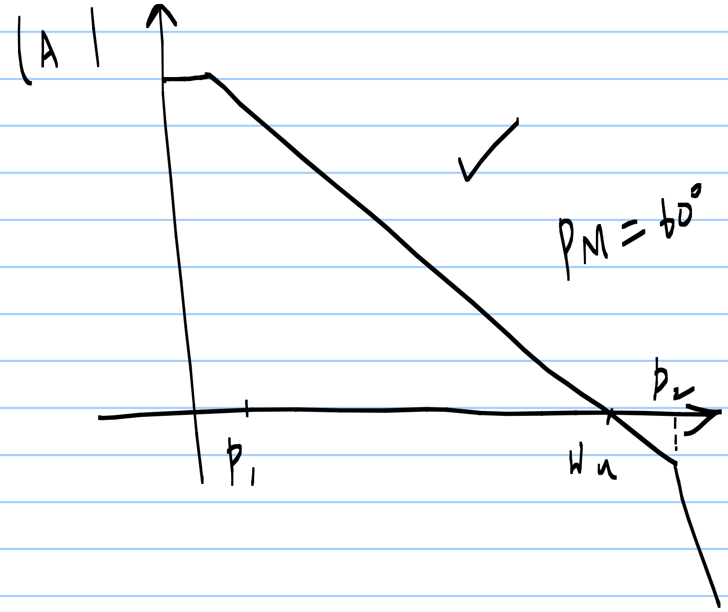
$L_1 = L_2$

$L_0 = L_6 = L_7$

6) $V_{Dsat_5} = V_{Dsat_6} = 0.5V$

7) $g_{m_5} \gg g_{m_1}$ $\{ z \gg \omega_u \}$

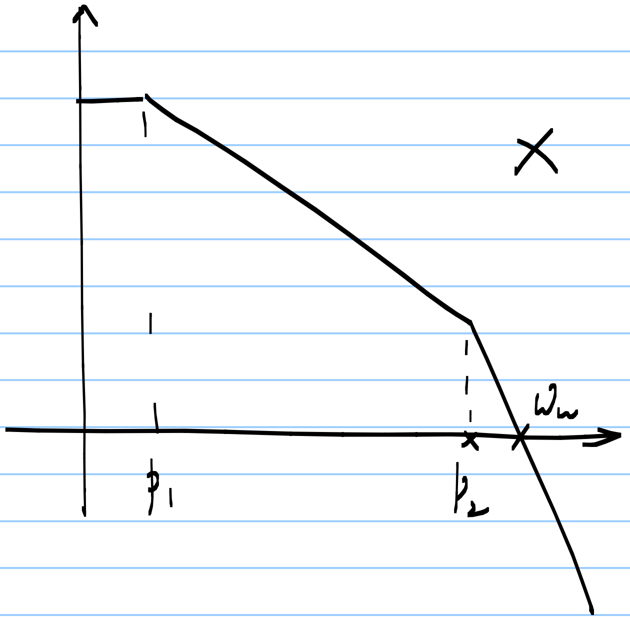
say, $g_{m_5} = 10\, g_{m_1}$



✓

$PM = 60°$

✓



×

* Assume C_1 is small (negligible)

* $C_L >> C_c$

$$p_2 = \sqrt{3}\,\omega_n$$

$$\left.\frac{1}{\underbrace{R_{eff}}_{R_{o2} \| \frac{1}{g_{m5}} \approx \frac{1}{g_{m5}}}\ \underbrace{C_{eff}}_{\approx C_L}} = \sqrt{3}\,\omega_n \right\} \Rightarrow \frac{g_{m5}}{C_L} \approx \sqrt{3}\,\omega_n$$

$$g_{m5} = \sqrt{3} \times 2\pi \times 5 \times 10^6 \times 10 \times 10^{-12}$$

$$\boxed{g_{m5} = 540\,\mu S}$$

$$\Rightarrow \boxed{g_{m1} = 54\,\mu S}$$

$$w_u = \frac{g_{m1}}{C_c}$$

$$C_c = \frac{g_{m1}}{w_u} = \frac{54\mu S}{2\pi \times 5 \times 10^6} = 1.72\, pF$$

$$SR = \frac{2I_0}{C_c}$$

$$2I_0 = SR \times C_c = 10\, V/\mu s \times 1.72\, pF$$

$$2I_0 = 10 \times 10^6 \times 1.72 \times 10^{-12} = 17.2\, \mu A$$

$$I_{D1} = I_{D2} = 8.6\, \mu A$$

$\left(\frac{W}{L}\right)_{1,2}$ from I_{D_1} & g_{m_1}

$$g_{m_{1,2}} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2} \cdot I_{D_{1,2}}}$$

$$\boxed{\left(\frac{W}{L}\right)_{1,2} = 3.4}$$

for M_5 : $g_{m_5} = \frac{2I_{D_5}}{V_{Dsat_5}}$

$$540\mu S = \frac{2I_{D_5}}{0.5} \Rightarrow I_{D_5} = I_6 = 135\mu A$$

$$g_{m5} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_5 \cdot I_{D5}}$$

$$\boxed{\left(\frac{W}{L}\right)_5 = 43.2}$$

$$V_{Dsat_6} = \sqrt{\frac{2 I_{D6}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_6}}$$

$$0.5V = \sqrt{\frac{2 \times 135 \mu A}{50 \mu A/V^2 \times \left(\frac{W}{L}\right)_6}} \Rightarrow \boxed{\left(\frac{W}{L}\right)_6 = 21.6}$$

$$\frac{2I_0}{(W/L)_0} = \frac{I_6}{(W/L)_6}$$

$$\Rightarrow \boxed{\left(\frac{W}{L}\right)_0 = 2.75}$$

$$\frac{2I_{3,4}}{(W/L)_{3,4}} = \frac{2I_5}{(W/L)_5} \Rightarrow \boxed{\left(\frac{W}{L}\right)_{3,4} = 2.75}$$

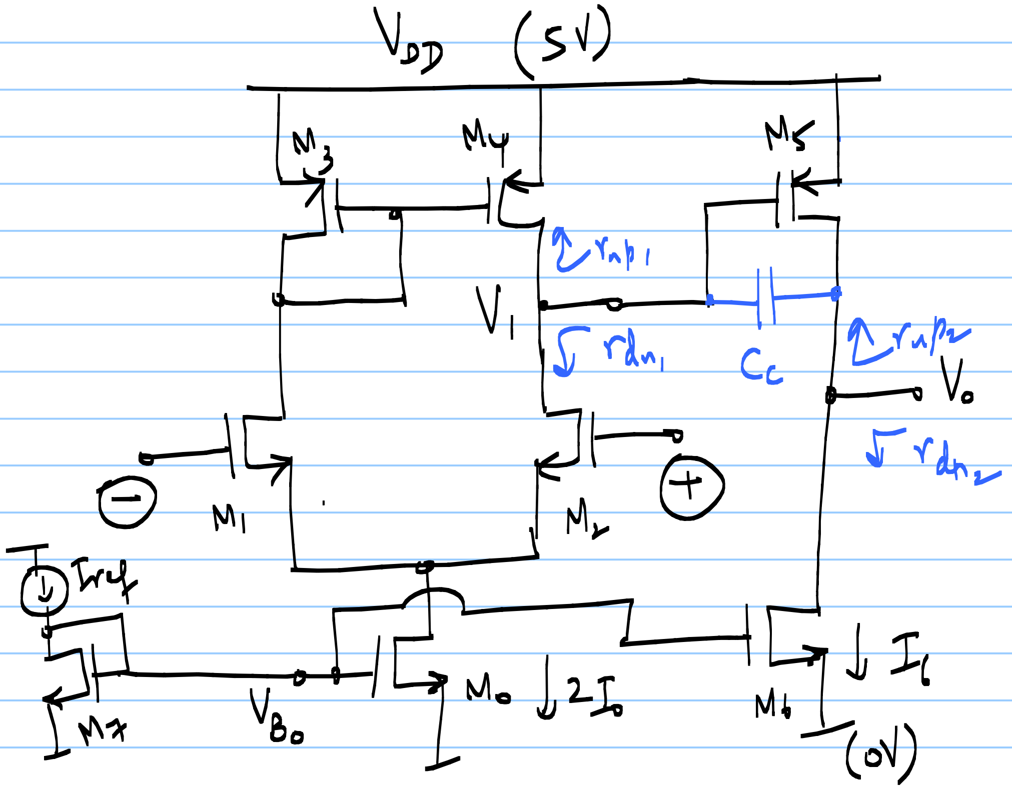
HW

find $L_1 - L_6$

- ↳ assume L_{min}
  - ↳ calculate r_{ds}'s
    - ↳ calculate R_{o1}, R_{o2}
      - ↳ calculate A_o
        - ↳ if $A_o < 10^4$,
          - ↑ L of device limiting R_{o1}/R_{o2} → calculate r_{ds}'s

14/11/23

Lec 20



Assume all L's = Lmin

$A_o = g_{m_1} r_{out_1} \cdot g_{m_5} r_{out_2}$

? ?

$r_{out_1} = r_{np_1} || r_{dn_1}$

$r_{out_2} = r_{np_2} || r_{dn_2}$

$$r_{up_1} = r_{ds_4} = \frac{1}{\lambda_4 \cdot I_{D_4}} = \frac{1}{\frac{0.1\,\mu m/V}{2\,\mu m} \times 8.6\,\mu A} = 2.33\,M\Omega$$

$$r_{dn_1} = r_{ds_2} = \frac{1}{\lambda_2 I_{D_2}} = \frac{1}{\frac{0.04\,\mu m/V}{2\,\mu m} \times 8.6\,\mu A} = 5.81\,M\Omega$$

$$\boxed{r_{out_1} = 1.66\,M\Omega} \Rightarrow A_1 = g_{m_1} \cdot r_{out_1} = 54\,\mu S \times 1.66\,M\Omega$$

$$\boxed{A_1 = 89.8\ V/V}$$

$$r_{up_2} = r_{ds_5} = \frac{1}{\lambda_5 \cdot I_{D_5}} = 148.4\,k\Omega \ ; \ r_{dn_2} = \frac{1}{\lambda_6 I_{D_6}} = 371\,k\Omega$$

$r_{out_2} = 105.71\ k\Omega$

$A_2 = g_{m_5} r_{out_2} = 540\mu S \times 105.71\ k\Omega$

$\boxed{A_2 = 57.1\ V/V}$

$\boxed{A_0 = A_1 \cdot A_2 = 5127.58}\ V/V < 10^4\ V/V$

* ↑ L of PMOS devices so that $r_{up_1} = r_{dn_1}$ & $r_{up_2} = r_{dn_2}$

$\Rightarrow r_{out_1} = \frac{r_{dn_1}}{2} \approx 2.9 M\Omega$; $r_{out_2} = \frac{r_{dn_2}}{2} \approx 185.5\ k\Omega$

$\Rightarrow A_1 = 156.6$; $A_2 = 100.2 \Rightarrow \boxed{A_0 = 15{,}686\ V/V}$ ✓

$$L_{3,4,5} = \frac{(\lambda L_{min})_p}{(\lambda L_{min})_n} \cdot L_{min} = \frac{0.1}{0.04} \times 2 = 5\,\mu m$$

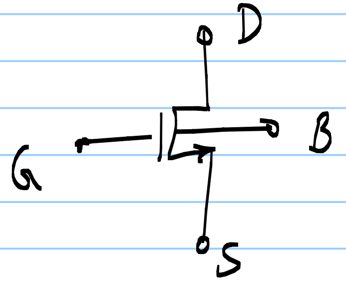
$$\Downarrow$$

$$A_o = 15,186 \ V/V$$

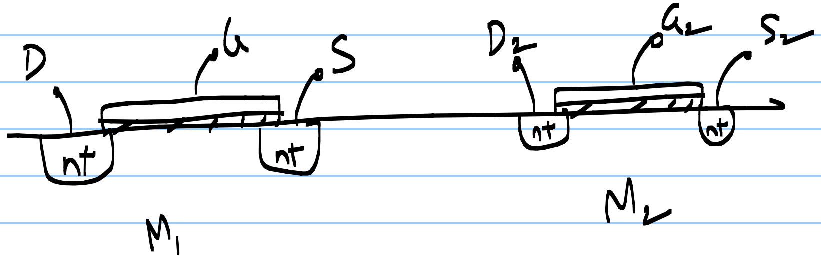
* Choose slightly shorter $L_{3,4,5}$ if parasitic cap is a concern

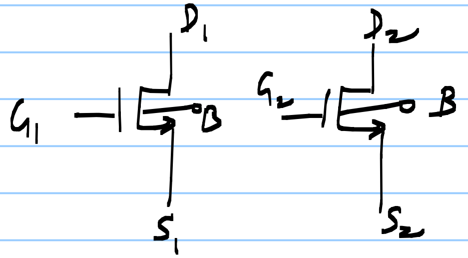
$$\begin{tikzcd} & W/L \arrow[r] & \text{small} \\ g_m \arrow[ur] \arrow[dr] & & \\ & I \arrow[r] & \text{large} \end{tikzcd}$$

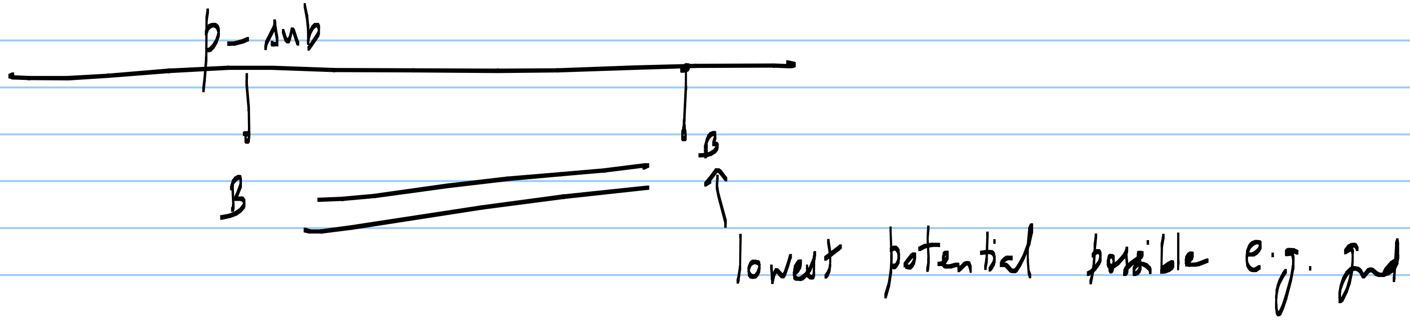
Body Effect

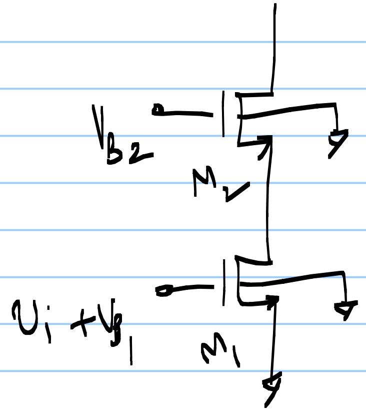


$$V_T = V_{T_0} + \gamma \left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right]$$



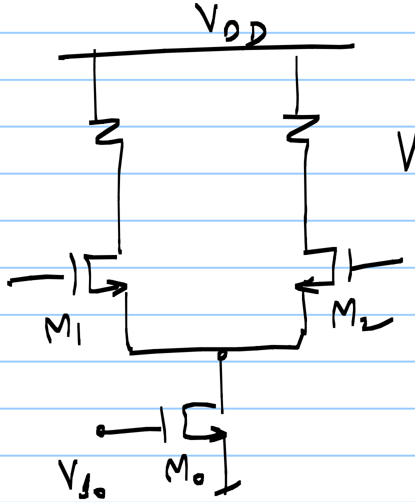




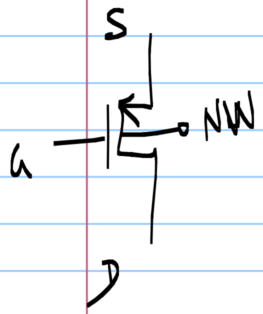


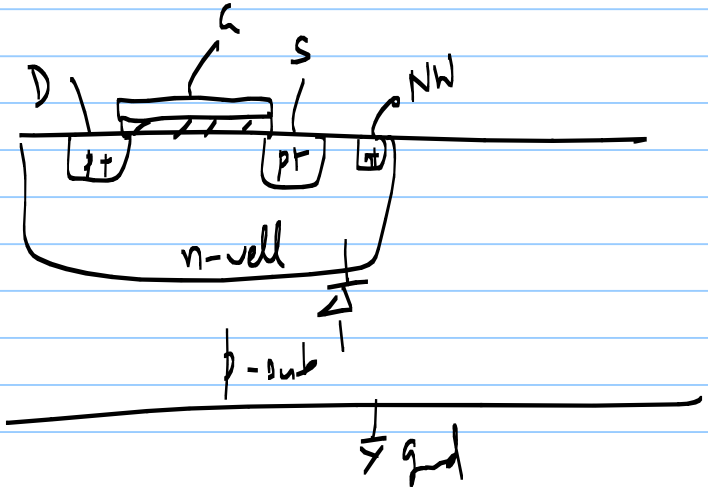
$V_{T_1} = V_{T0}$

$V_{T_2} > V_{T0}$ $(V_{S_2B} > 0)$

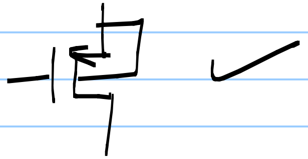


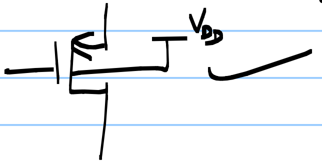
$V_{T_{1,2}} > V_{T0}$





$V_{nwell} > 0V$ → well-sub diode is reverse biased.





$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T\right)^2 \quad \left(1+\lambda V_{DS}\right)$$

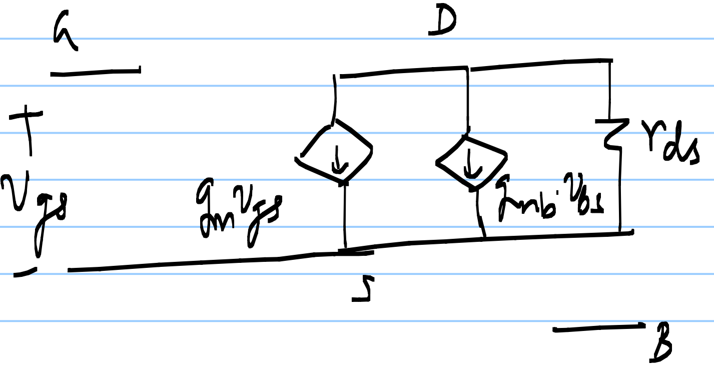
g_m

r_{ds}

$$\frac{\partial I_D}{\partial V_{SB}} = -g_m \cdot \frac{\partial V_T}{\partial V_{SB}} = -\alpha g_m$$

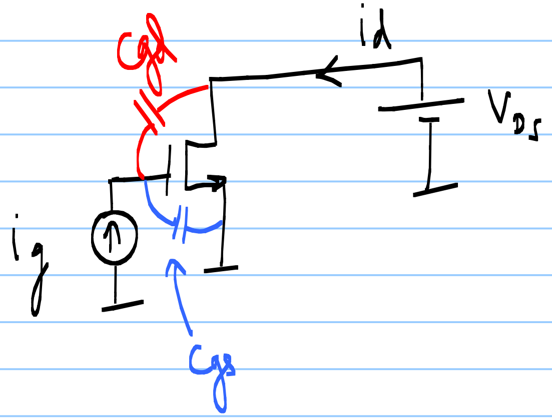
$$\alpha < < 1$$

$$\frac{\partial I_D}{\partial V_{BS}} = \alpha g_m = g_{mb} \text{ or } g_{mbs}$$



Transition Frequency (ω_T or f_T)

* freq. at which "current gain" of device = 1 or 0dB



freq. at which $\left|\frac{i_d}{i_g}\right| = 1$

$$v_{gs} = \frac{i_g}{sC_{gs}} \quad ; \quad i_d = g_m v_{gs} = \frac{g_m i_g}{sC_{gs}}$$

$$\frac{i_d}{i_g}(\omega) = \frac{g_m}{j\omega C_{gs}} \Rightarrow \boxed{\omega_T = \frac{g_m}{C_{gs}}} = \frac{g_m}{C_{gs}+C_{gd}}$$

$$\omega_T \approx \frac{g_m}{C_{gs}} = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)}{\frac{2}{3} W \cdot L \cdot C_{ox}}$$

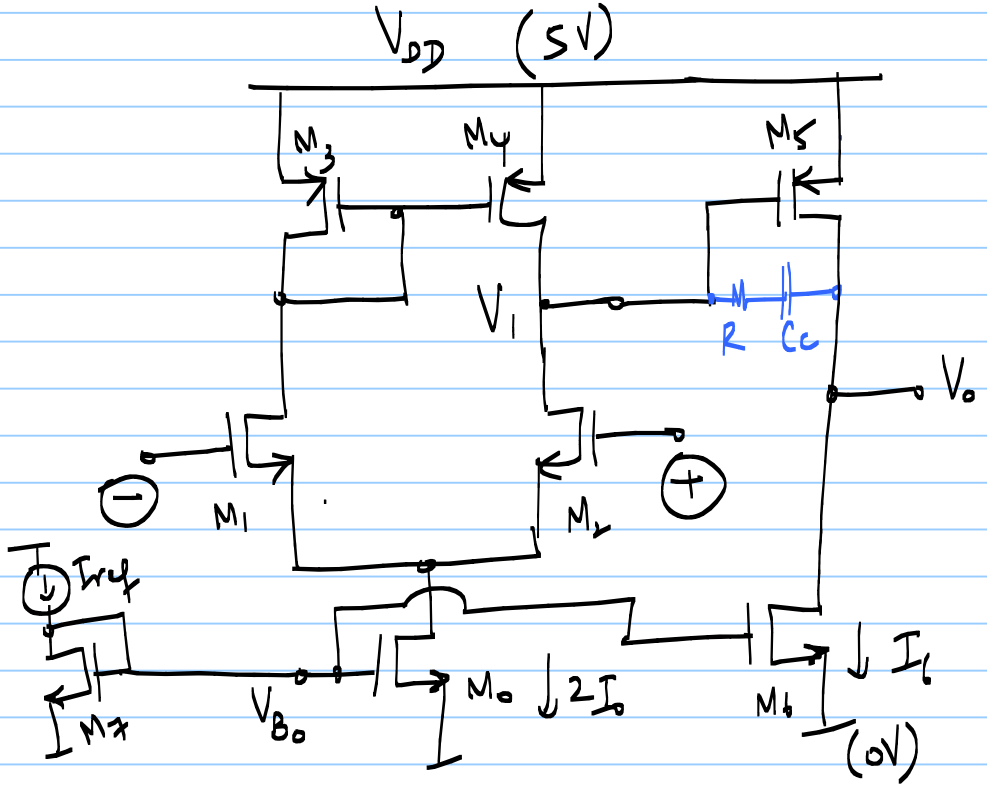
$$\boxed{\omega_T \approx \frac{3}{2} \frac{\mu_n \cdot (V_{GS} - V_T)}{L^2}}$$

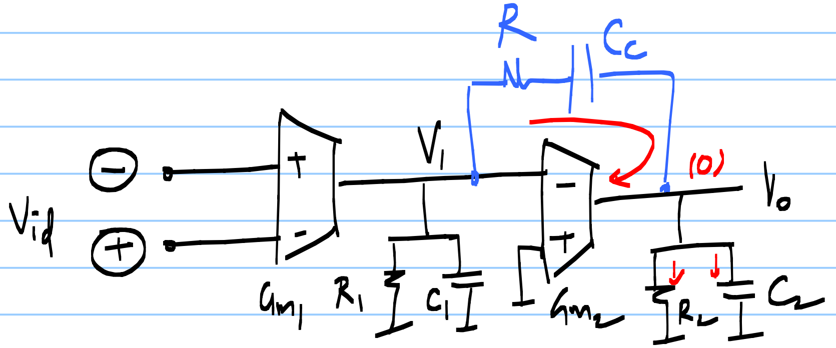
→ keep $L = L_{min.}$ (low intrinsic gain)

→ keep $(V_{GS} - V_T)$ = large {low swing limits}

i.e. $\frac{I_D}{W}$ = large

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)$$



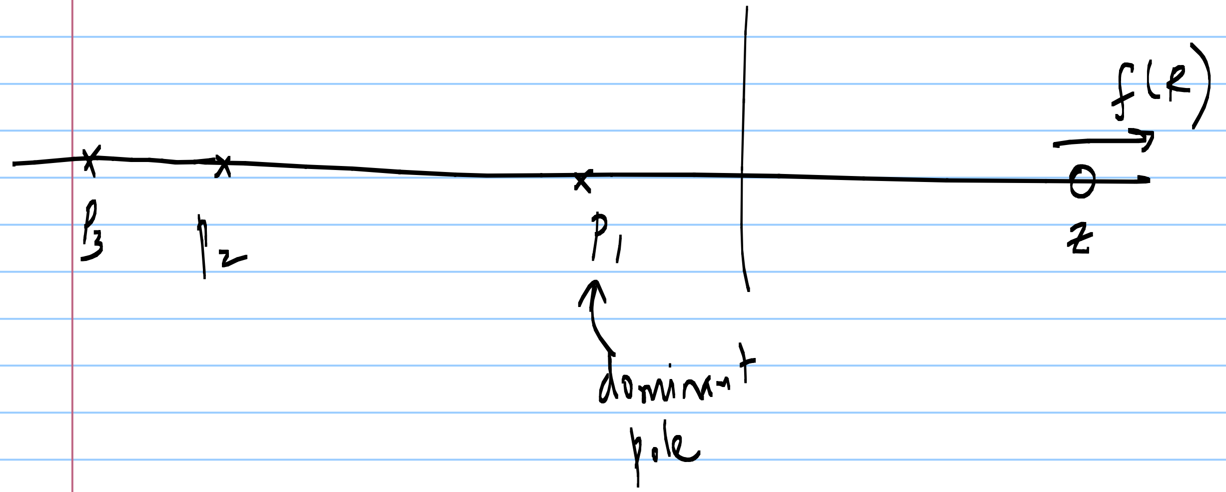


$$Z = \frac{G_{m2}}{C_c} \rightarrow f(R)$$

↓

$$\omega_u = \frac{G_{m_1}}{C_c}$$

R changes phase/freq. resp. of I_{C_c}



$$\boxed{\text{HW: find out } \frac{V_o}{V_{id}}(s)}$$

$$\frac{V_o}{V_{id}}(s) = \frac{N(s)}{D(s)} \begin{array}{l} \leftarrow \text{degree } 1 \\ \leftarrow \text{degree } 3 \end{array}$$

$$p_1 \simeq \frac{-1}{R_1(G_{m2}R_2)C_c}$$

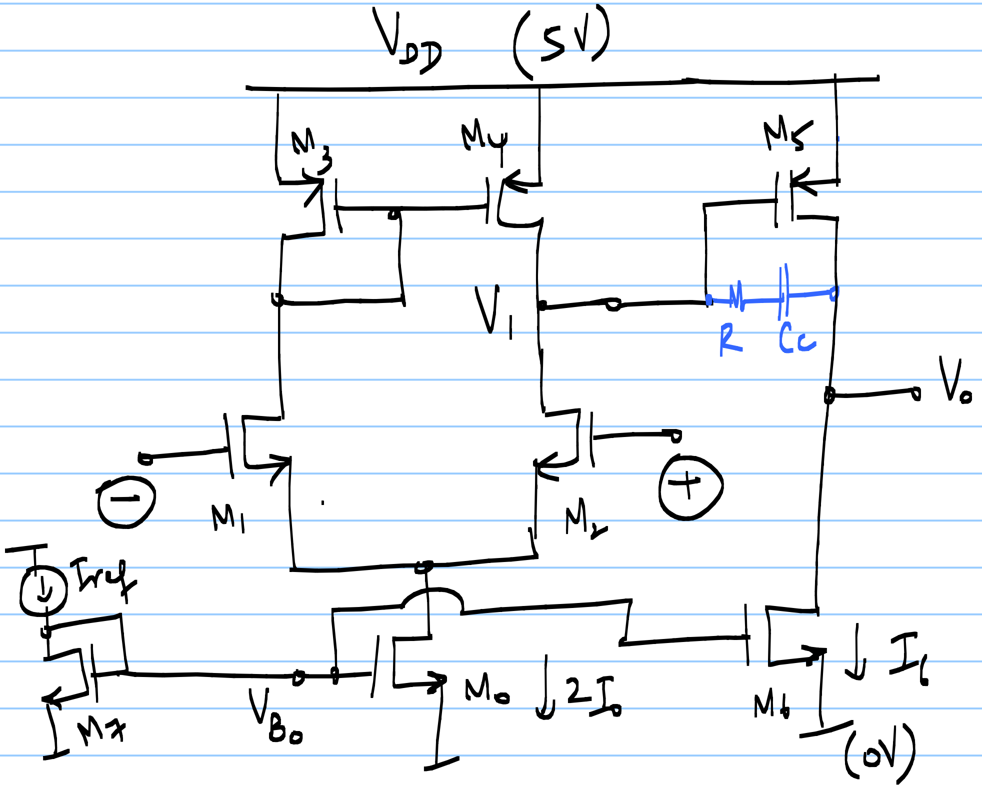
$p_2 = ?$ $\boxed{\text{HW}}$ $\qquad p_3 = ?$

$z = ? \rightarrow G_{m2} \cdot V_1 = V_1 \cdot \frac{1}{R + \frac{1}{sC_c}}$

$$\boxed{z = \frac{1}{C_c\left(\frac{1}{G_{m2}} - R\right)}}$$

16/11/23

Lec 21



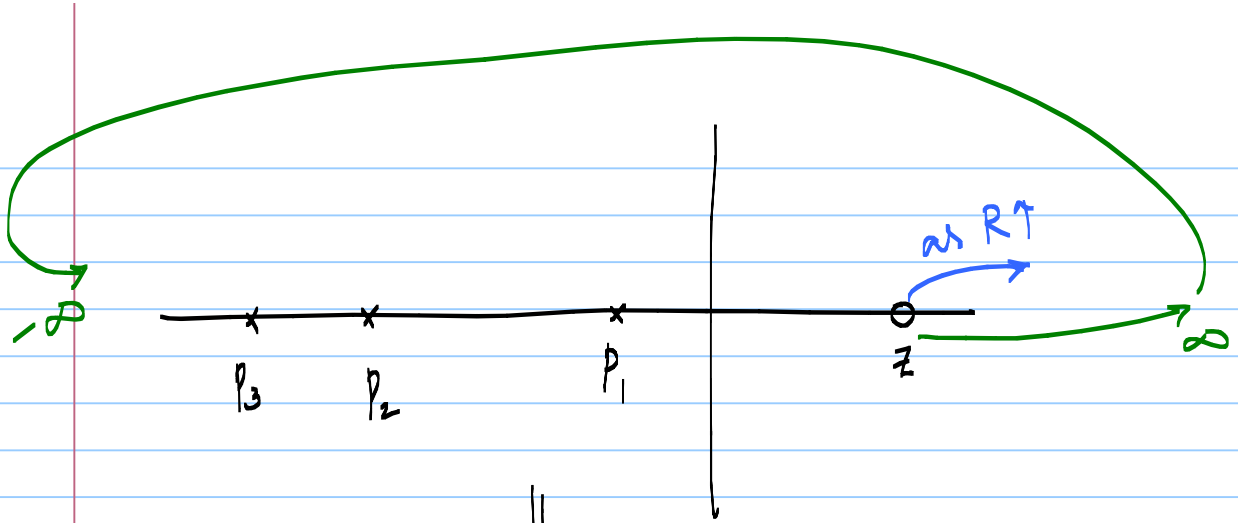
$$p_1 \approx \frac{-1}{R_1 \cdot (G_{m_2} R_2) C_c} = p_d$$

$$Z = \frac{+1}{C_c\left(\frac{1}{G_{m_2}} - R\right)} = f(R)$$

$$p_2 \approx \frac{-G_{m_2}}{(C_1 + C_2 + \ldots)} = f(G_{m_2})$$

$$p_3 \approx \frac{-1}{R(C_1 \| C_2)}$$

$$\omega_u = \frac{G_{m1}}{C_c}$$

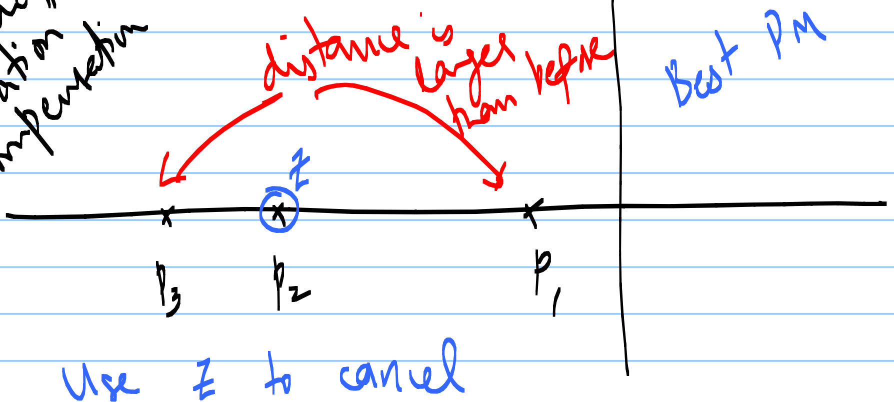


as $R\uparrow \rightarrow \left(\frac{1}{G_{m_2}} - R\right)$ reduces

1) Set $R = \frac{1}{G_{m_2}} \Rightarrow Z = \infty$

↳ p_3 limits phase margin (apart from p_2)

"Pole-zero cancellation" compensation



2) Set $R > \frac{1}{G_{m_2}} \rightarrow Z$ moves into LHP (from $-\infty$)

Use Z to cancel first ND pole ⟶ $R = f(G_{m_2}) = f(g_{m5})$

$p_2 = f(g_{m5})$

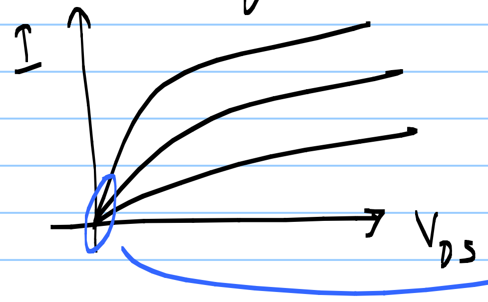
You want $R = f(g_{m5})$

→ MOSFET

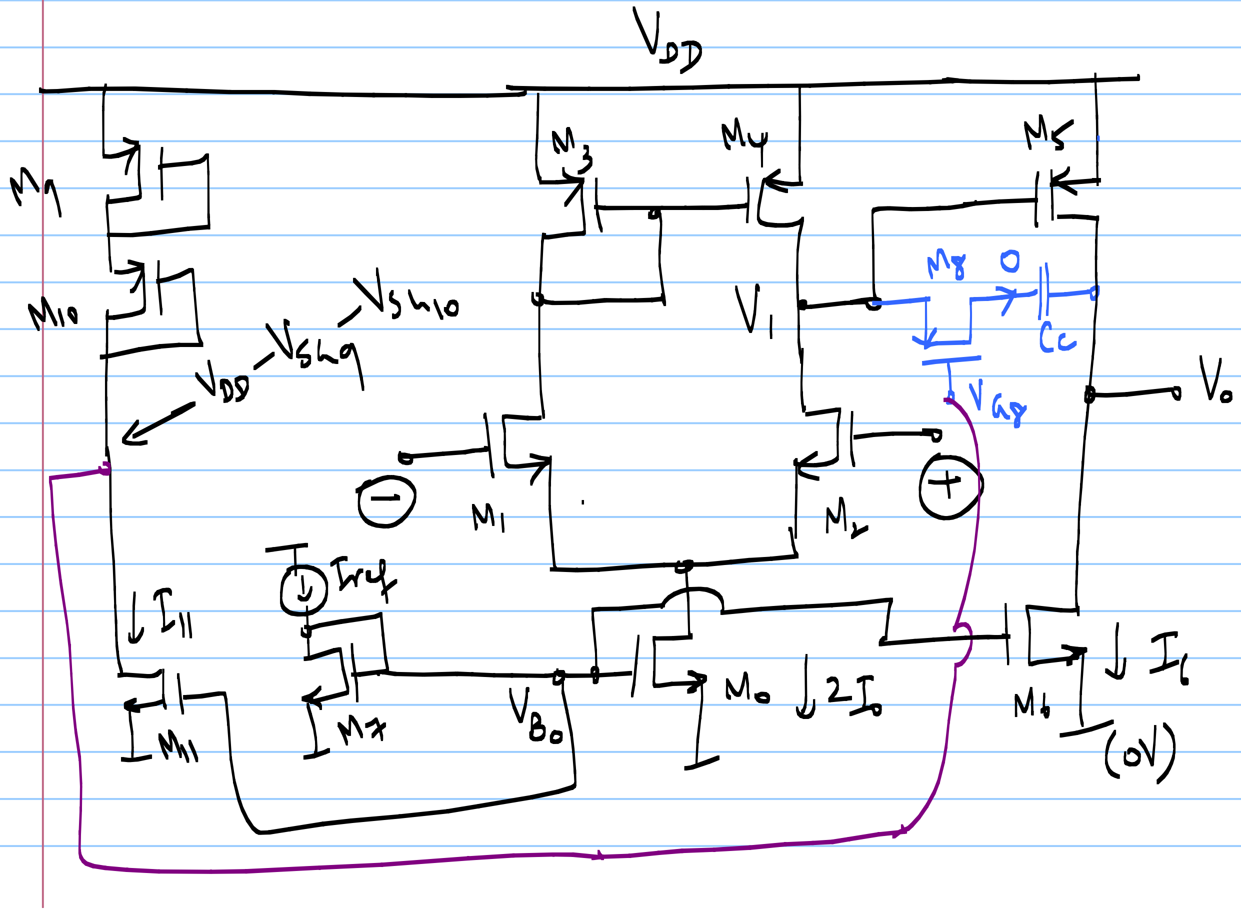
→ Passive resistor

} $p_2 = z$ cannot be guaranteed over PVT variations

1) Use a MOSFET in triode regim to create R



$$g_{ds_0} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\text{origin}}$$



$I_{DS_8} = 0$

Need: $V_{G_8} = V_{DD} - V_{SG_5} - V_{SG_8}$

||

Set $V_{G_8} = V_{DD} - V_{SG_9} - V_{SG_{10}}$

* Set $\dfrac{I_{11}}{(W/L)_9} = \dfrac{I_6}{(W/L)_5}$

$\Rightarrow V_{SG_9} = V_{SG_5}$

$\Rightarrow \boxed{V_{SG_8} = V_{SG_{10}}}$

Issue: $M_8 \Rightarrow$ linear

$M_{10} \Rightarrow$ saturation

$$I_{D_8} = \mu_p C_{ox} \left(\frac{W}{L}\right)_8 \left[\left(V_{SG_8} - V_{Tp}\right)\left(V_{SD_8}\right) - \frac{V_{SD_8}^{2}}{2} \right]$$

$$g_{ds_8}\Big|_{V_{SD}=0} = \mu_p C_{ox} \left(\frac{W}{L}\right)_8 \left[V_{SG_8} - V_{Tp} - V_{SD_8} \right]\Bigg|_{V_{SD}=0}$$

$$g_{ds_8} = \mu_p C_{ox} \left(\frac{W}{L}\right)_8 \left(V_{SG_8} - V_{Tp}\right) = \underbrace{\mu_p C_{ox} \left(\frac{W}{L}\right)_8 \left(V_{SG_{10}} - V_{Tp}\right)}_{\propto g_{m_{10}}}$$

$\Rightarrow g_{ds_8} \propto g_{m_{10}}$

$g_{m_{10}} \propto g_{m_5}$ $\{ I_{11} \propto I_6 \}$

$$\left.\begin{array}{l} \Rightarrow g_{ds_8} \propto g_{m_{10}} \\ g_{m_{10}} \propto g_{m_5} \end{array}\right\} \quad \boxed{g_{ds_8} \propto g_{m_5}}$$

$$\boxed{R \propto \frac{1}{g_{m_5}}}$$

Set R

so that

$z = 1/2$

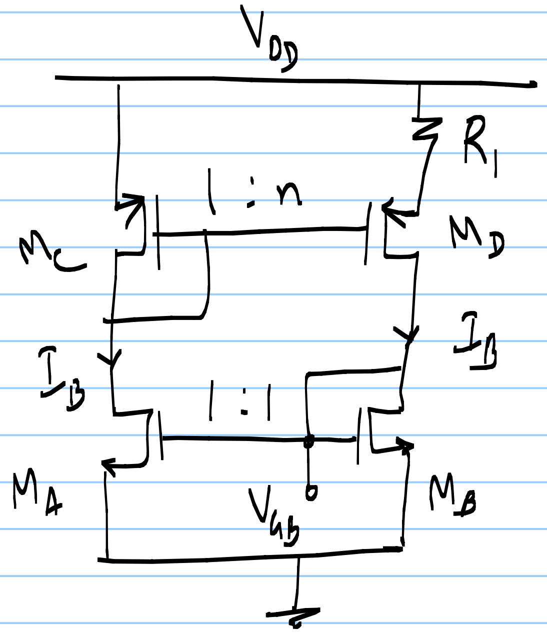
2) Use a passive Resistor R

Ensure that g_{m_5} tracks $\frac{1}{R}$

$$g_{m_5} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_5 I_{D_5}}$$

$$\begin{tikzcd} & I_{D_5} & \propto \frac{1}{R^2} \arrow[l] \\ g_{m_5} \arrow[ur] \arrow[dr] & & \\ & \left(\frac{W}{L}\right)_5 & \text{fixed} \arrow[l] \end{tikzcd}$$

"constant-g_m" reference



$$M_A : M_B = 1 : 1 \Rightarrow I_{DA} = I_{DB} = I_B$$

$$= I_C = I_D$$

$$\left(\frac{W}{L}\right)_D = n \left(\frac{W}{L}\right)_C$$

$$\left(V_{SG_C} - V_{Tp}\right) = \sqrt{n}\left(V_{SG_D} - V_{Tp}\right)$$

$$V_{SG_C} = V_{SG_D} + I_B \cdot R_1$$

$$\left(V_{SG_C} - V_{Tp}\right) = \left(V_{SG_D} - V_{Tp}\right) + I_B R_1$$

$$\frac{2I_B}{g_{m_c}} = \frac{1}{\sqrt{n}} \cdot \frac{2I_B}{g_{m_c}} + I_B R_1 \longrightarrow I_B = 0 \text{ is one solution}$$

This circuit needs a startup circuit

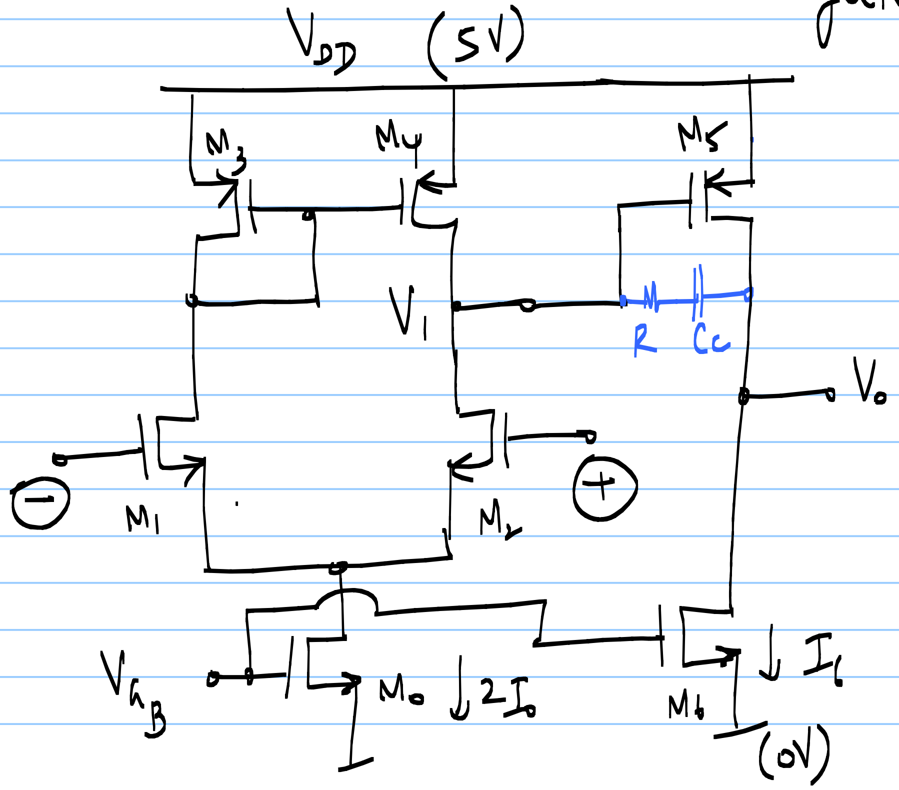
If $I_B \neq 0$:

$$\frac{2}{g_{mc}} = \frac{2}{\sqrt{n}\, g_{mc}} + R_1$$

$$\frac{2}{g_{mc}}\left(1 - \frac{1}{\sqrt{n}}\right) = R_1 \Rightarrow \boxed{g_{mc} = \frac{2}{R_1}\left(1 - \frac{1}{\sqrt{n}}\right)}$$

$$g_{mc} \propto \frac{1}{R_1}$$

* Use this to bias the op amp



gate of M_B → biases M_0 & M_6

1st stage

2nd stage

↳ $I_6 \propto I_B \propto \frac{1}{R_1^2}$

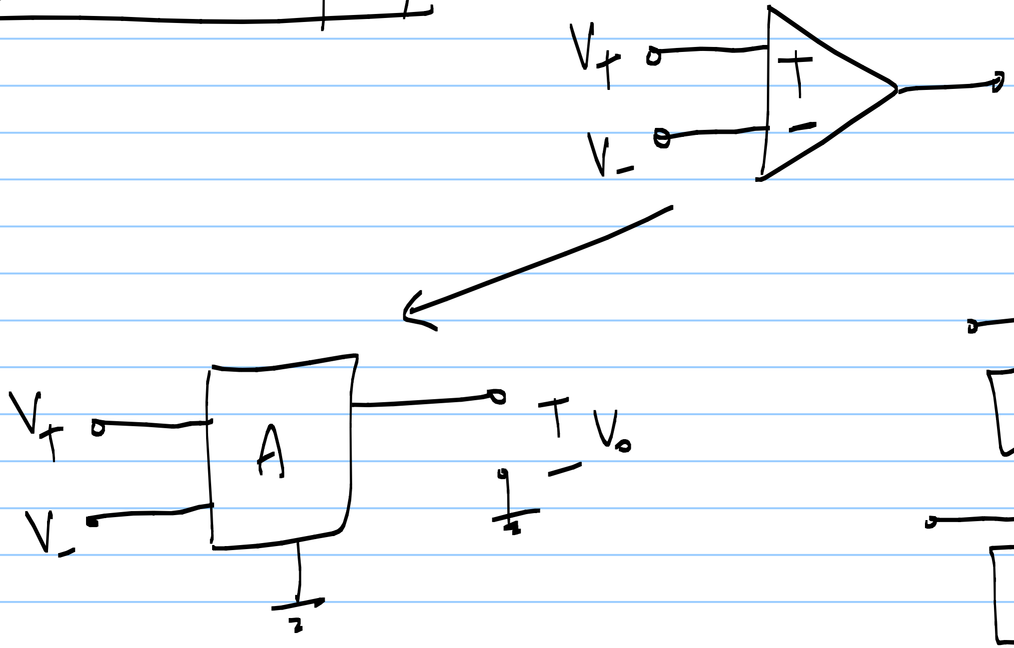
$g_{m5} \propto \frac{1}{R_1}$

set R & R_1 to be same type of resistor

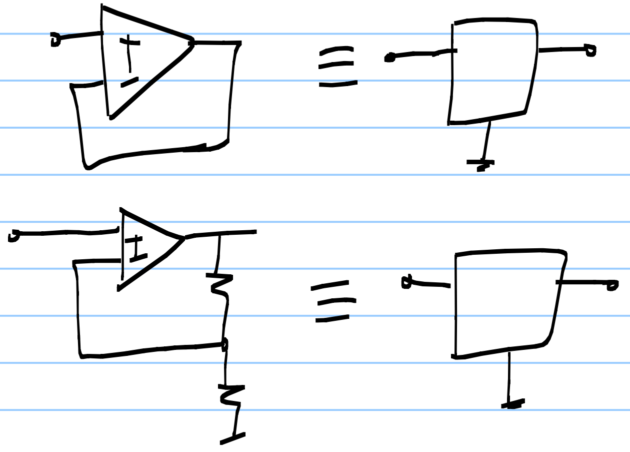
21/11/2023

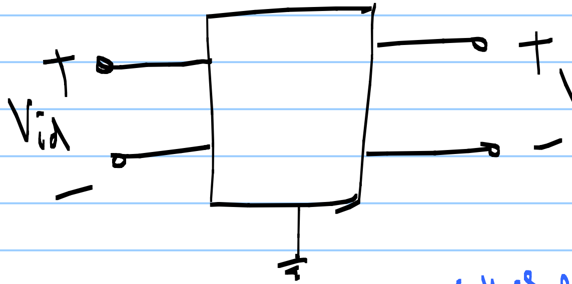
Lec 22

Fully Differential Opamps



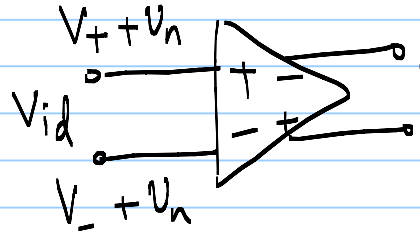
$$V_o = A(V_+ - V_-)$$





$V_{od} = A(V_{id})$

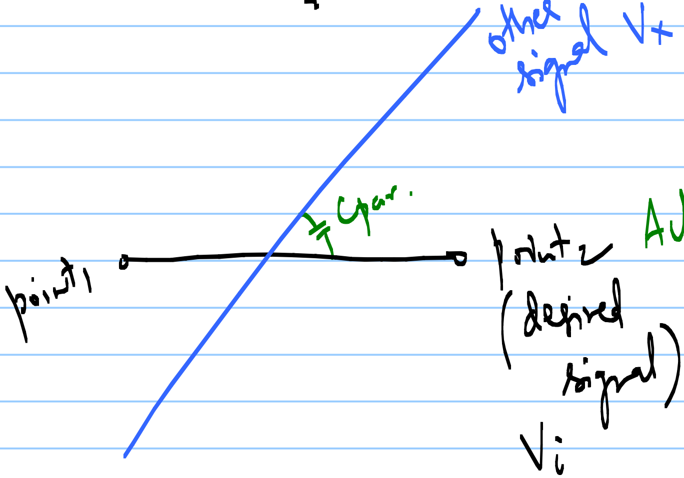
1)



$V_{od} = A V_{id}$

$= A(V_+ - V_-)$

CM noise immunity



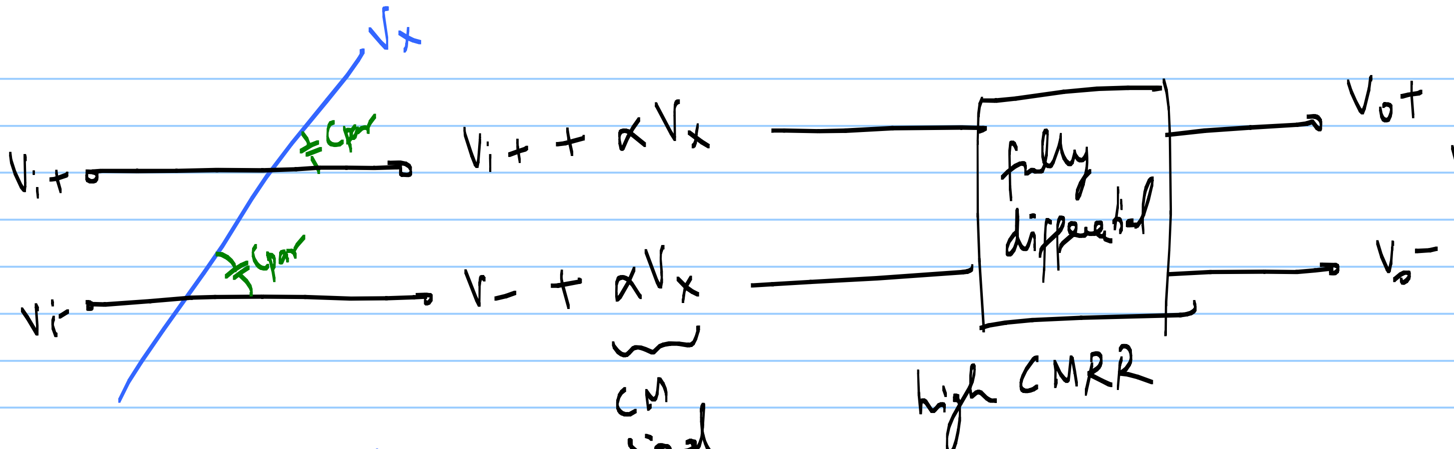
Actual voltage

$= V_i + \underbrace{\alpha V_x}_{\text{unwanted}}$

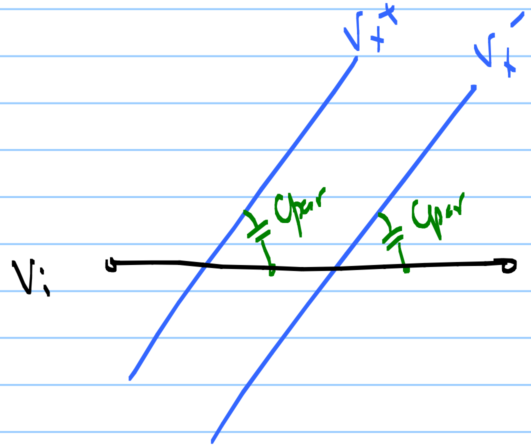
2) Not referred explicitly to gnd

(V_{i+}, V_{i-}, V_{o+}, V_{o-})

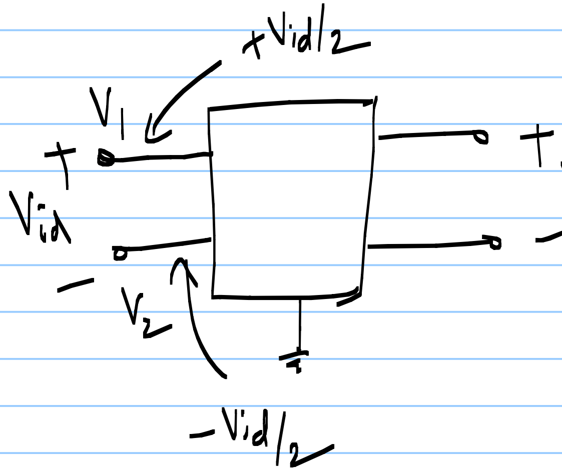
(processed differentially)



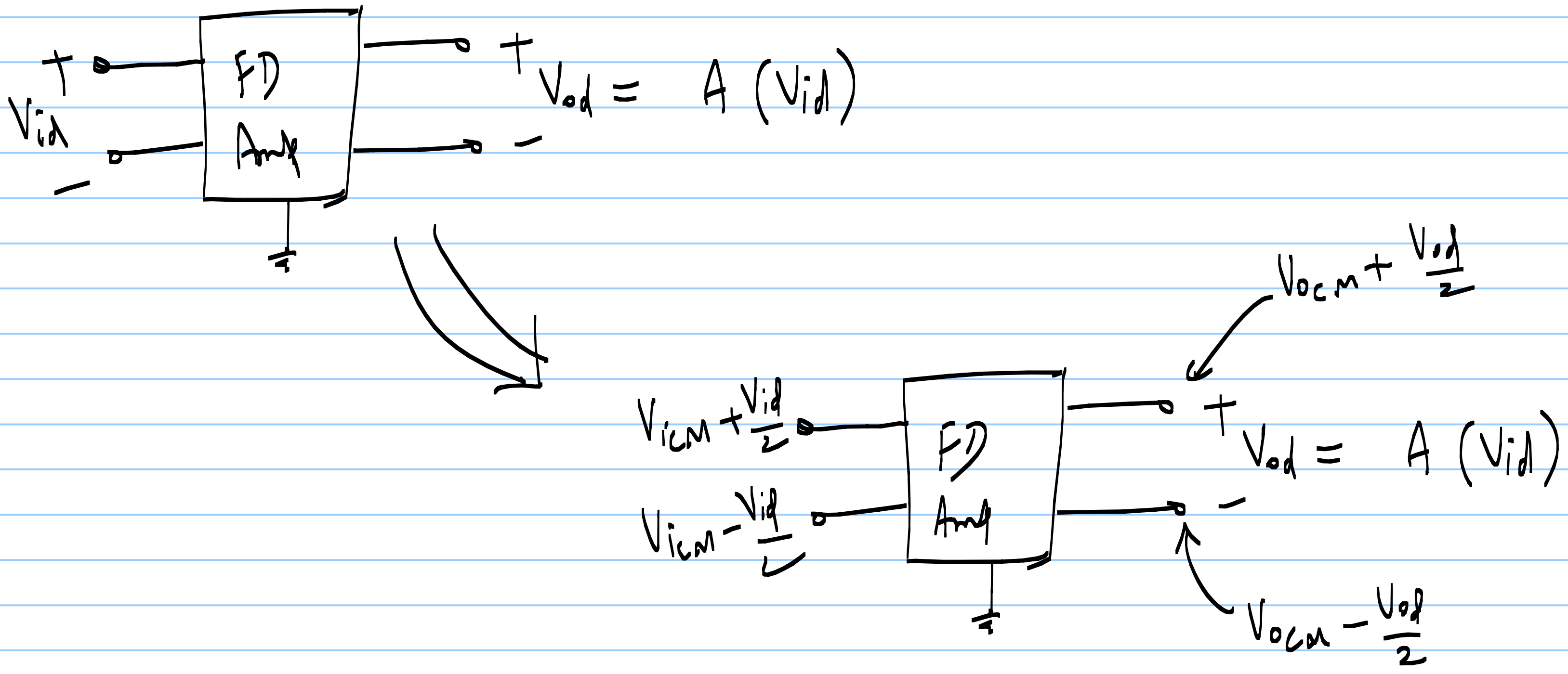
$V_{od} = A(V_{id})$

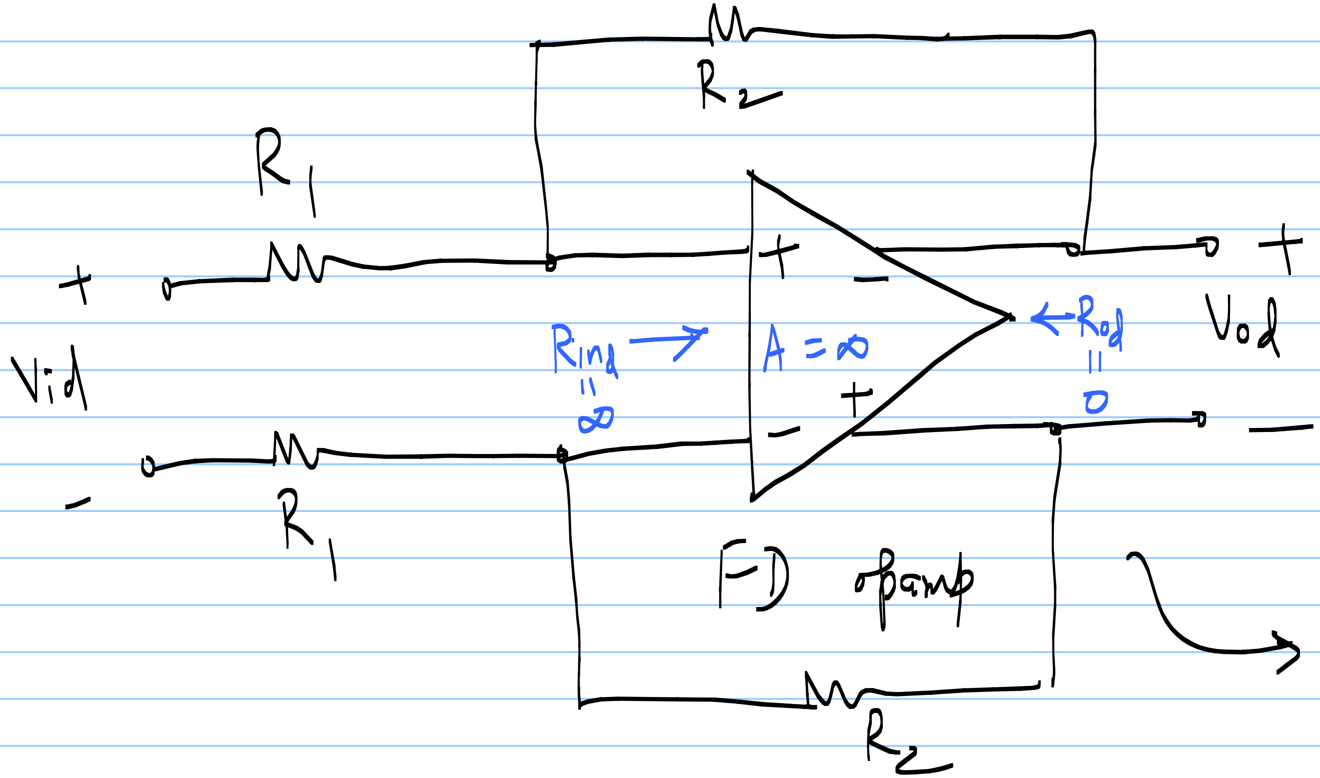


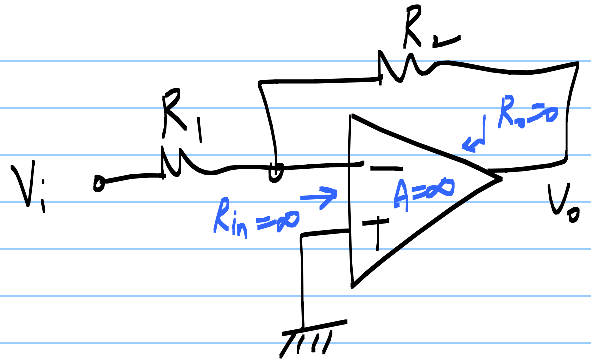
fully



$V_{od} = A(V_{id})$

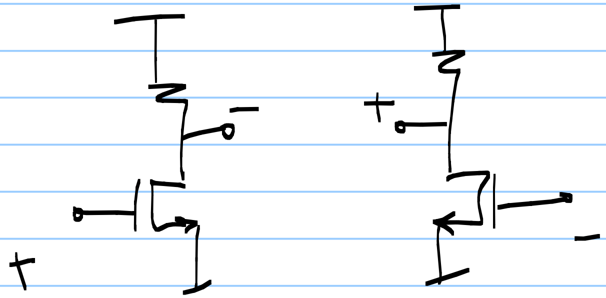

+
V_{id}
−
FD Amp
+
$V_{od} = A(V_{id})$
−
$V_{ocm} + \frac{V_{od}}{2}$
$V_{icm} + \frac{V_{id}}{2}$
$V_{icm} - \frac{V_{id}}{2}$
FD Amp
+
$V_{od} = A(V_{id})$
−
$V_{ocm} - \frac{V_{od}}{2}$



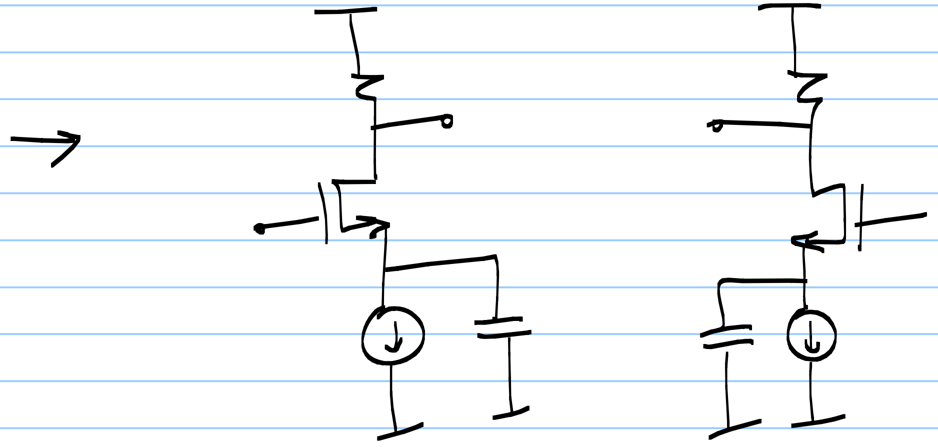


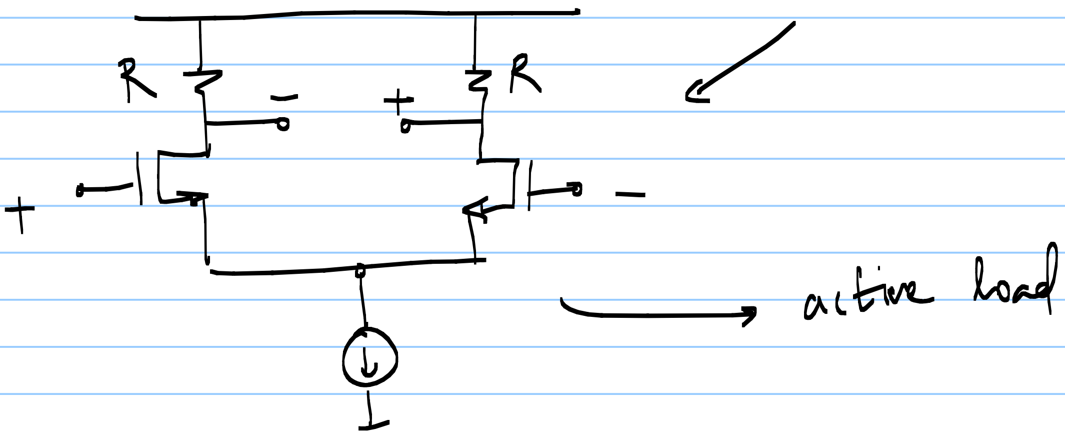
$$\frac{V_o}{V_i} = \frac{-R_2}{R_1}$$

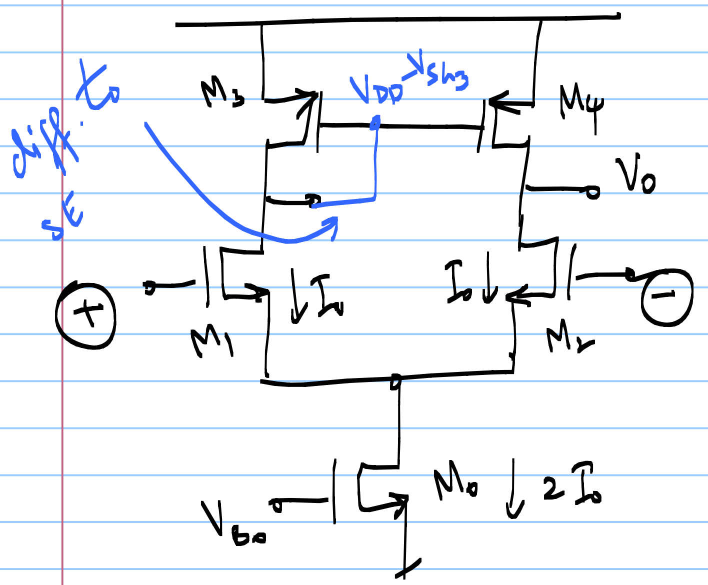
$$\frac{V_{od}}{V_{id}} = \frac{-R_2}{R_1}$$



pseudo differential amplifier

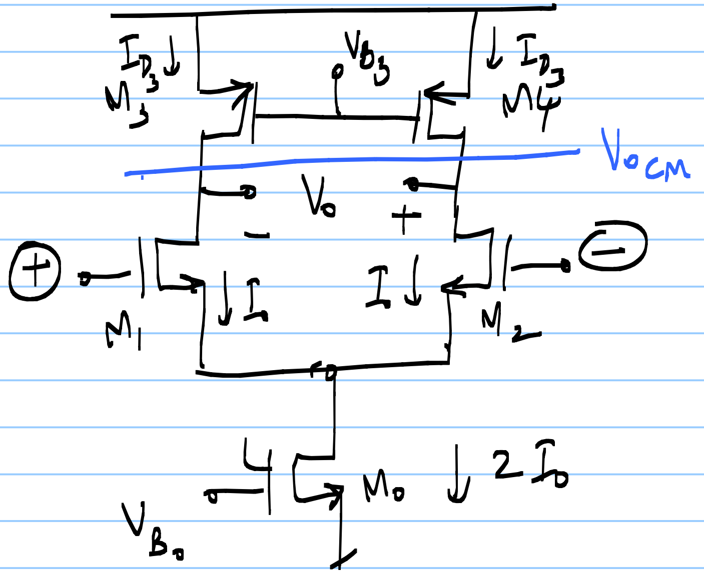






one-stage opamp

devices can easily move out of saturation



fully differential one-stage opamp

1) Set V_{B_3} so that $I_{D_3} = I_0$

2) $V_{OCM} = ?$

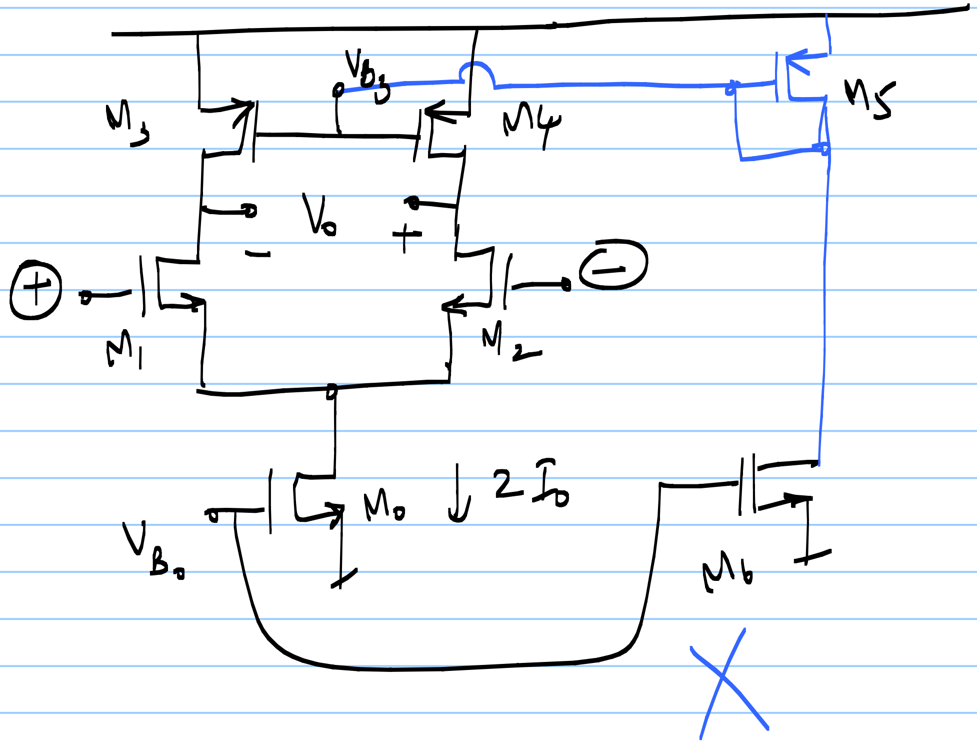
undefined

3) What happens if V_{B_3} varies, or $V_{T_{3,4}}$ changes etc.?

$\Rightarrow I_{D_3} \neq I_0$

$I_{D_3} > I_0 \Rightarrow V_{OCM} \uparrow$

$I_{D_3} < I_0 \Rightarrow V_{OCM} \downarrow$



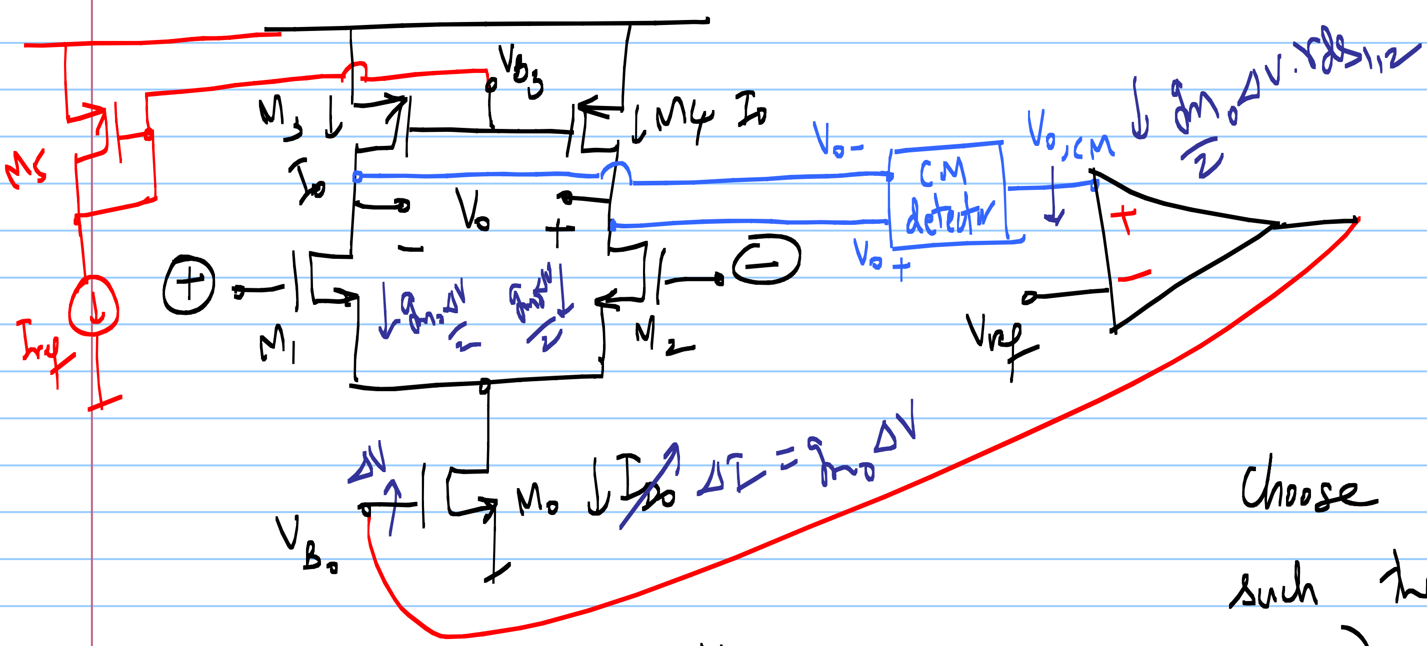
- V_{DS} mismatches are still an issue
- $V_{o,cm}$ is still ill-defined

⇒ Set $V_{o,cm}$ so that all devices are in saturation

↳ Negative f.b. → 1) Sense $V_{o,cm}$

2) Compare w/ $V_{o,cm}$ desired

3) Drive actual $V_{o,cm}$ → $V_{o,cm}$ (desired)



Set V_{B3} independently
Set V_{B0} via neg. fb. } ✓

$$V_{o+} = V_{o,CM} + \frac{V_{od}}{2}$$

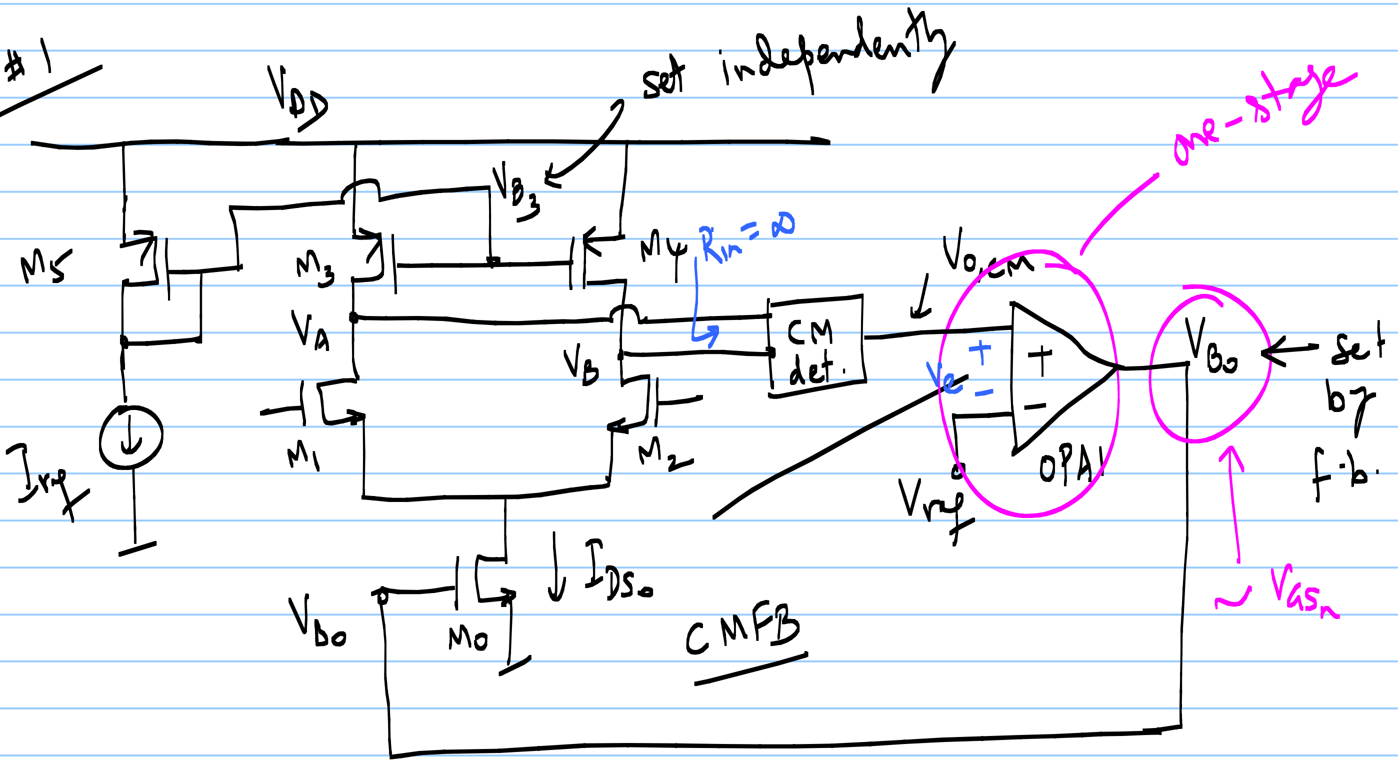
$$V_{o-} = V_{o,CM} - \frac{V_{od}}{2}$$

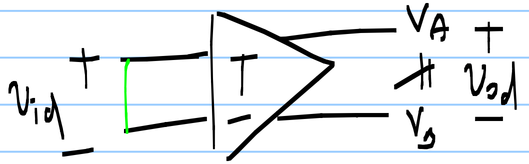
Choose $V_{o,cm}$ (desired) = Vref
such that all devices are in sat. AND
e.g. 1) Max output signal swing
2) Set by V_{icM} of next stage

23/11/2023

Lec 23

Circuit # 1





* CMFB

= common-mode feedback

* Steady State:

$V_{o,CM} = V_{ref}$

* errors → finite A_o

finite V_{os}

* ICMR of OPA1 needs to include V_{ref} by design

→ $V_{Bo} \sim V_{GSn}$

→ OPA1 OCMR needs to cover $\sim V_{GSn}$ (low)

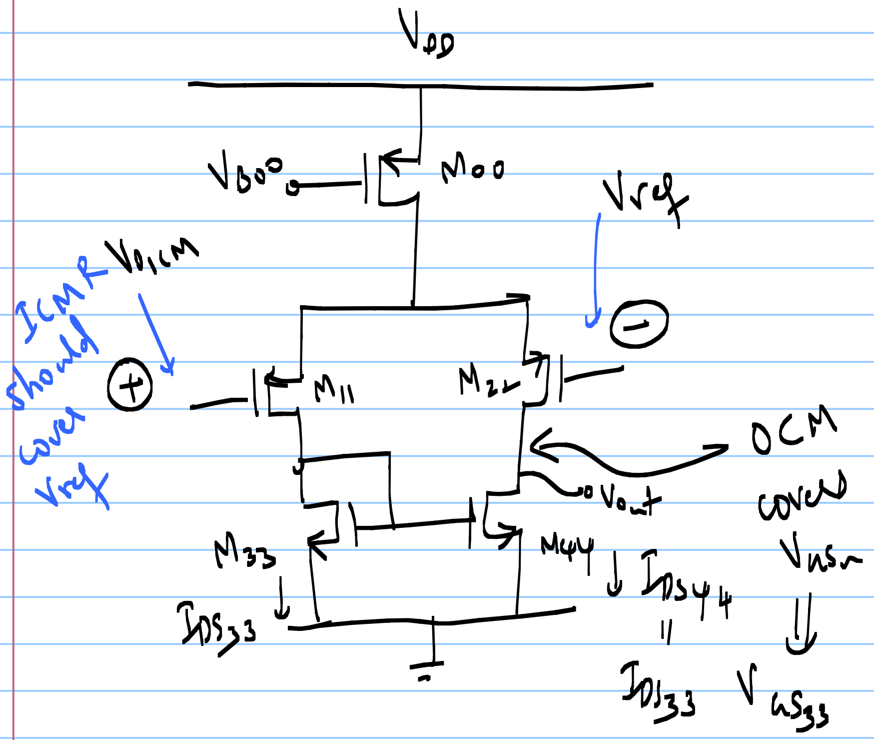
* OPA1 → one stage opamp → 2-stage loop → adequate gain in most cases

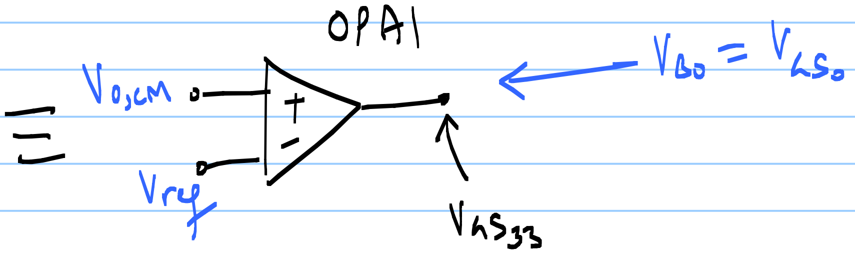
* OCM_{low} of OPA1 ⟺ ICM of OPA1

OPA1 with NMOS input pair → $OCM_{low} = ICM - V_{Tn}$

$$= \underbrace{V_{o,cm} - V_{Tn}}_{}$$

OPA1 with PMOS input pair ← { may not be low enough to cover V_{Bo}





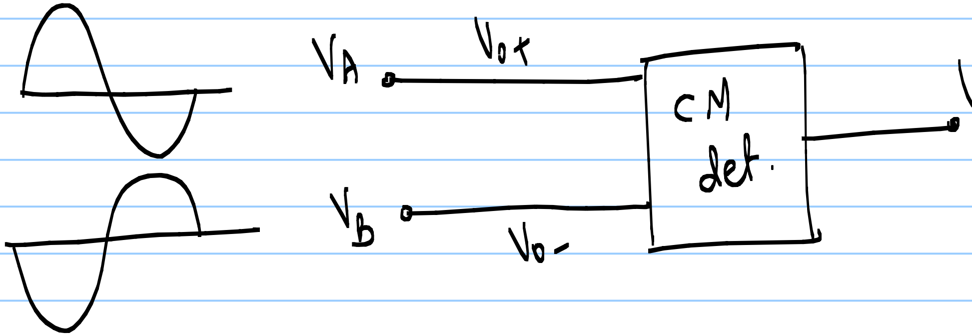
If $V_{GS_{33}} \neq V_{B_0}$

e.g. $V_{GS_{33}} < V_{B_0} \Rightarrow$ mismatch inside OPA1

* Design OPA1 so that: $V_{GS_{33}} = V_{GS_0} = V_{B_0}$

$\boxed{L_{33} = L_{44} = L_0}$; $\boxed{\frac{I_{DS_{33}}}{(W/L)_{33}} = \frac{I_{DS_0}}{(W/L)_0}}$

* CM detector



$V_{o,cm} = \frac{V_A + V_B}{2}$

1) Out = avg. of inputs

passive

2) Linear in the presence of FD swings @ V_A & V_B

active circuit using MOSFETs

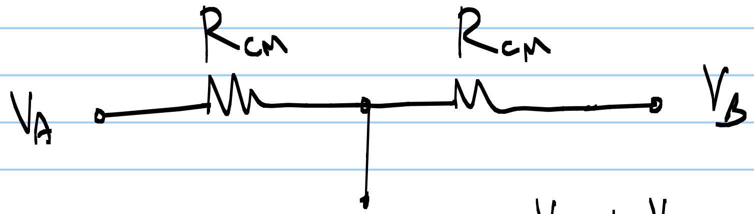
3) $Rin_{\text{CM det.}} = \infty$ so that $\frac{V_{od}}{V_{id}}$ of FD opamp is not affected

Circuit #1 : FD one-stage opamp

- * $\frac{V_{od}}{V_{id}}$ = gain from 1-stage
- * Don't want resistive loads ✓
- * V_A & V_B have large swings ✓

For circuit #1 :

1) Choose fully passive CM det.
2) Choose R_{in} as high as possible
3) Deal with effects of large R_{in}



$$V_{0,CM} = \frac{V_A + V_B}{2}$$

⇒ Resistive CM detector — —

perfectly linear

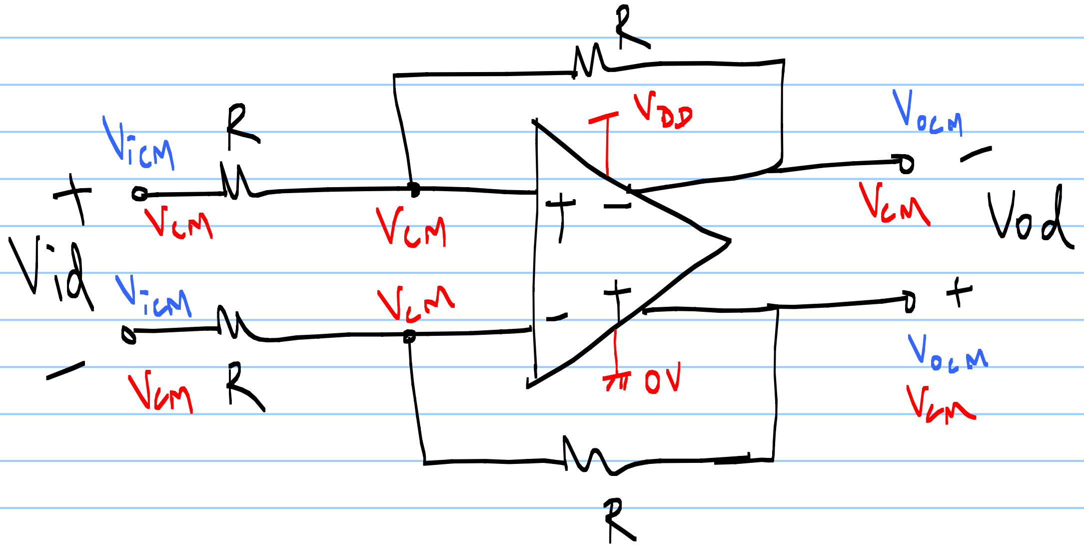
(if R's are linear)

R_{CM} as high as possible

e.g. $R_{CM} = 10 \times r_{out}$ of FD opamp

28/11/23

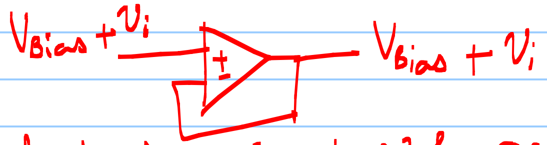
Lec 24



- * V_{icm}, V_{ocm} etc. need to be set carefully
- * Need to have all devices in saturation
- * Need to satisfy the opamp ICMR & OCMR

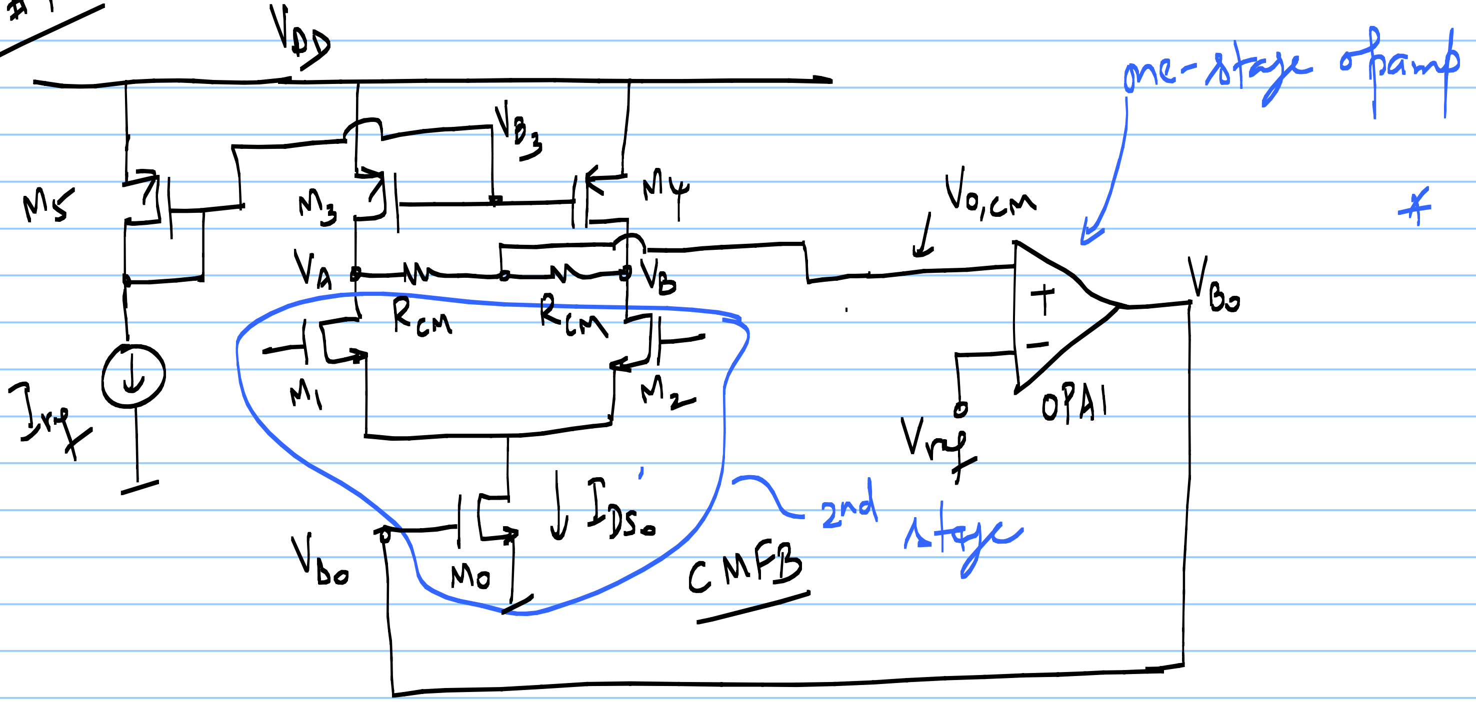
e.g. Choose $V_{icm} = V_{ocm} = V_{cm}$

V_{cm} needs to satisfy ICMR & OCMR

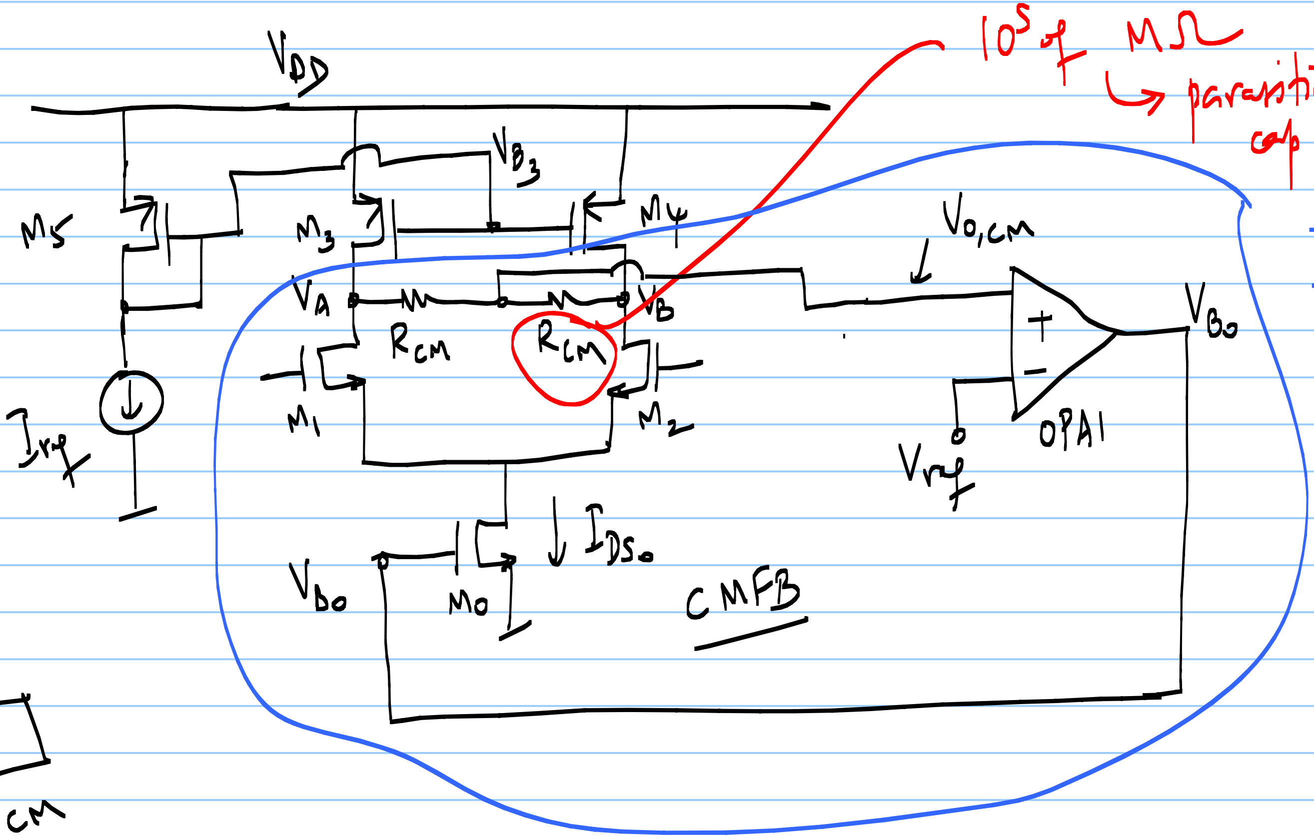


V_{Bias} needs to be in ICMR & OCMR

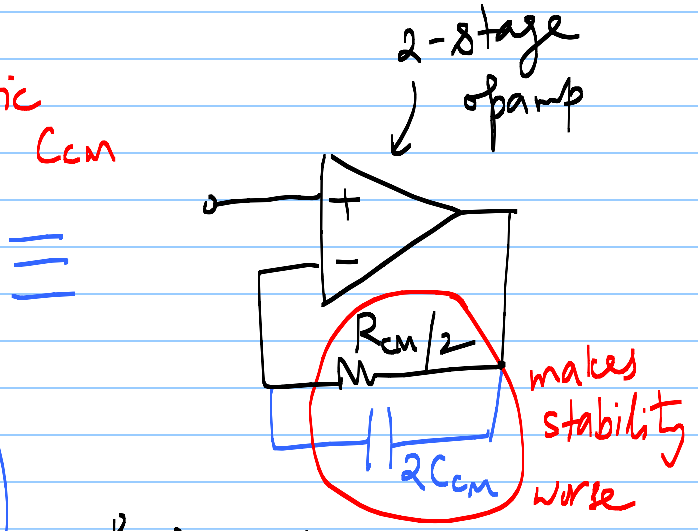
Circuit #1



* 2-stage loop

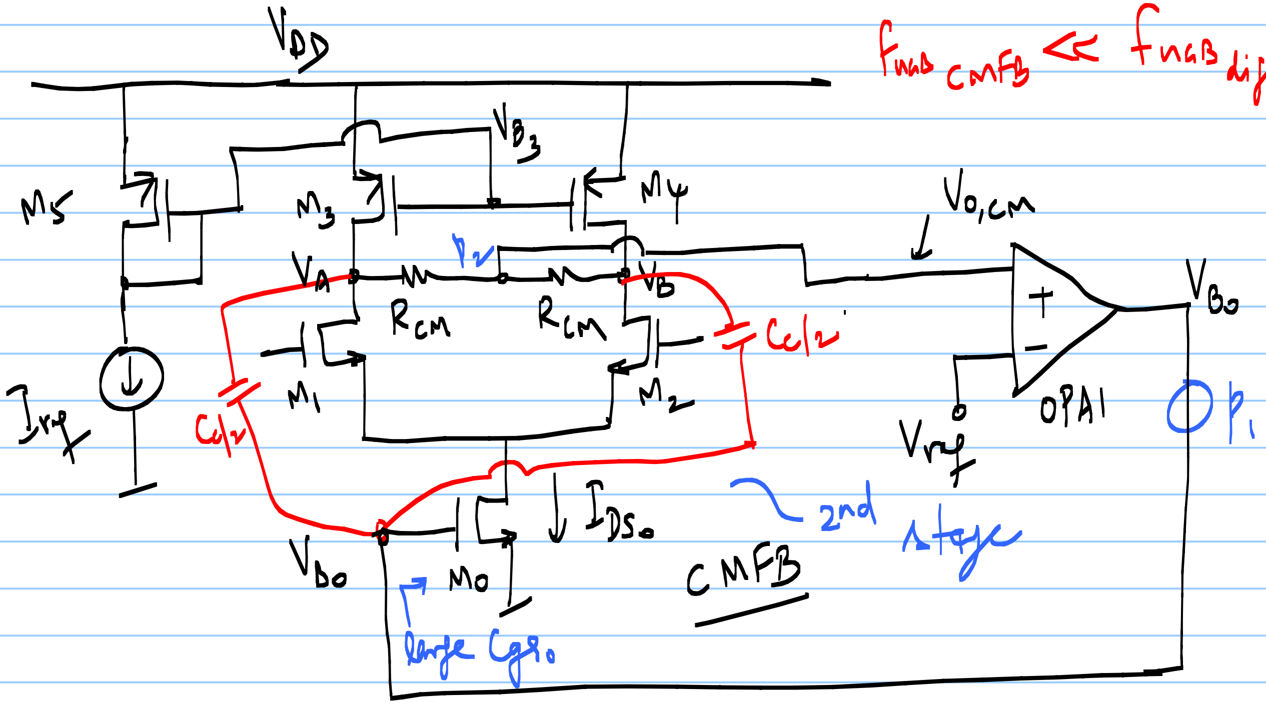


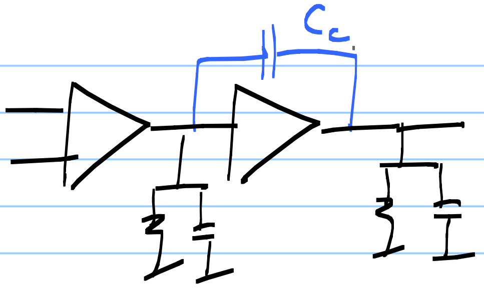
10s of MΩ ↳ parasitic cap Ccm



makes stability worse

$f_{UGB,CMFB} \ll f_{UGB,diff.}$





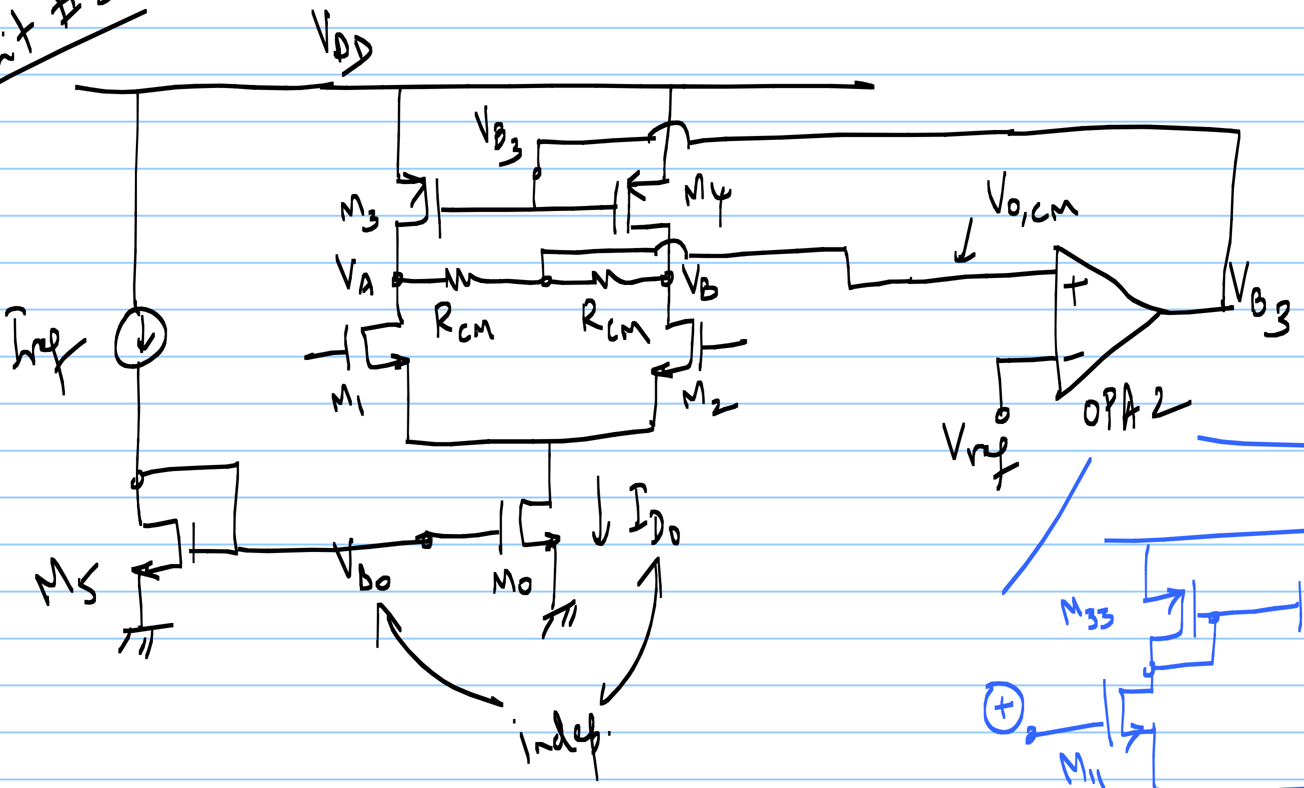
$p_1 \ll p_2$

$M_o \rightarrow$ large L due to 1) good CMRR 2) I_{DS_o} indep of V_{icm}

→ large W/L to achieve low $V_{icm,min}$

Choose C_c so that CMFB loop has good PM

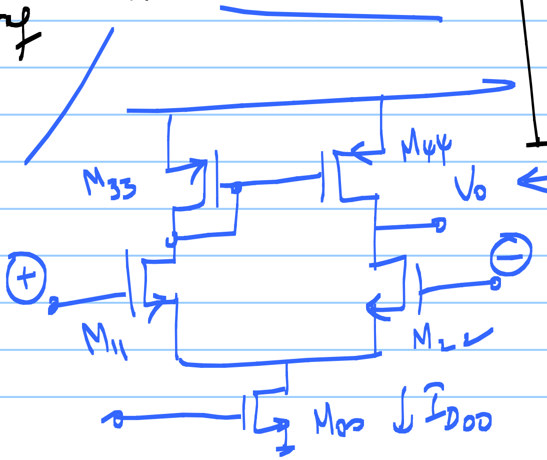
circuit #2



* $V_{o,cM} = V_{ref}$

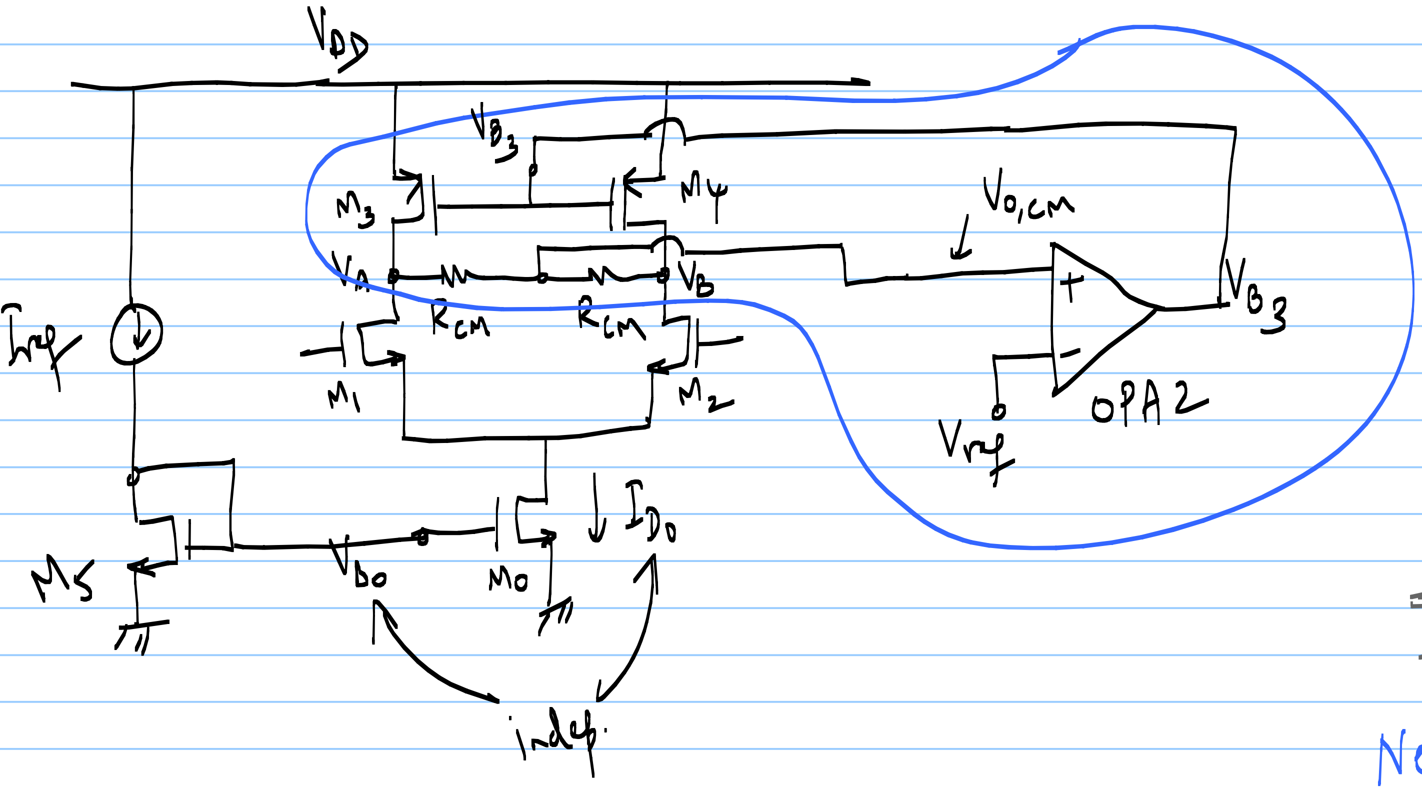
↳ $I_{D_3} = I_{D_4} = \frac{1}{2} I_{D_0}$

* $$\boxed{V_{SG_{33}} = V_{SG_{3,4}} \quad \hookrightarrow \frac{I_{D_{33}}}{(W/L)_{33}} = \frac{I_{D_{3,4}}}{(W/L)_{3,4}}}$$

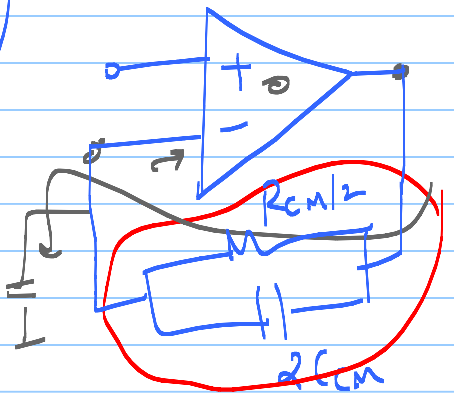


$V_o \leftarrow V_{B_3}$

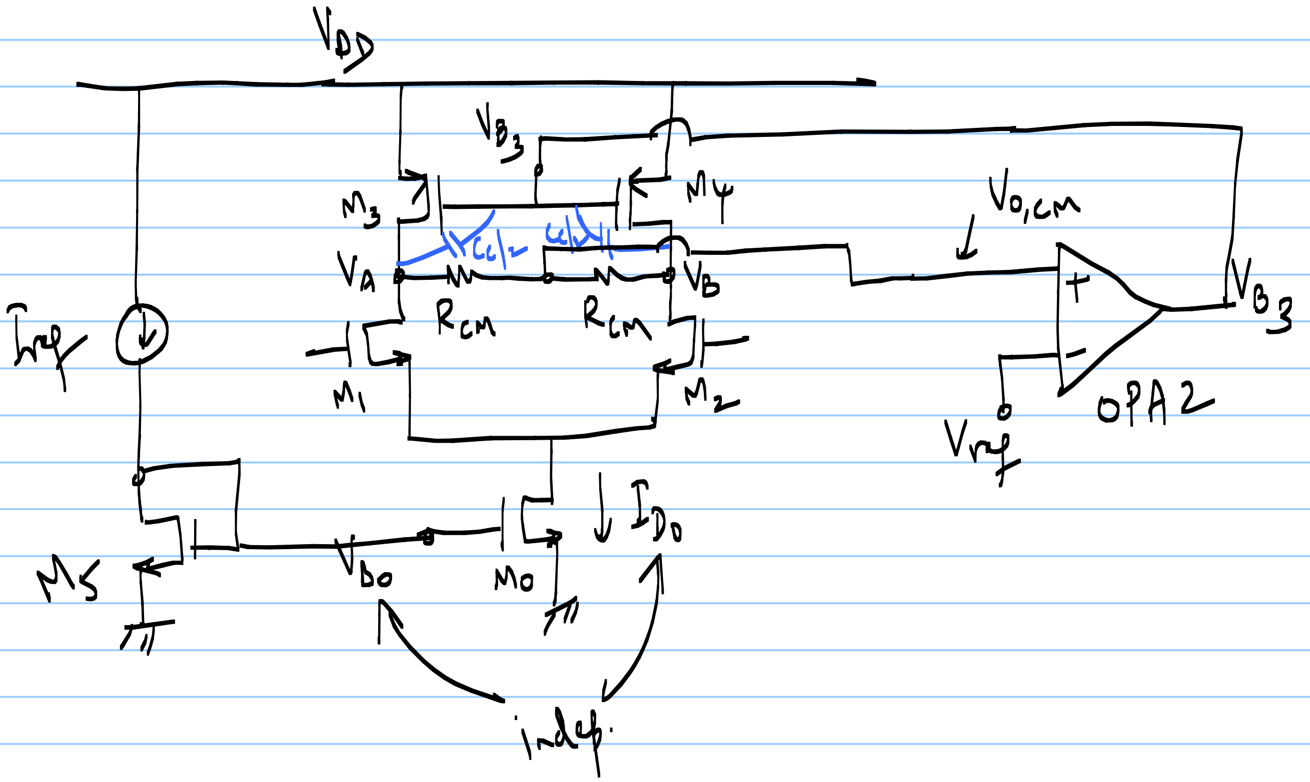
i.e. $V_{DD} - V_{SG_{33}} = V_{B_3}$

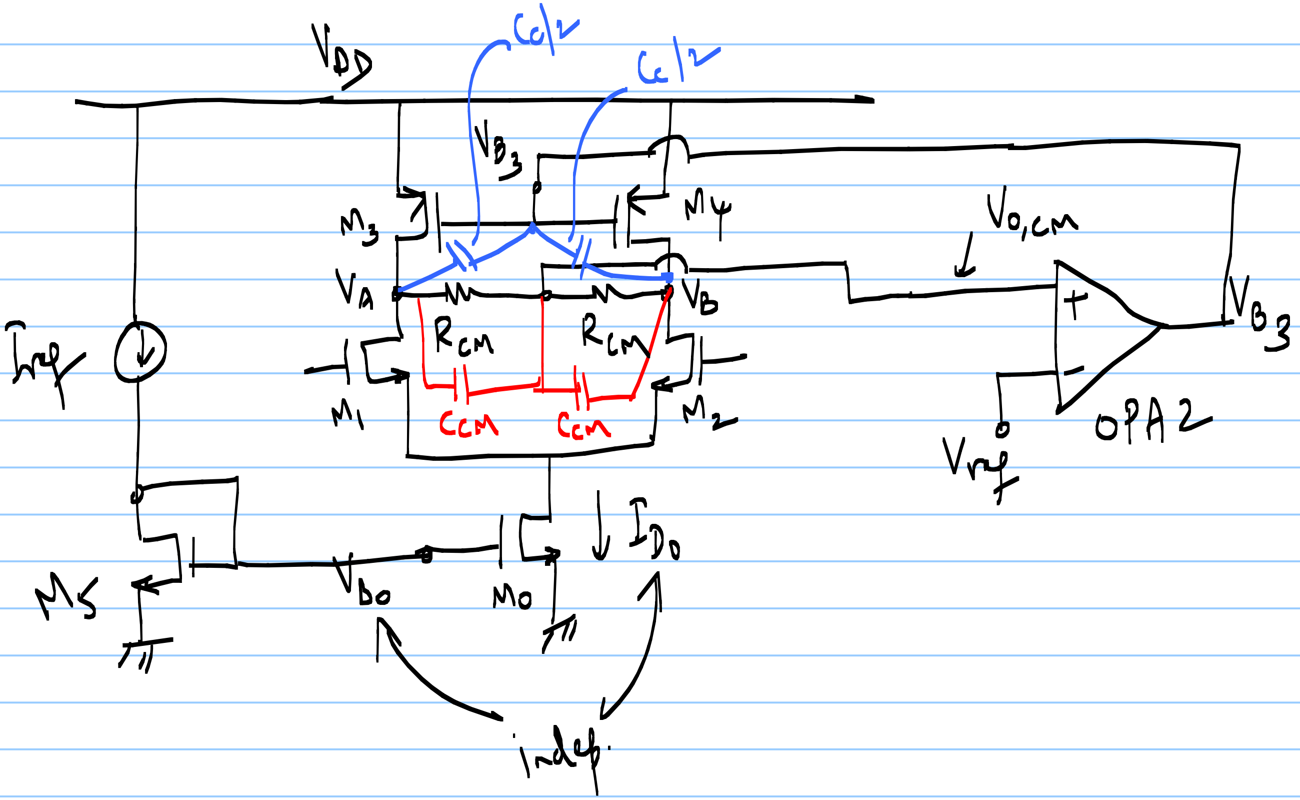


2-stage
CMFB loop

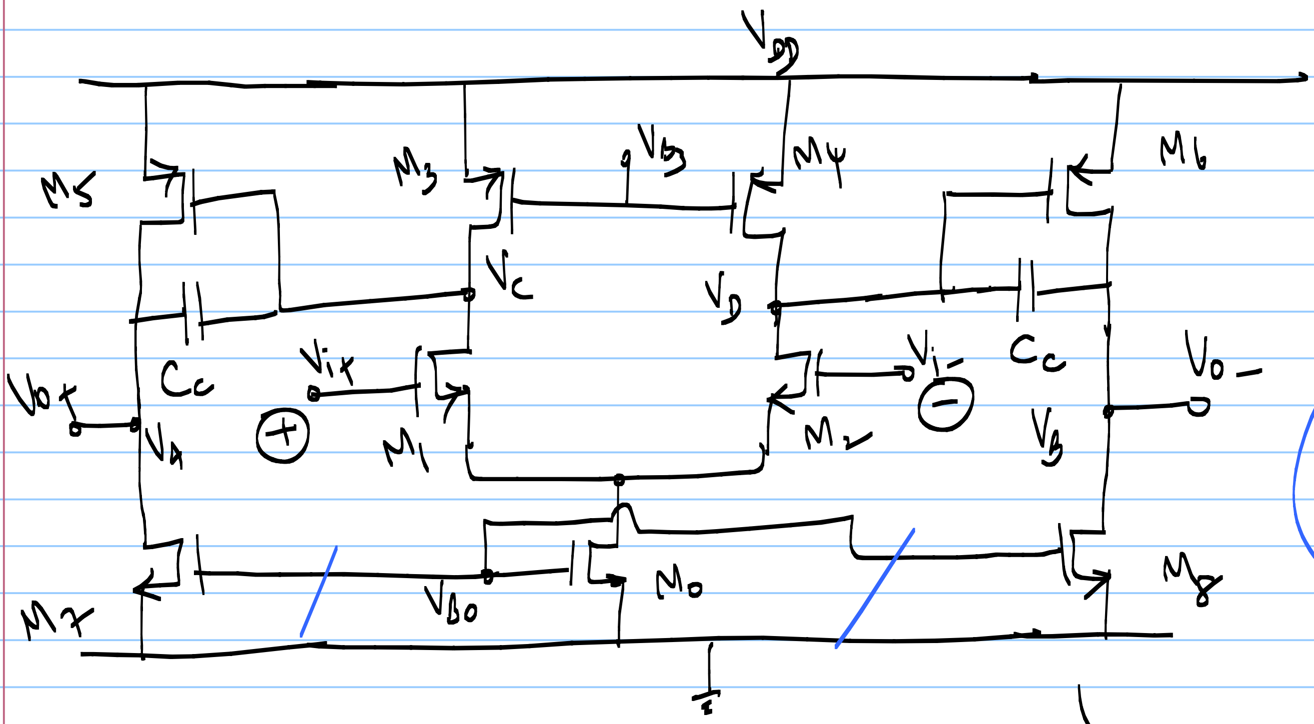


Need to compensate


V_{DD}
V_{B3}
M_3
M_4
$C_c/2$
$C_c/2$
V_A
V_B
R_{CM}
R_{CM}
I_{ref}
M_1
M_2
$V_{o,CM}$
+
−
OPA2
V_{B3}
V_{ref}
M_5
V_{B0}
M_0
I_{D0}
indep.



Cc/2
Cc/2
VDD
VB3
M3
M4
VA
VB
Vo,cm
RCM
RCM
CCM
CCM
M1
M2
Iref
+
-
OPA 2
VB3
Vref
ID0
M5
VB0
M0
indep.

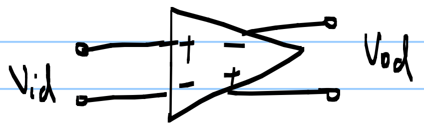


1) $C_c \rightarrow$ compensation cap for FD mode

2) Set $\frac{V_A + V_B}{2}$ i.e. V_{oCM}

3) Set $\frac{V_C + V_D}{2}$ i.e. V_{iCM}

→ 2 CMFB loops
independent

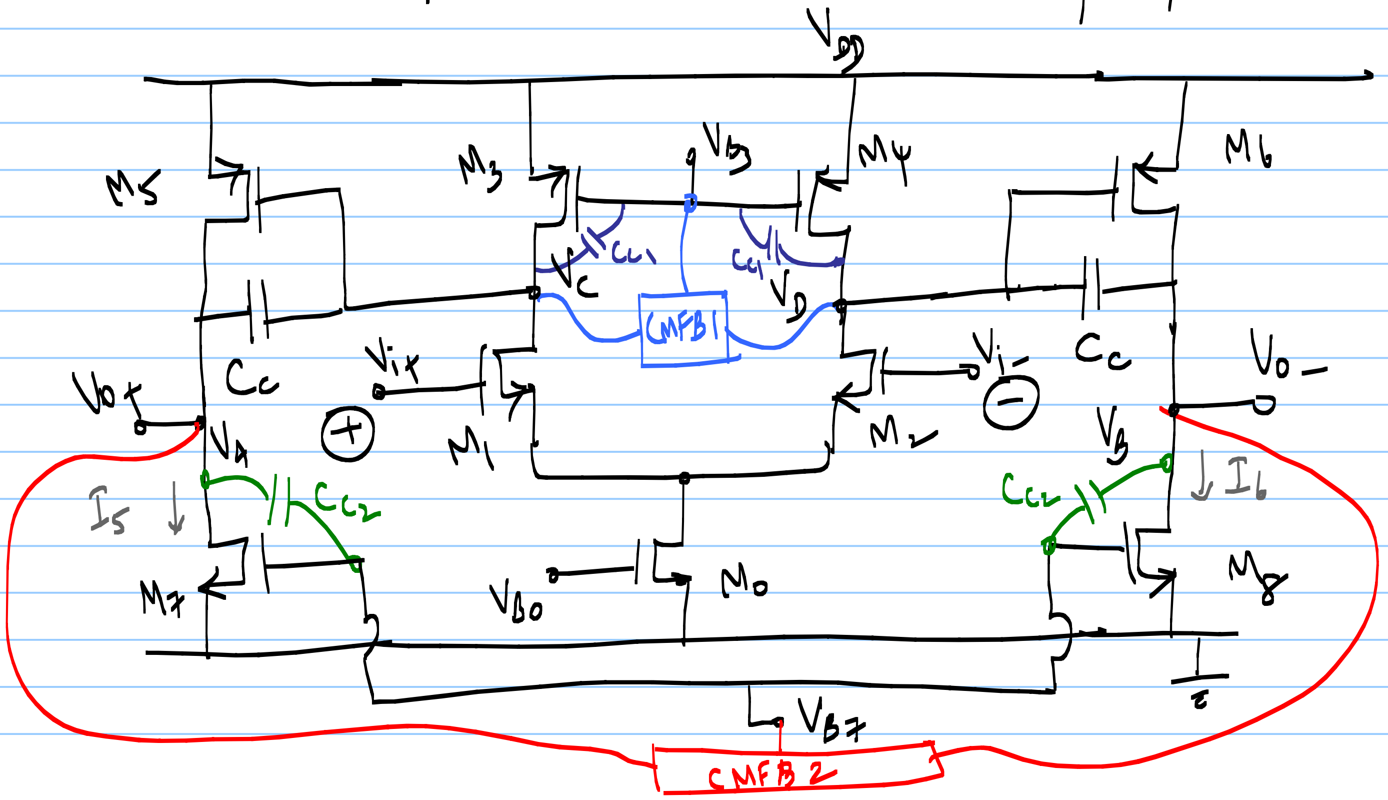


* disconnect V_{G7} & V_{G8} from V_{B0}?

30/11/2023

Lec 25

CMFB for two-stage FD opamp



C_c – compensation cap for FD path

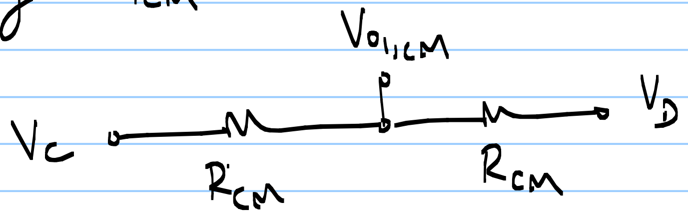
$$V_{o1,cm} = \frac{V_c + V_D}{2}$$

$$V_{o,cm} = \frac{V_A + V_B}{2}$$

Simple: Set $V_{o1,cm}$ & $V_{o,cm}$ independently

Setting $V_{o1,CM} = \frac{V_C + V_D}{2} = V_{ref1}$

* Sensing V_{o1CM}



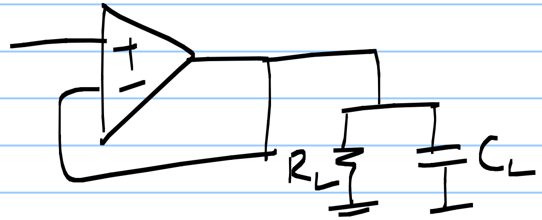
* V_C & V_D swings are low (output swing ÷ gain of 2nd stage)

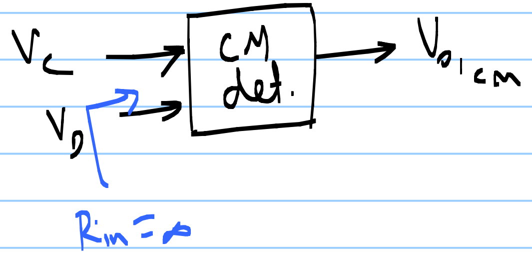
↳ (Resistive) CM detector is not mandatory

linear

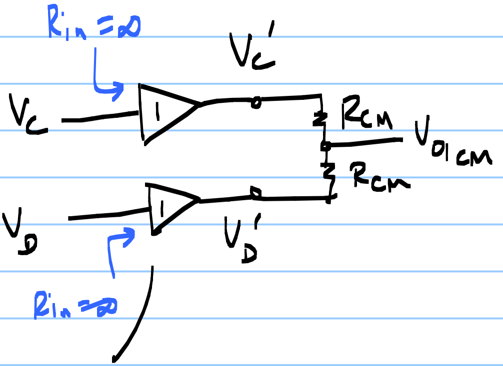
* If driving Resistive loads → most of the gain comes from 1st stage ← don't use R_{cm}

@ opamp o/p $(R_L||C_L)$



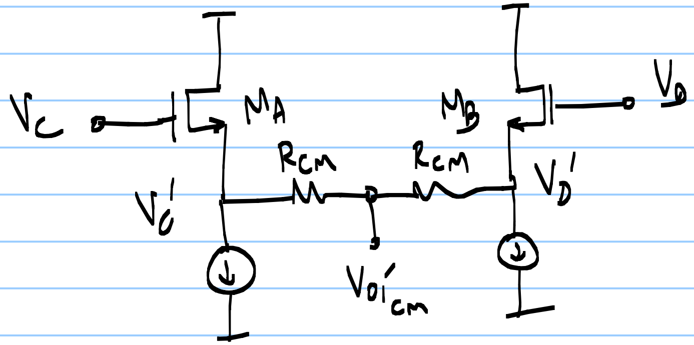


$\rightarrow$



$$V_{o1cm} = \frac{V_C + V_D}{2}$$

small-signal VCVS of gain = 1

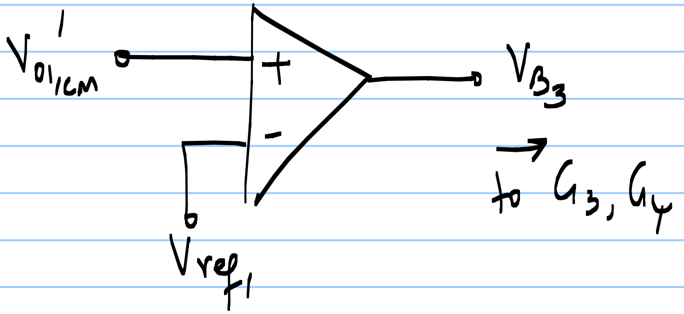


$$V_C' = V_C - V_{GS_A}$$

$$\parallel$$

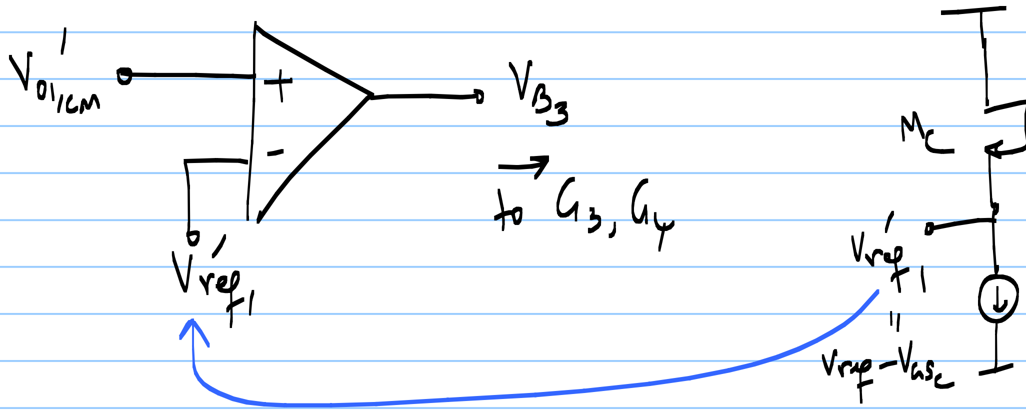
$$V_D' = V_D - V_{GS_B}$$

$$V_{o1cm}' = \frac{V_C + V_D}{2} - V_{GS_{AB}} = V_{o1cm} - V_{GS_{A,B}}$$



$\Rightarrow$ sets $V'_{o,CM} = V_{ref_1}$

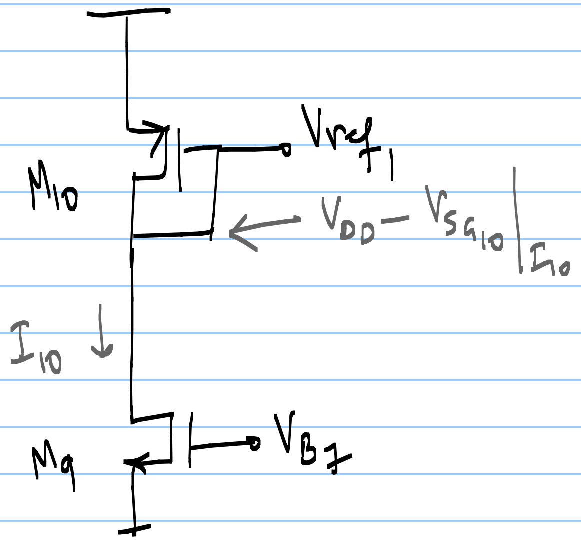
$V_{o1,CM} = V_{ref_1} + V_{GS_{A,B}}$ ✗



set $V_{GS_A} = V_{GS_B} = V_{GS_C}$

$V'_{o1,CM} = V'_{ref_1}$

$V_{o1,CM} = V_{ref_1}$



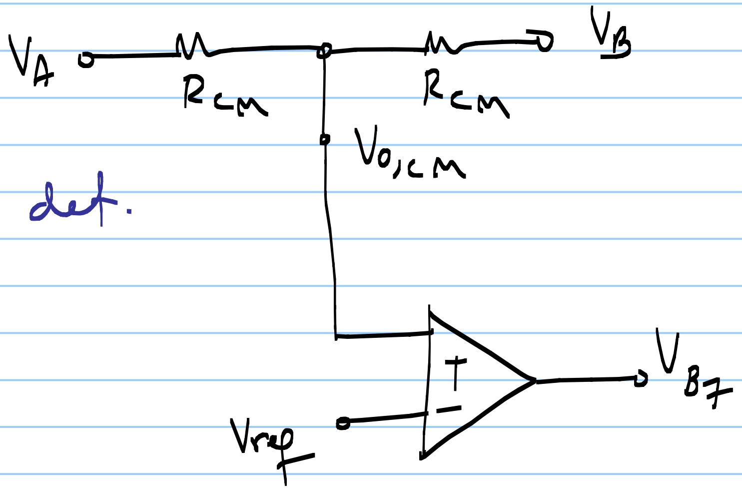
$V_{O1,CM} = V_{ref1}$ → need this to be $= V_{DD} - V_{SG_{5,6}}\Big|_{I_{5,6}}$

* Set $\quad V_{DD} - V_{SG_{10}}\Big|_{I_{10}} = V_{DD} - V_{SG_{5,6}}\Big|_{I_{5,6}}$

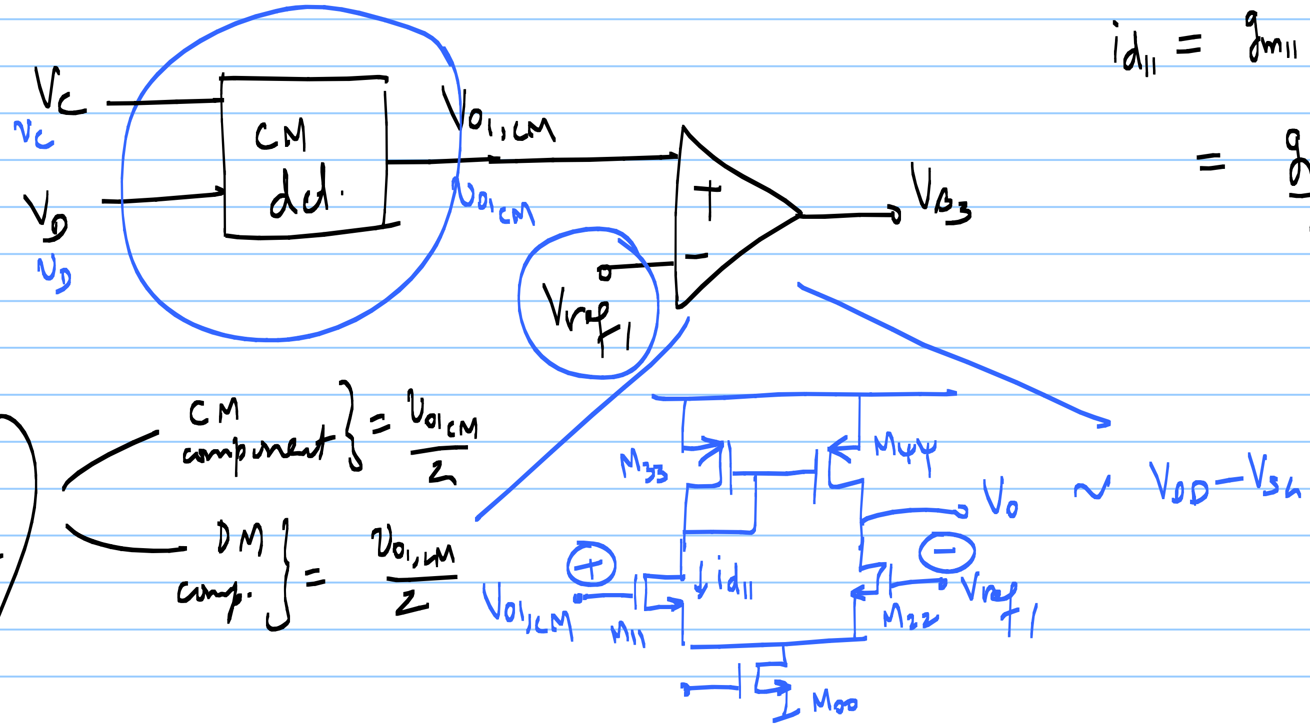
$$\Rightarrow \boxed{\frac{I_{10}}{(W/L)_{10}} = \frac{I_{5,6}}{(W/L)_{5,6}}}$$

Set $\frac{V_A + V_B}{2} = V_{o,cm} = V_{ref}$

High swing → use res. CM det.



Setting $V_{o1,CM} = \frac{V_C + V_D}{2} = V_{ref1}$ – Circuit 2



$$id_{11} = g_{m11} \cdot \frac{v_{o1cM}}{2} = g_{m11}\left(\frac{v_c + v_D}{4}\right)$$

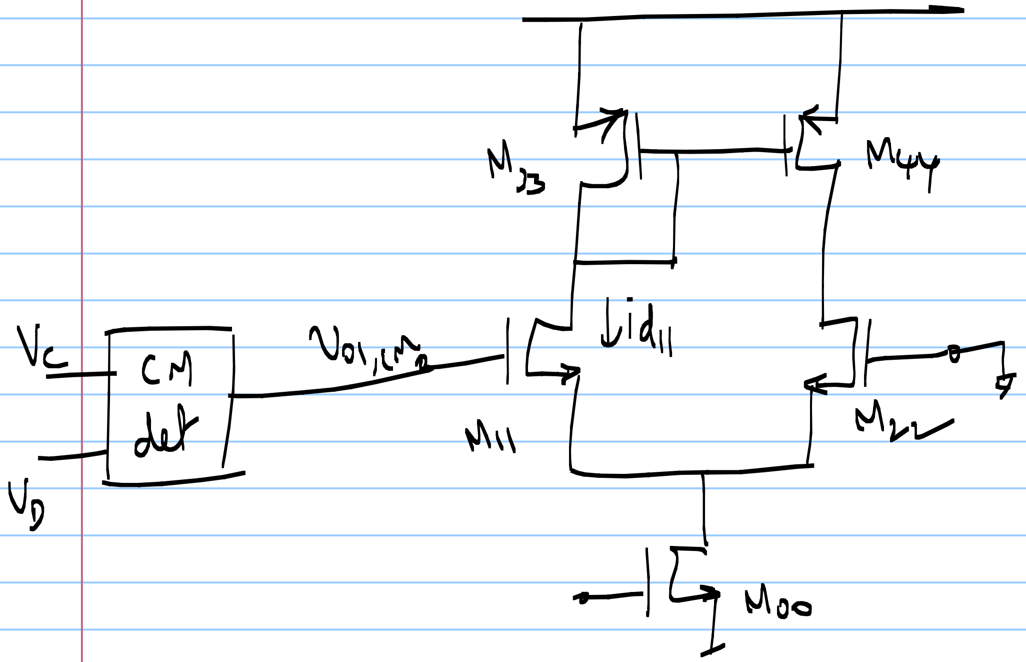
$$= \frac{g_{m11}}{4} \cdot v_c + \frac{g_{m11}}{4} \cdot v_D$$

$v_{o1,cM} \rightarrow G_{11}$

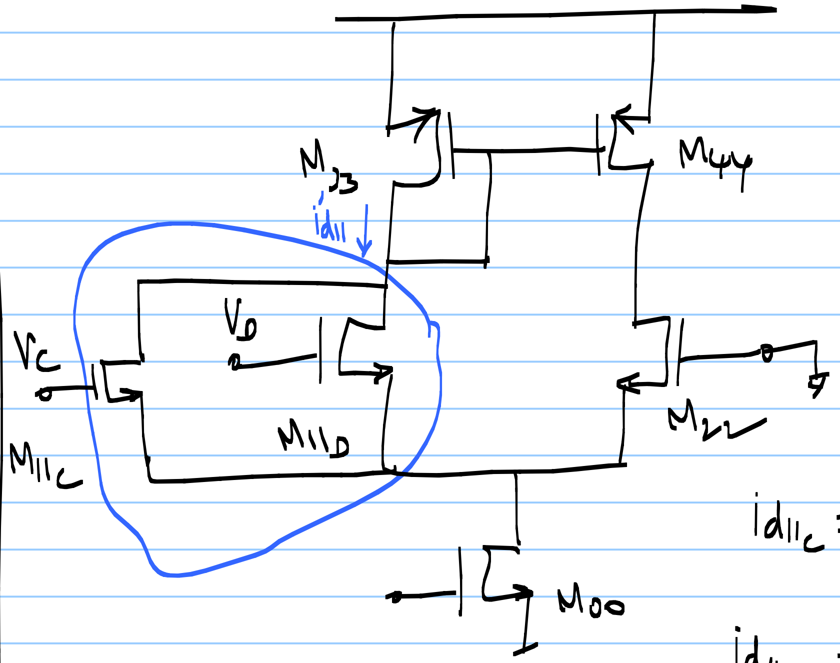
$0 \rightarrow G_{22}$

CM component $\} = \frac{v_{o1cM}}{2}$

DM comp. $\} = \frac{v_{o1,cM}}{2}$



$$i_{d11} = \frac{g_{m11}}{4} \cdot v_c + \frac{g_{m11}}{4} \cdot v_D$$



$$i_{d11}' = i_{d11_C} + i_{d11_D}$$

$$= g_{m11}/_2 \, (v_{o1CM})$$

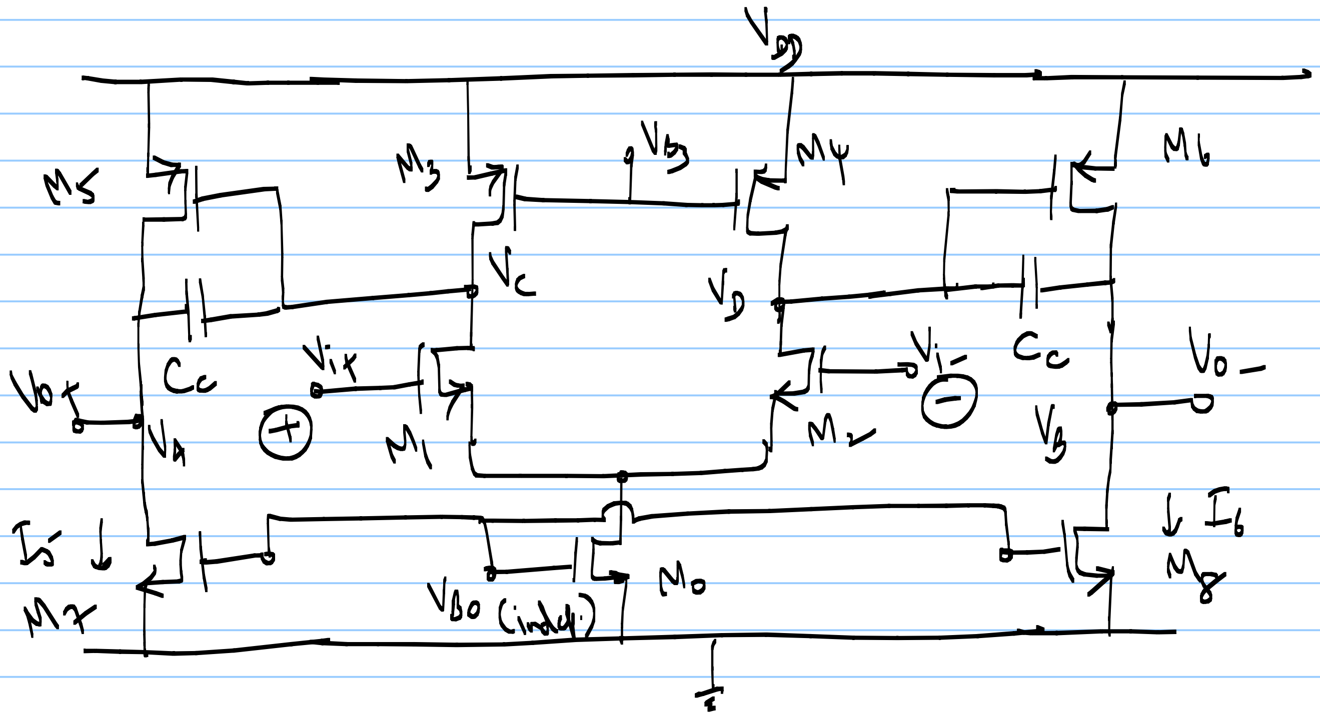
$g_{m11}/_2$

$$i_{d11_C} = g_{m11_C} \cdot \frac{v_c}{2}$$

$$i_{d11_D} = g_{m11_D} \cdot \frac{v_D}{2}$$

$g_{m11}/_2$

$$M_{11C} = M_{11D} = M_{11}/_2$$



V_{B3} sets $V_{o1,CM}$

Set $\frac{V_A+V_B}{2} = V_{o,CM}$

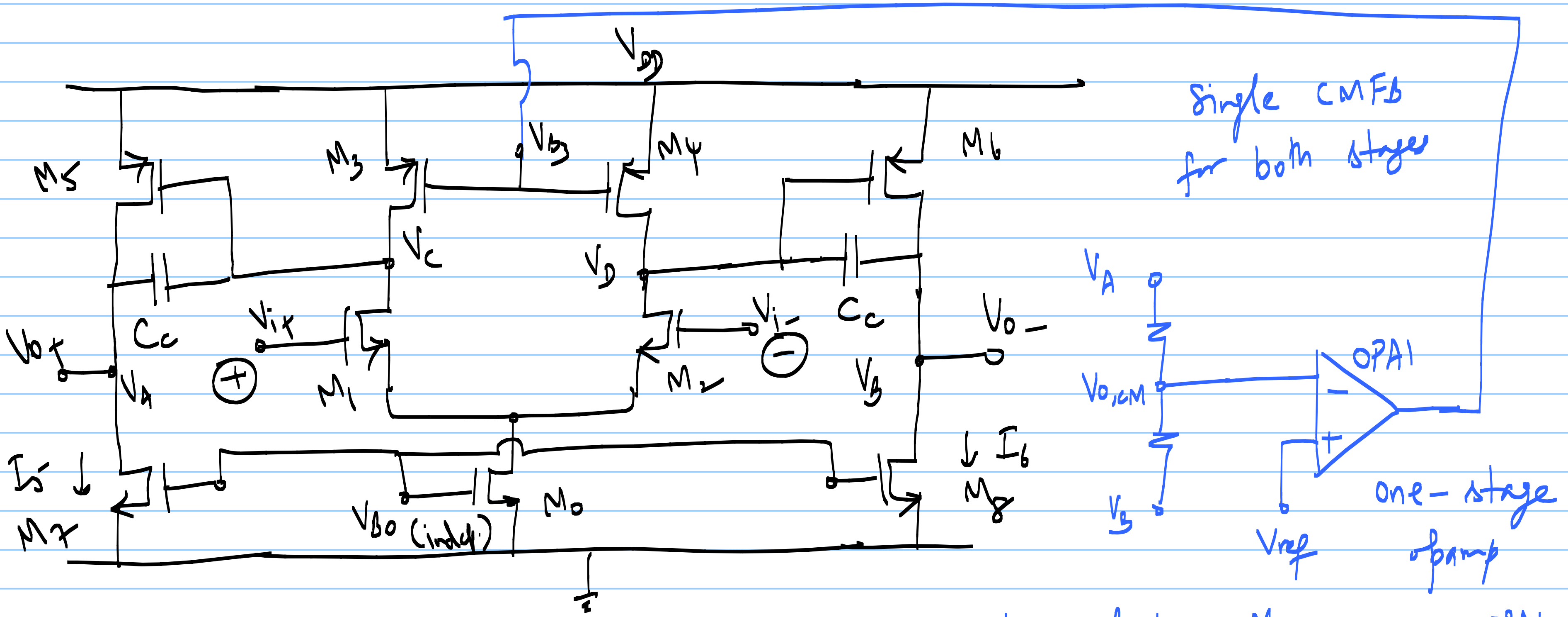
via M_5, M_6

* M_5 & M_6 are pseudo-differential amplifiers

↳ they have CM gain

↳ $\frac{V_C+V_D}{2}$ controls $\frac{V_A+V_B}{2}$

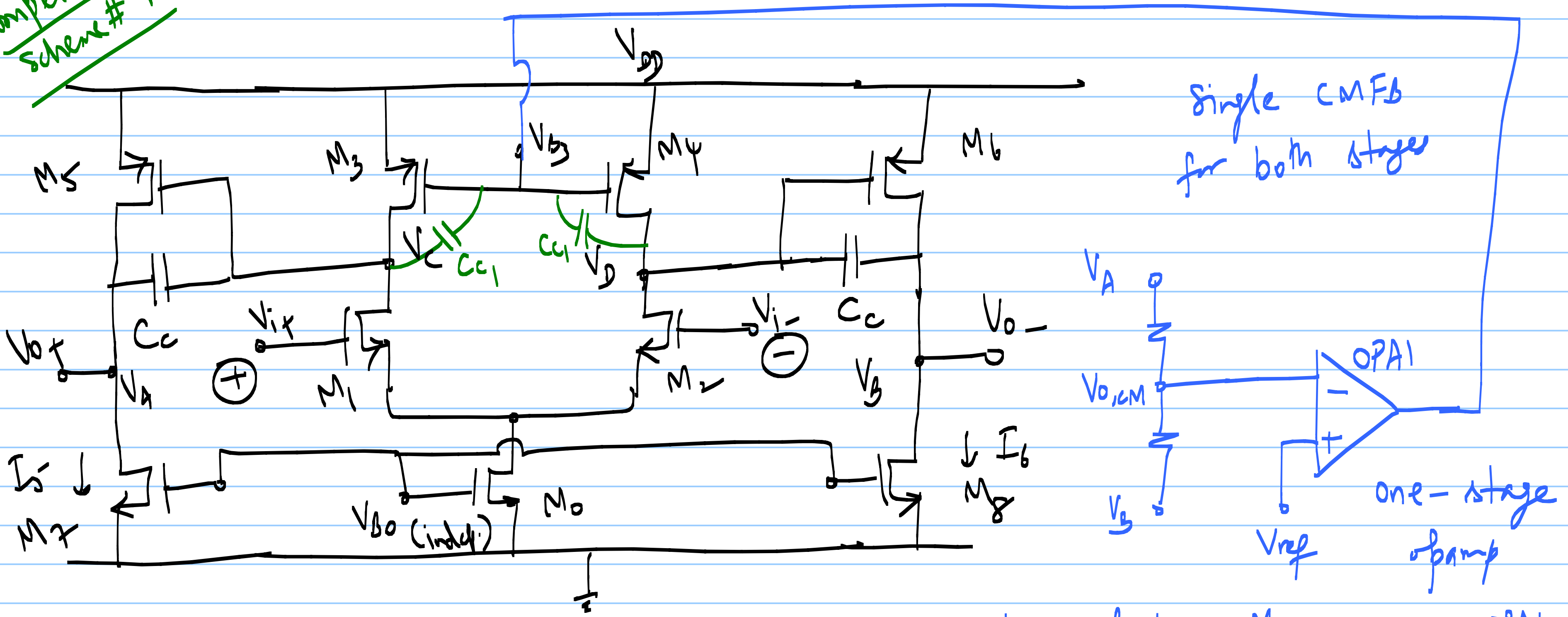
($V_{o1,CM}$) ($V_{o,CM}$)



Single CMFB for both stages

3-stage loop → M3,4; M5,6; OPA1 needs to be compensated

Compensation scheme # 1

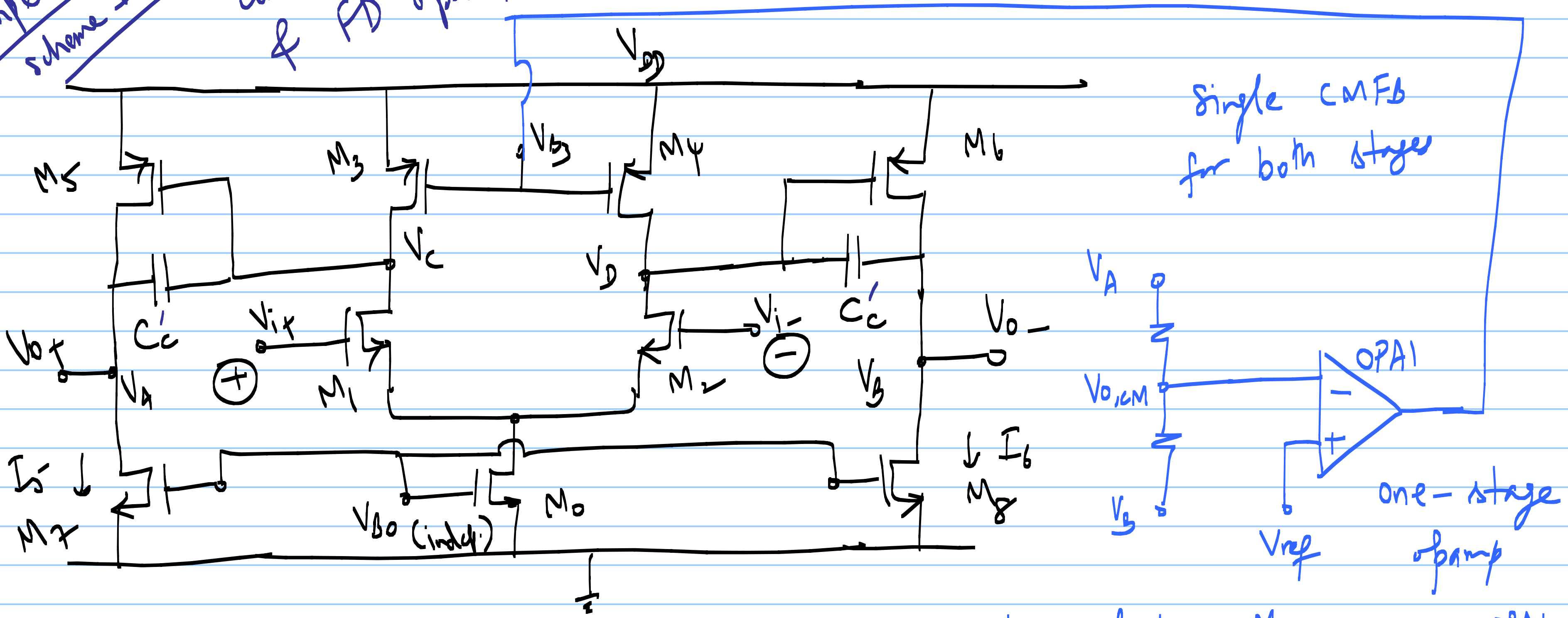


Single CMFB for both stages

3-stage loop → M3,4; M5,6; OPA1 needs to be compensated

Compensation scheme #2

combine CMFB compensation & FD opamp compensation into a single cap C_c'

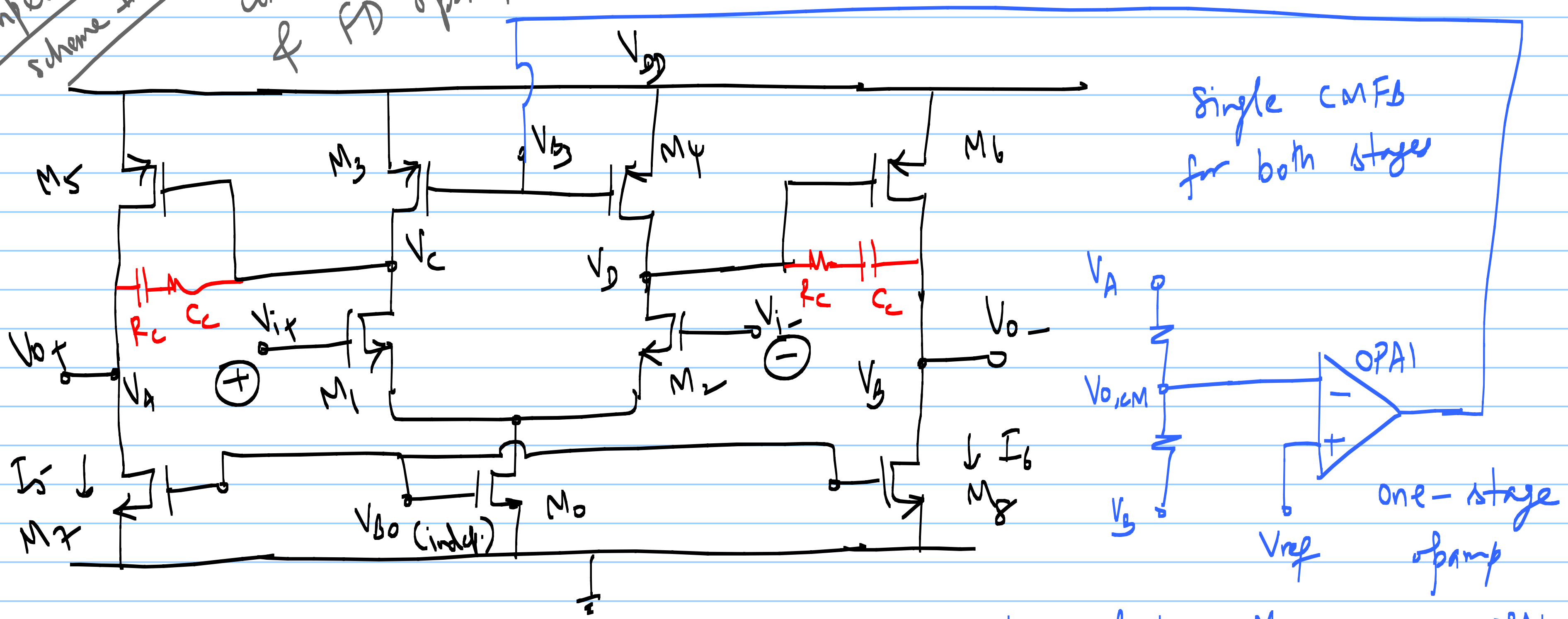


Single CMFB for both stages

3-stage loop → $M_{3,4}$; $M_{5,6}$; OPA1 needs to be compensated

Compensation scheme #3

combine CMFB compensation & FD opamp compensation into a single cap C_c' + R_c for pole-zero cancellation



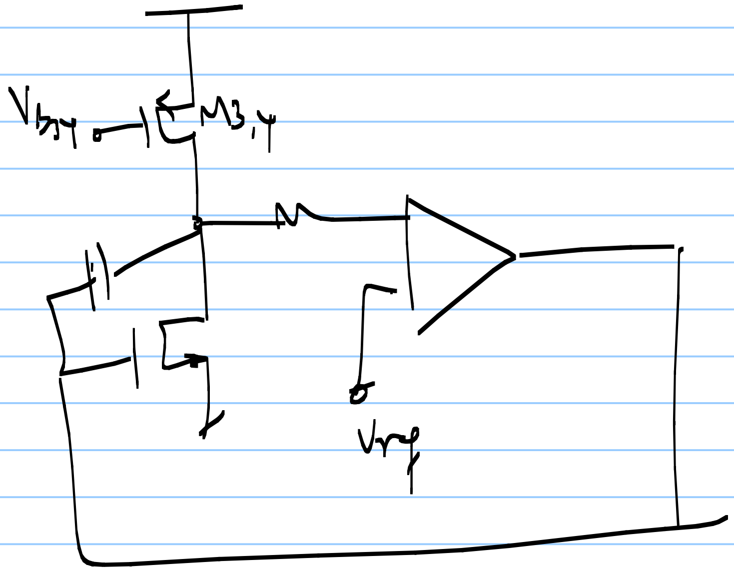
Single CMFB for both stages

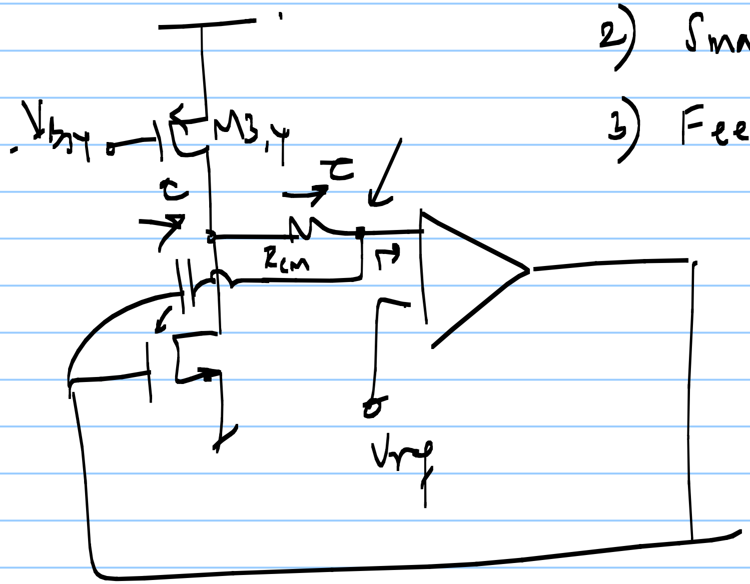
3-stage loop → $M_{3,4}$; $M_{5,6}$; OPA1 needs to be compensated

8/12/2023

Lec 26

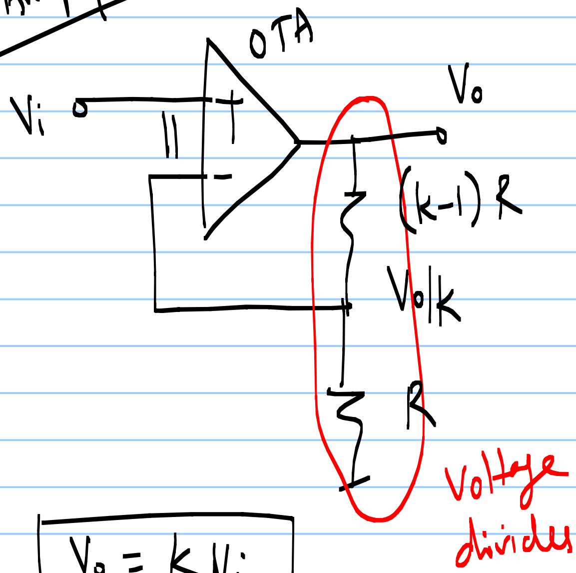
1) 3-stage opamps
2) Small R-loads
3) Feed forward opamps





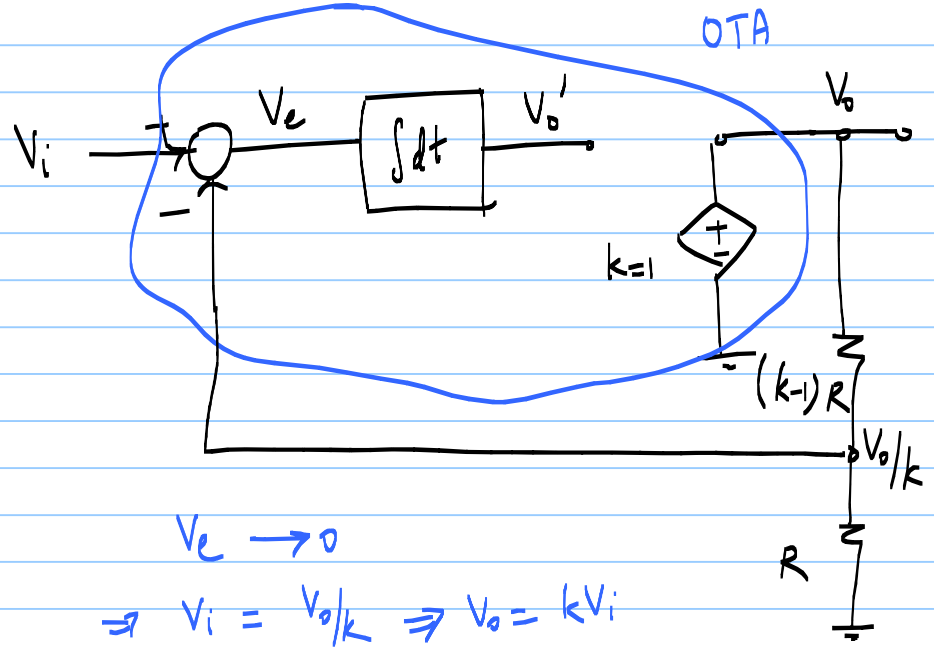
Phase Locked Loops → Set phase based on a negative f/b. loop

Voltage Amplifier



Want $\boxed{V_o = kV_i}$

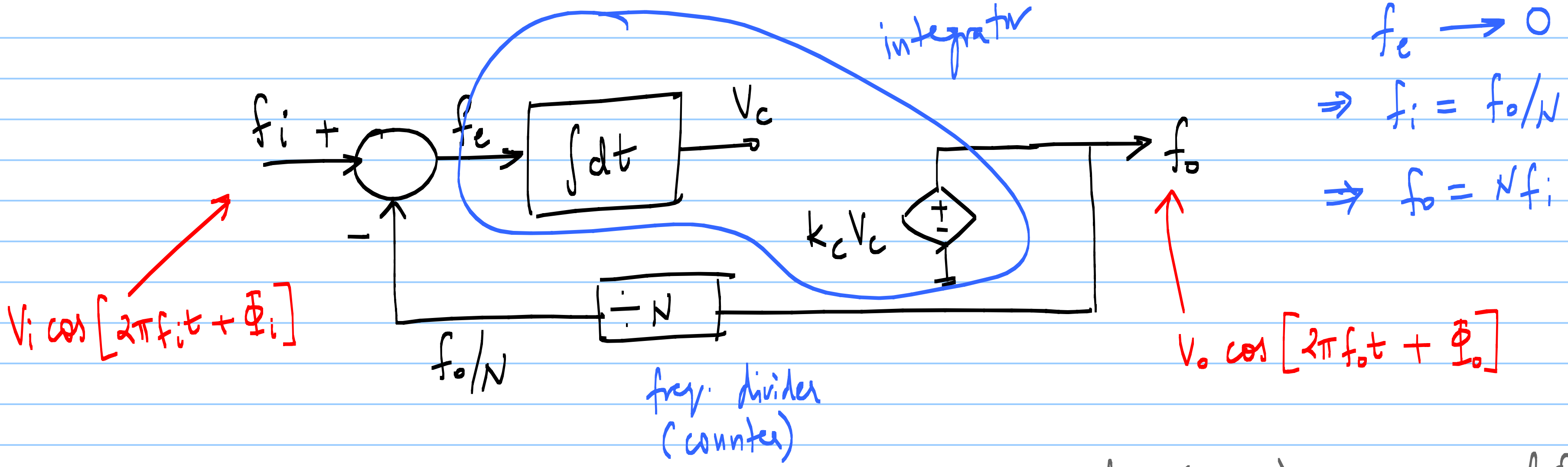
→



$V_e \to 0$

$\Rightarrow V_i = V_o/k \Rightarrow V_o = kV_i$

Frequency Multiplier : Want $f_o = N f_i$



$f_e \to 0$

$\Rightarrow f_i = f_o/N$

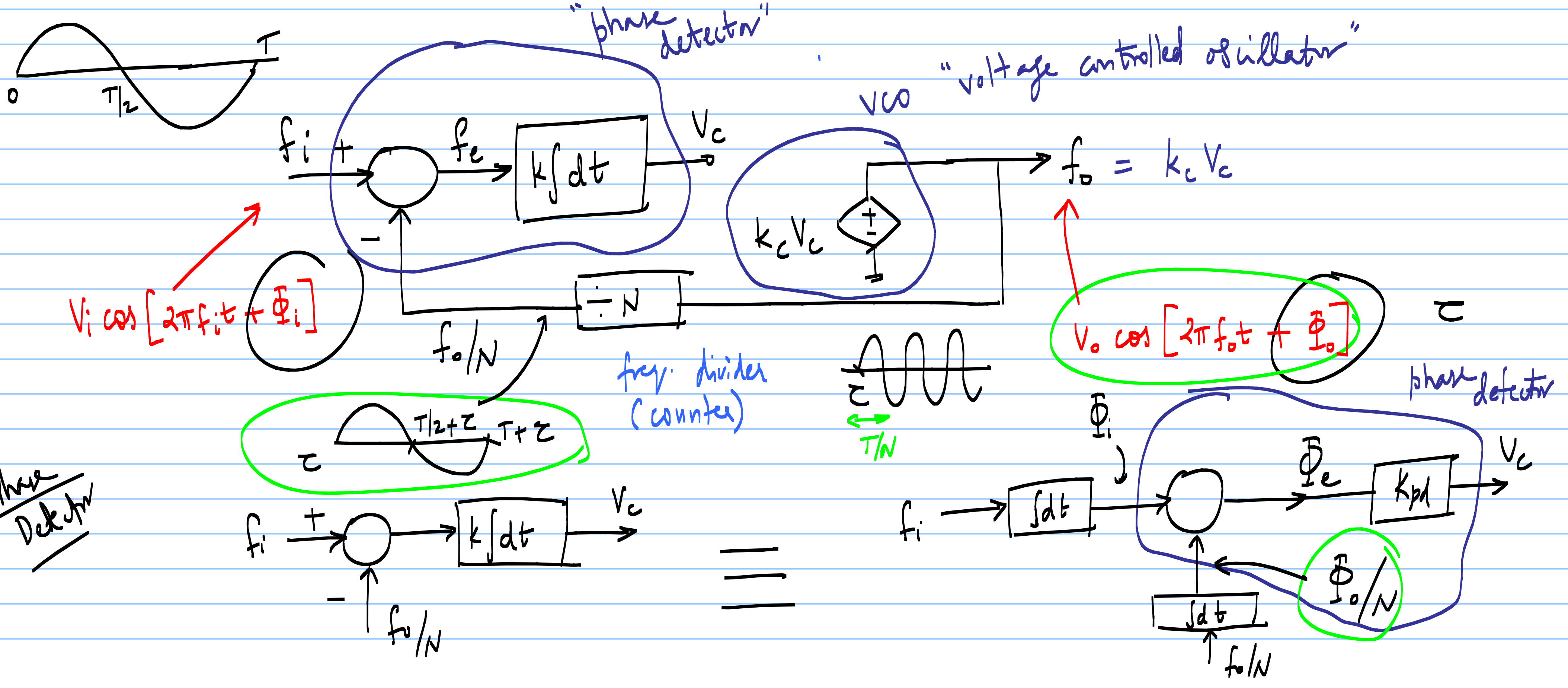
$\Rightarrow f_o = N f_i$

$$A \cos(\omega_o t + \varphi_o)$$

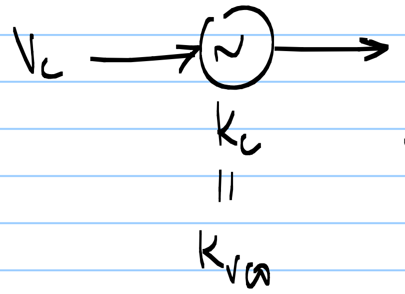
$$\text{phase} = \omega_o t + \varphi_o = 2\pi f_o t + \varphi_o$$

$$\text{angular freq.} = \frac{d}{dt}(\text{phase}) = \omega_o + \frac{d\varphi_o}{dt}$$

$$\text{freq.}(\text{Hz}) = \frac{1}{2\pi}\frac{d}{dt}(\text{phase}) = \frac{\omega_o}{2\pi} + \frac{1}{2\pi}\frac{d\varphi_o}{dt}$$


0
T/2
T
"phase detector"
f_i
+
−
f_e
$k\int dt$
V_c
VCO "voltage controlled oscillator"
$k_c V_c$
+
−
$f_o = k_c V_c$
$V_i \cos[2\pi f_i t + \Phi_i]$
÷N
f_o/N
freq. divider (counter)
τ
T/2+τ
T+τ
τ
T/N
$V_o \cos[2\pi f_o t + \Phi_o]$
τ
phase detector
Phase Detector
f_i
+
−
$k\int dt$
V_c
f_o/N
≡
Φ_i
f_i
$\int dt$
Φ_e
K_{pd}
V_c
Φ_o/N
$\int dt$
f_o/N

VCO



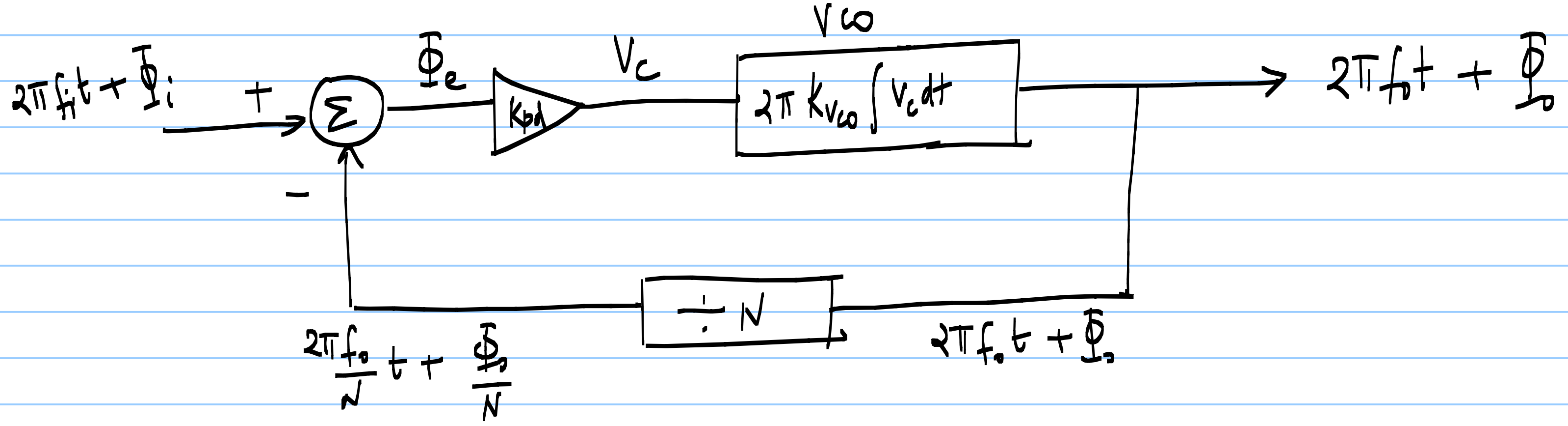
$f_{vco} = K_c V_c = K_{vco} \cdot V_c$

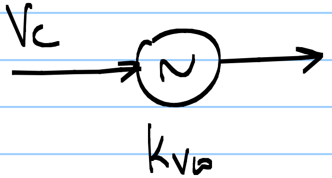
$V_0 \cos(2\pi f_{vco} t)$

$\Phi_{vco} = 2\pi K_{vco} \int V_c dt$

Phase domain model

gain = N

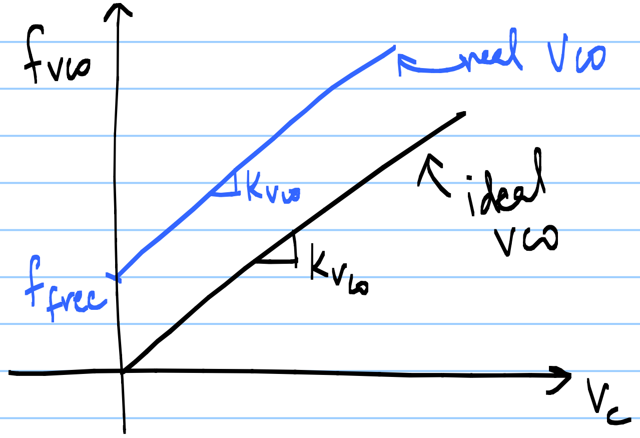




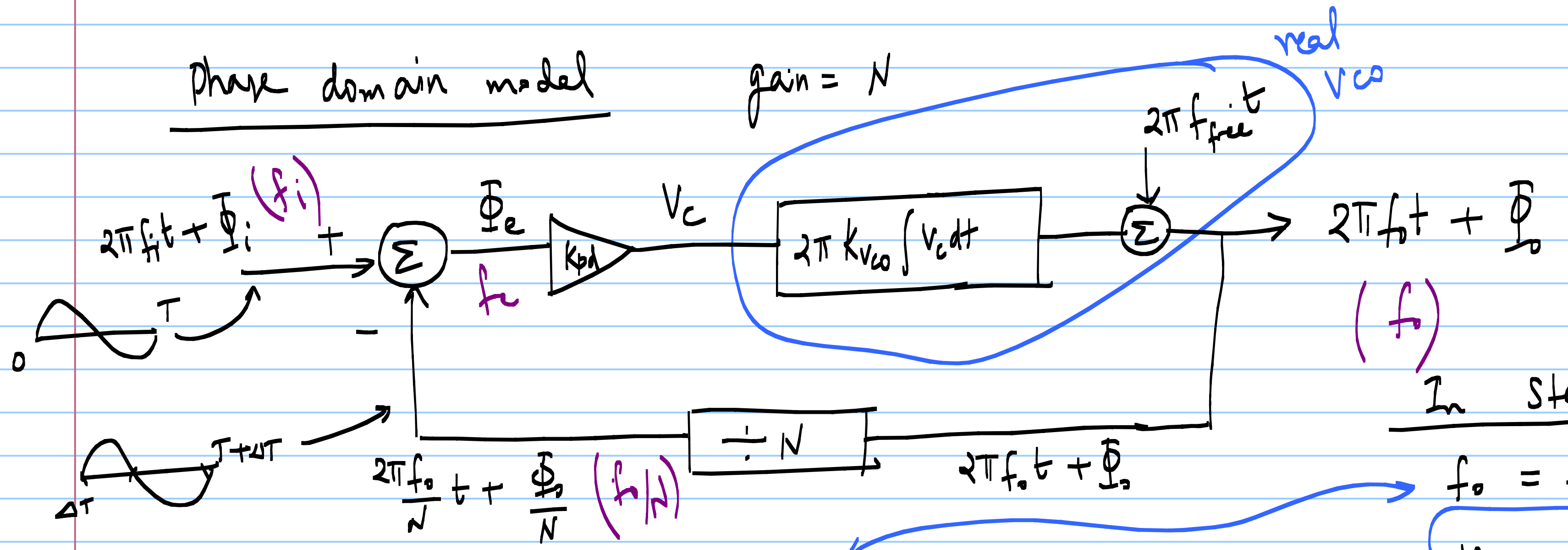
$$f_{VCO} = K_{VCO} \cdot V_c$$

$$f_{VCO} = f_{free} + K_{VCO} \cdot V_c$$

free running freq. of VCO ($V_c = 0$)



Phase domain model



$$\Phi_e = \left(2\pi \cancel{f_i t} + \Phi_i\right) - \left(2\pi \cancel{\frac{f_o}{N} t} + \frac{\Phi_o}{N}\right) = \Phi_i - \Phi_o/N$$

$$f_o = f_{free} + K_{vco} \cdot V_c = N f_i \Rightarrow \boxed{V_c = \frac{N f_i - f_{free}}{K_{vco}}}$$

In Steady State

$f_o = f_i \cdot N \leftarrow f_e = 0$

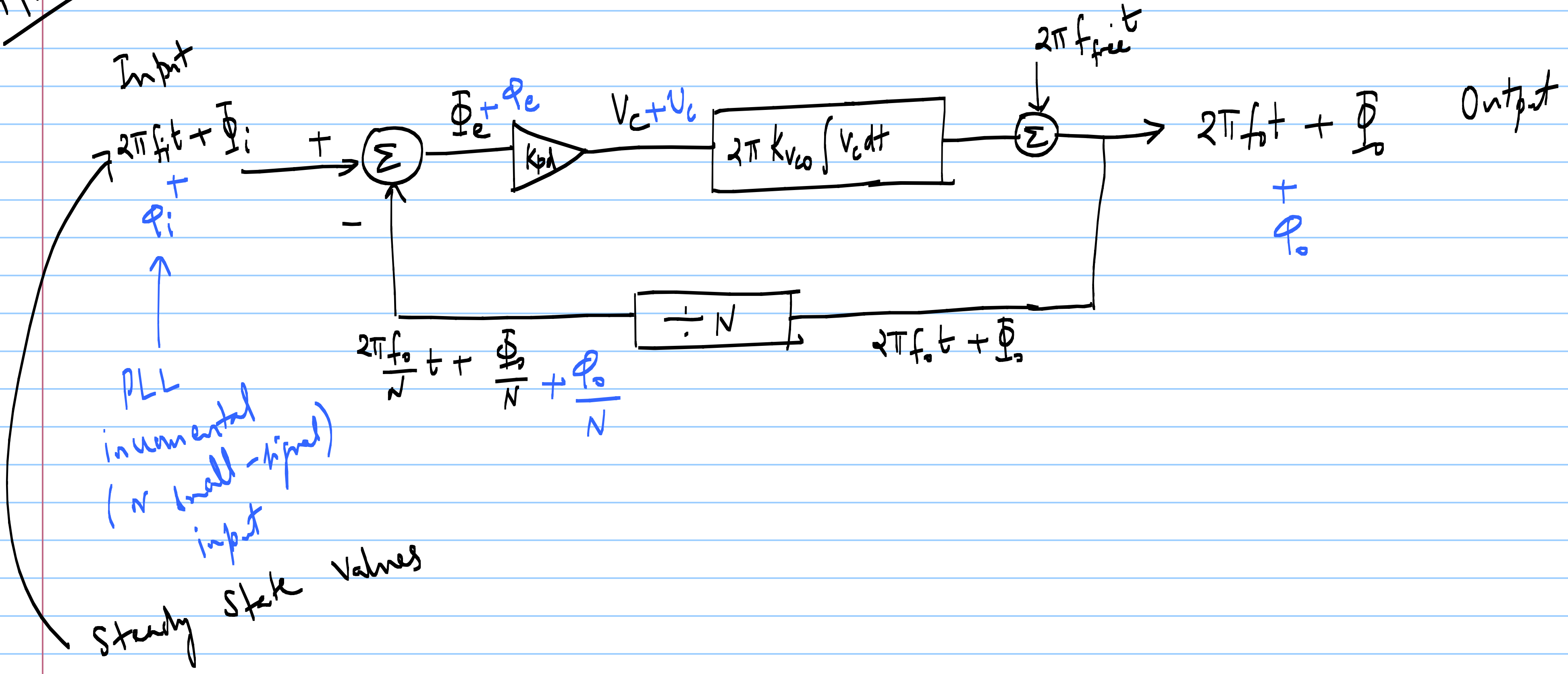
$$\boxed{V_c = K_{pd} \cdot \Phi_e} = K_{pd}\left(\Phi_i - \Phi_o/N\right)$$

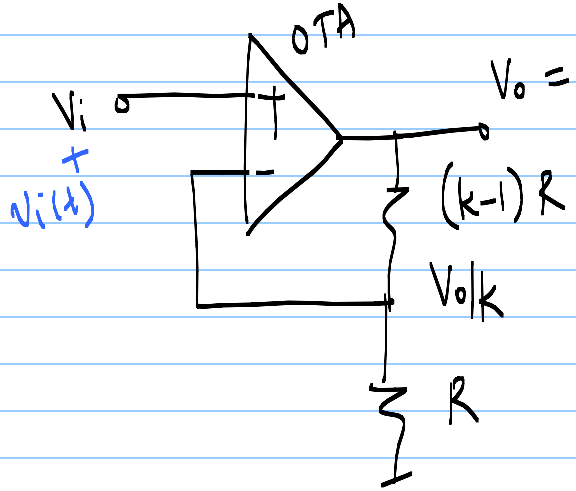
If $N f_i > f_{free} \rightarrow$ non-zero V_c

non-zero Φ_e

14/12/23

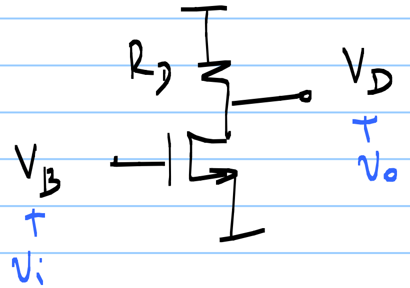
Lec 27



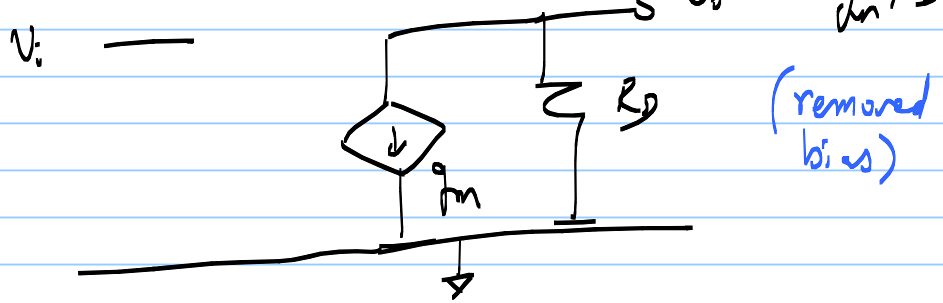


$V_o = kV_i + k\,v_i(t)$

→ incremental eq. ckt.

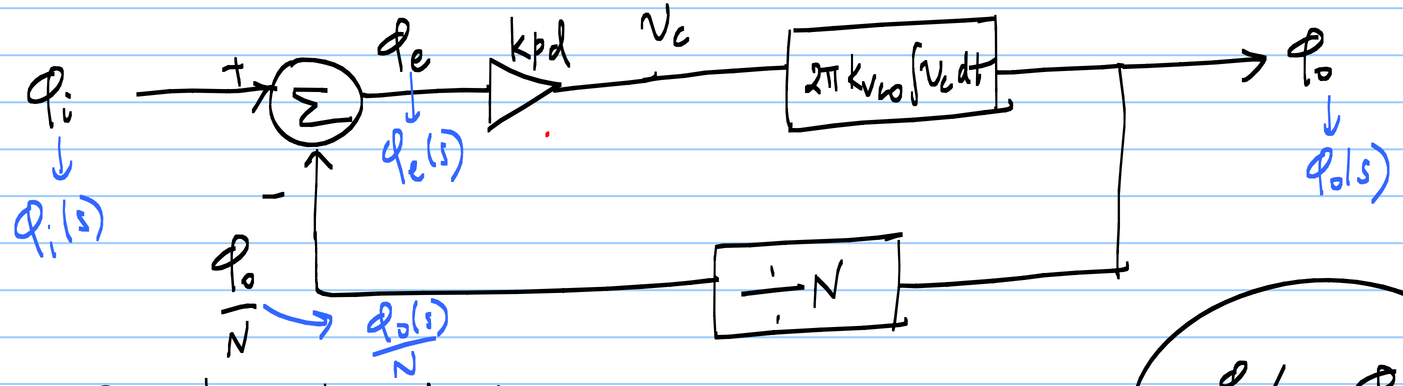


→



$v_o = -g_m R_D \cdot v_i$

Incremental Phase Domain Model of PLL ← phase locked loop



$$\underbrace{\varphi_i,\ \varphi_o,\ \frac{\varphi_o}{N},\ \varphi_e,\ v_c}$$

all small signal values

$$\boxed{\varphi_o/N = \varphi_{fb}}$$

Type-I phase locked loop

1) Closed loop gain as a function of freq.
2) $LG(s) \rightarrow$ PM, GM from Bode plots
3) Step response
4) −3dB BW, UGB

...

1) Loop Gain

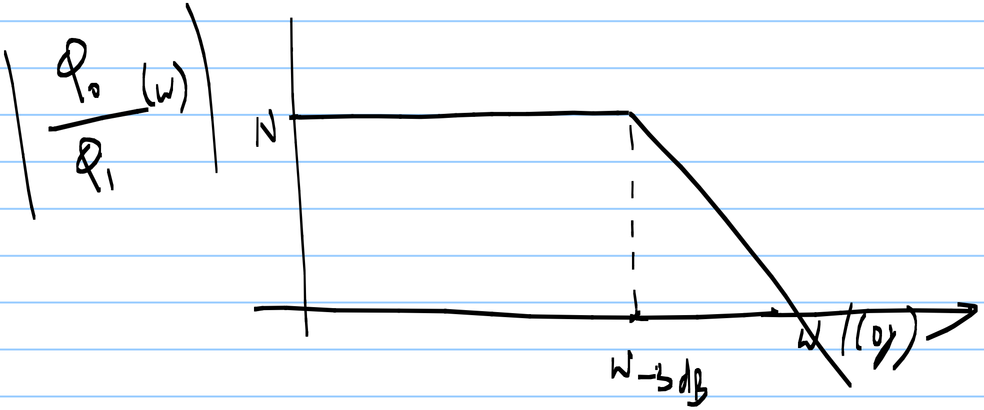
$$LG(s) = \frac{2\pi K_{pd} K_{vco}}{N s}$$

2) Closed loop gain

$$CLG(s) = \frac{\varphi_o}{\varphi_i}(s) = N \cdot \frac{LG(s)}{1 + LG(s)}$$

$$= N \cdot \frac{1}{1 + \frac{N s}{2\pi K_{pd} K_{vco}}} = N \cdot \frac{1}{1 + \frac{s}{\omega_{3dB}}}$$

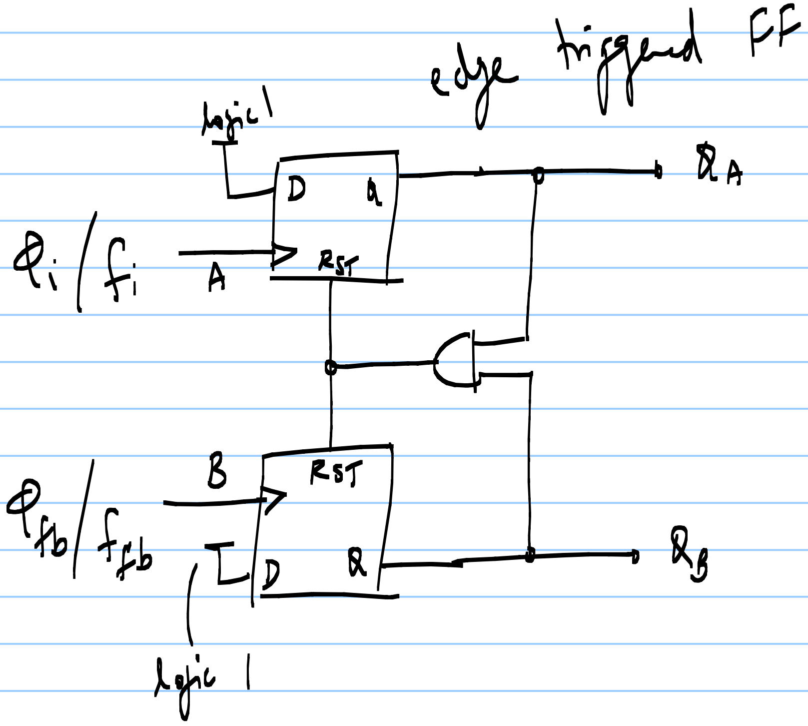
3) $\underset{(CL)}{\omega_{-3dB}} = \frac{2\pi K_{pd} K_{vco}}{N}$; $\underset{(CL)}{f_{-3dB}} = \frac{K_{pd} K_{vco}}{N}$

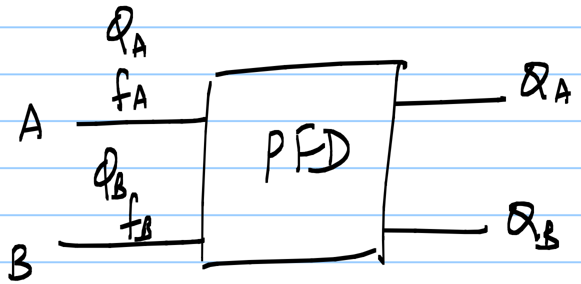


$$w_n = \frac{2\pi K_{pd} K_{vco}}{N}$$

HW : step response

Phase Detector (Tristate Phase-frequency Detector) (PFD) (Very common)

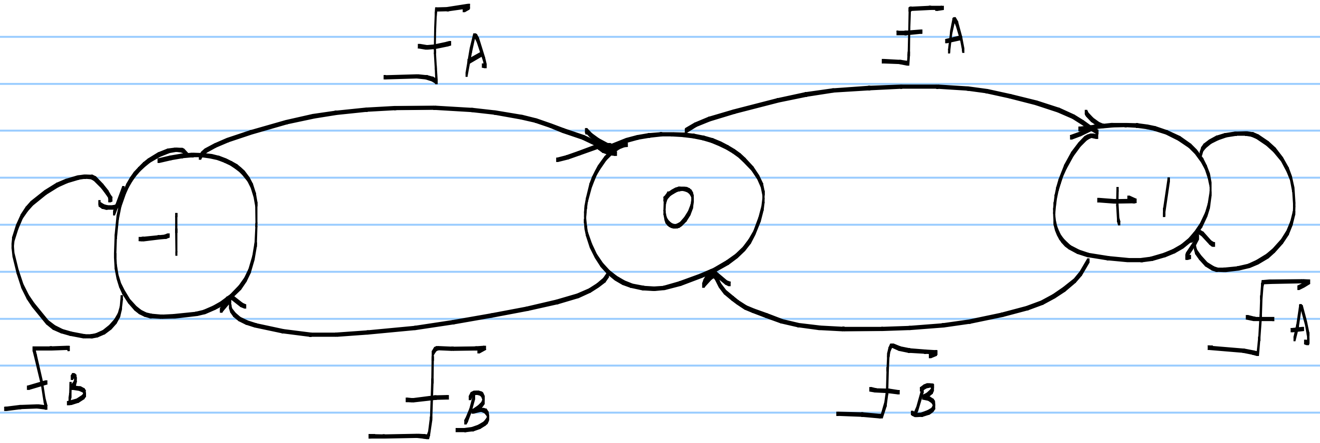


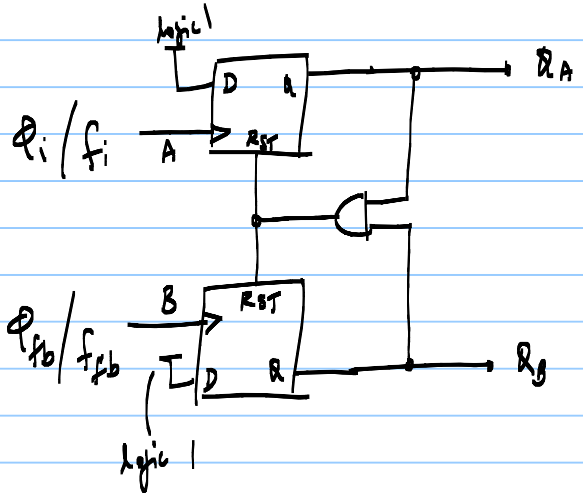


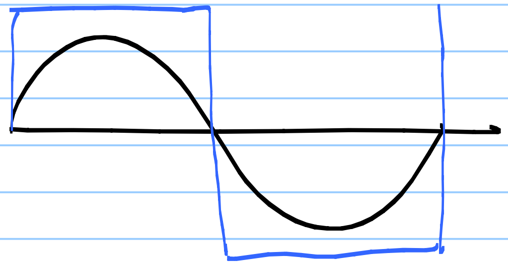
$$\frac{1}{N} \cdot \Phi_o = \Phi_{fb}$$

$$\left.\begin{array}{l}\text{Desired}\\ \text{output}\end{array}\text{PFD}\right\} = \underset{\text{in volts}}{\text{avg.}\,(Q_A - Q_B)} \qquad \text{"}V_c\text{"}$$

Q_A	Q_B	State
0	0	0
0	1	−1
1	0	+1
1	1	Reset





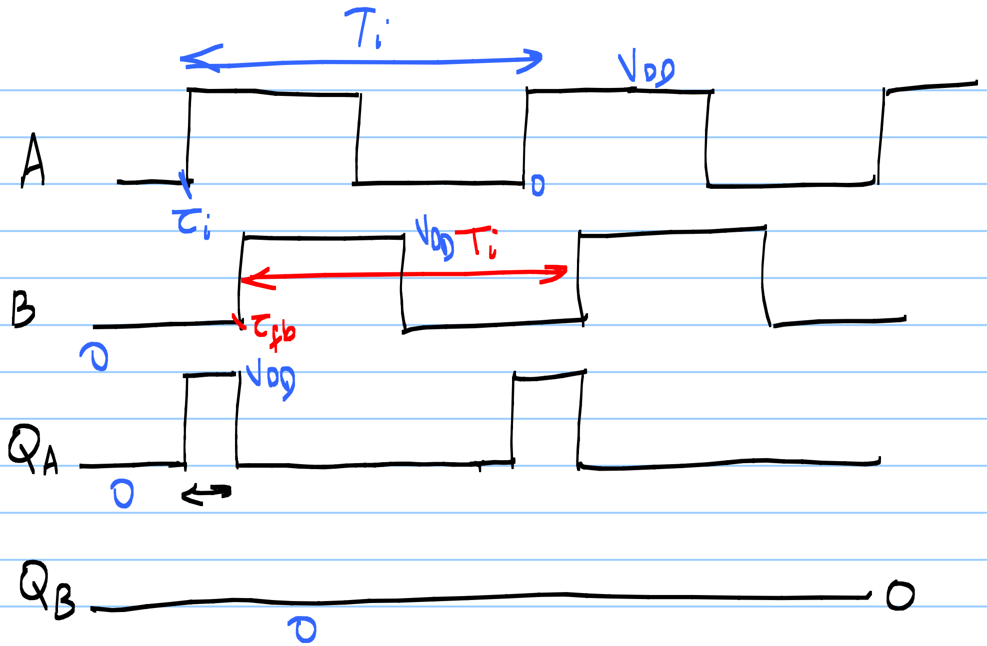


+ve edge triggered DFFs

No gate delays

$f_A = f_B$

$\varphi_A \neq \varphi_B$



If $\tau_{fb} < \tau_i$

$\Downarrow$

Q_A remains @ zero

Q_B goes high for $(\tau_i - \tau_{fb})$

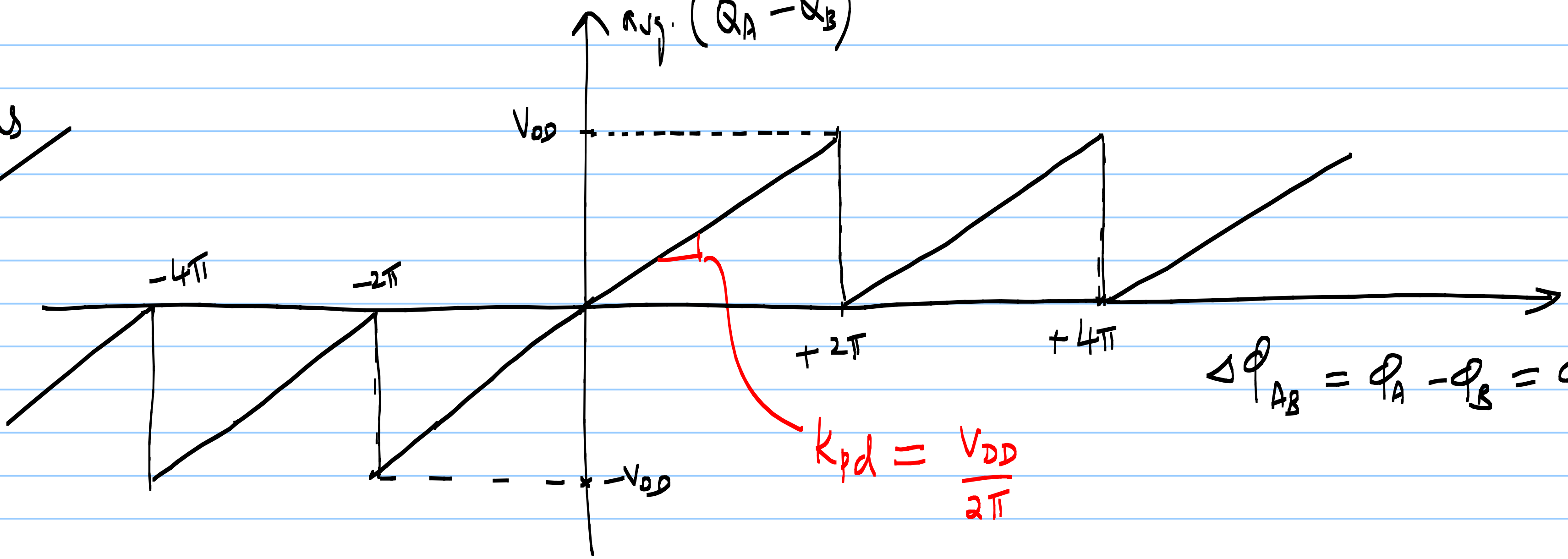
PFD characteristic plot

↳ plot avg. $(Q_A - Q_B)$ ← output voltage

vs

$\varphi_A - \varphi_B$ ← input phase diff.

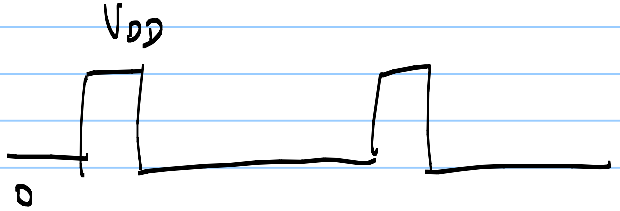
PFD characteristics



$k_{pd} = \frac{V_{DD}}{2\pi}$

$\Delta\varphi_{AB} = \varphi_A - \varphi_B = \varphi_e$

$(Q_A - Q_B)$

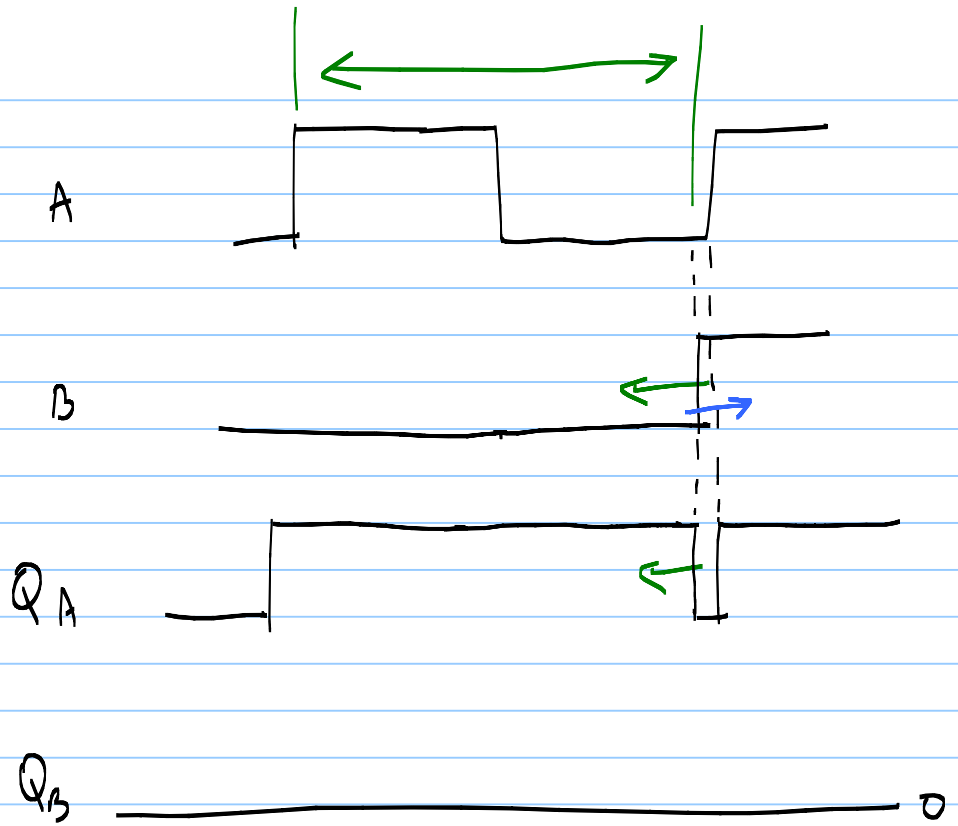


. . .

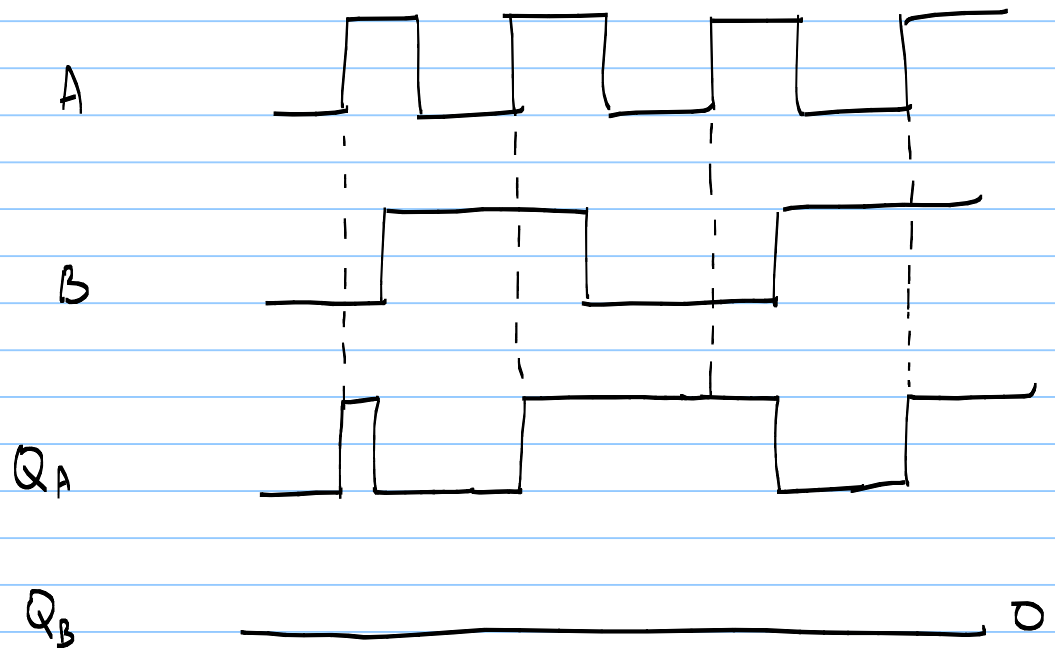
→ DC value avg. $(Q_A - Q_B)$

→ fundamental $\frac{1}{T_i}$ $(= f_i)$

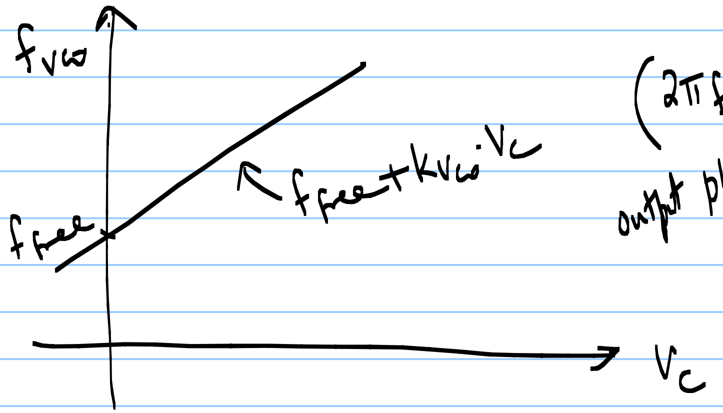
→ harmonics $\frac{k}{T_i}$ $(= k f_i)$

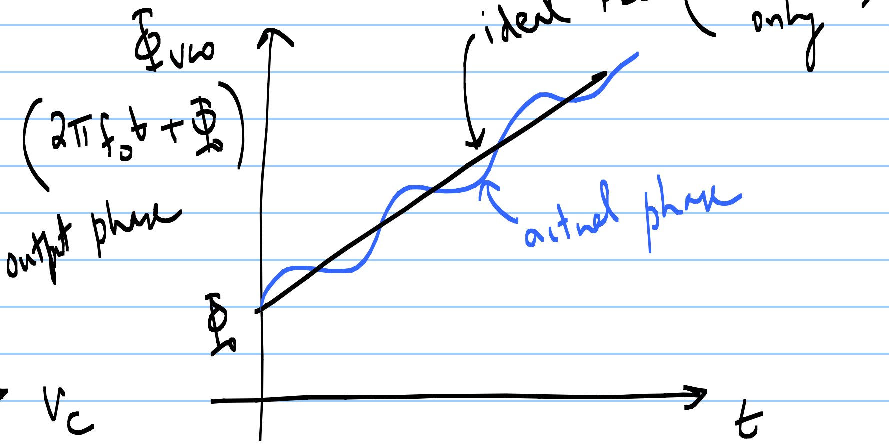

A
B
QA
QB
0

$f_A >> f_B$



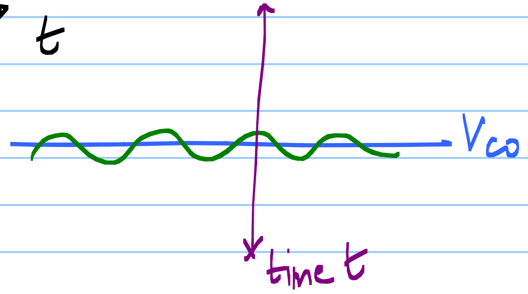
$V_c(t) = \underset{(DC)}{V_{c_0}} + \underset{v_{c_1}(t)}{\text{fundamental}} + \underset{v_{c_k}(t)}{\text{harmonics}}$ → output freq. will have avg. value = $f_{free} + K_{vco} \cdot V_{co}$





instantaneous freq.) − dep. on $V_c(t)$

$$\Phi_{vco} = 2\pi f_{vco} t + \Phi_0 = \Phi_{out}$$



assume only fundamental exists

$$V_c(t) = V_{co} + v_c(t)$$

$$f_o = f_{vco} = f_{free} + K_{vco}\cdot V_{co} + K_{vco}\cdot v_c(t)$$

PLL output

$$V_o(t) = A\cos\left(2\pi f_o t + \Phi_o\right)$$

$$= A\cos\left[2\pi\left(\underbrace{f_{free} + K_{vco}\cdot V_{co}}_{N\cdot f_i}\right) + 2\pi\int_{-\infty}^{t}\underbrace{K_{vco}\cdot v_c(t)}_{FM}\,dt + \Phi_o\right]$$

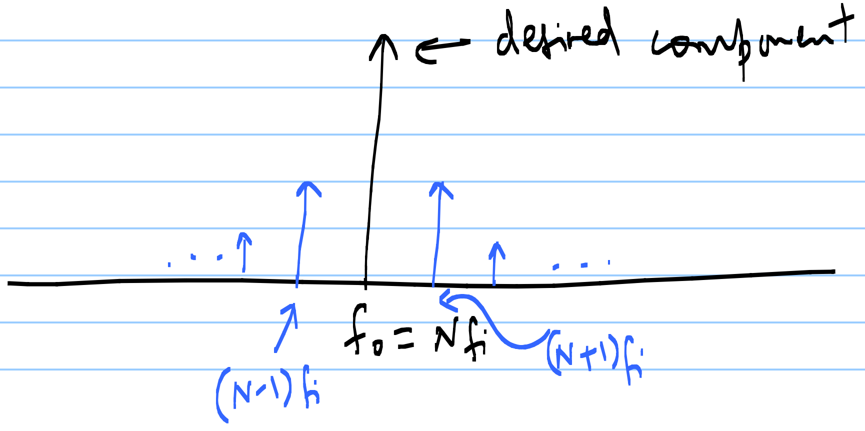
- e.g. $v_c(t) = a\cos(2\pi f_i t)$

$a =$ small

$$v_o(t) = A\cos\left(2\pi N f_i t + 2\pi K_{vco}\cdot a\,\sin(2\pi f_i t) + \Phi_o\right)$$

output components @ Nf_i ; $(N\pm1)f_i$; $(N\pm k)f_i$

VCO spectrum



Type-I PLL output

$(N\mp k)f_i$ $(k\neq 0)$ } reference feedthrough

* filter $V_c(t)$ to give V_{co} ← affects ω_n, PM, ...

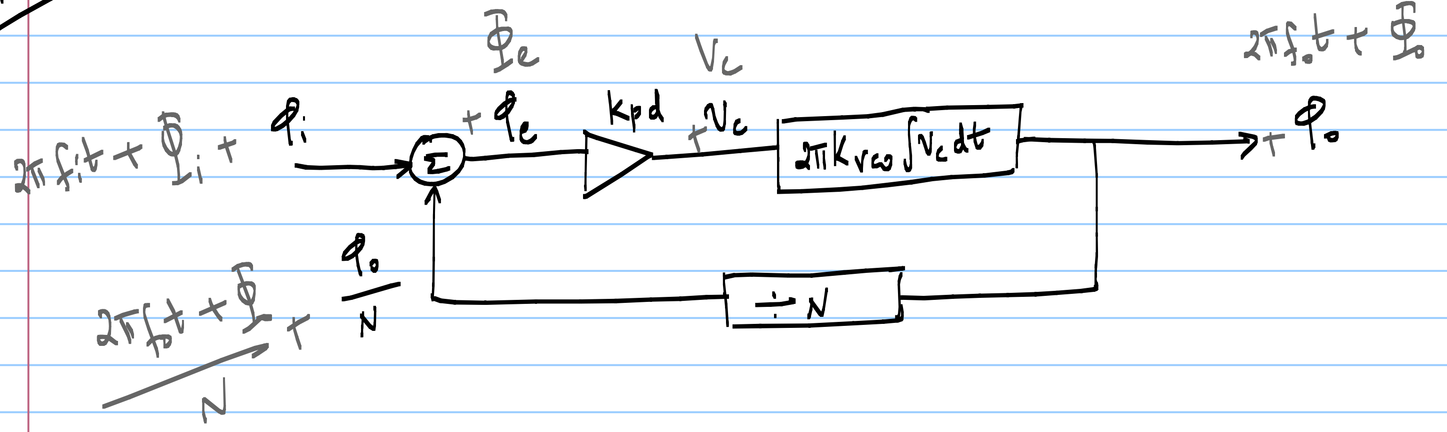
* Reduce ω_n of loop ↔ tradeoff loop speed/BW vs. ref. feedthrough.

* Type I loop is simple, stable

* find use in clocks of digital systems

19/12/23

Lec 28



$f_o \neq f_{free}$

$\Downarrow$

$V_c \neq 0$

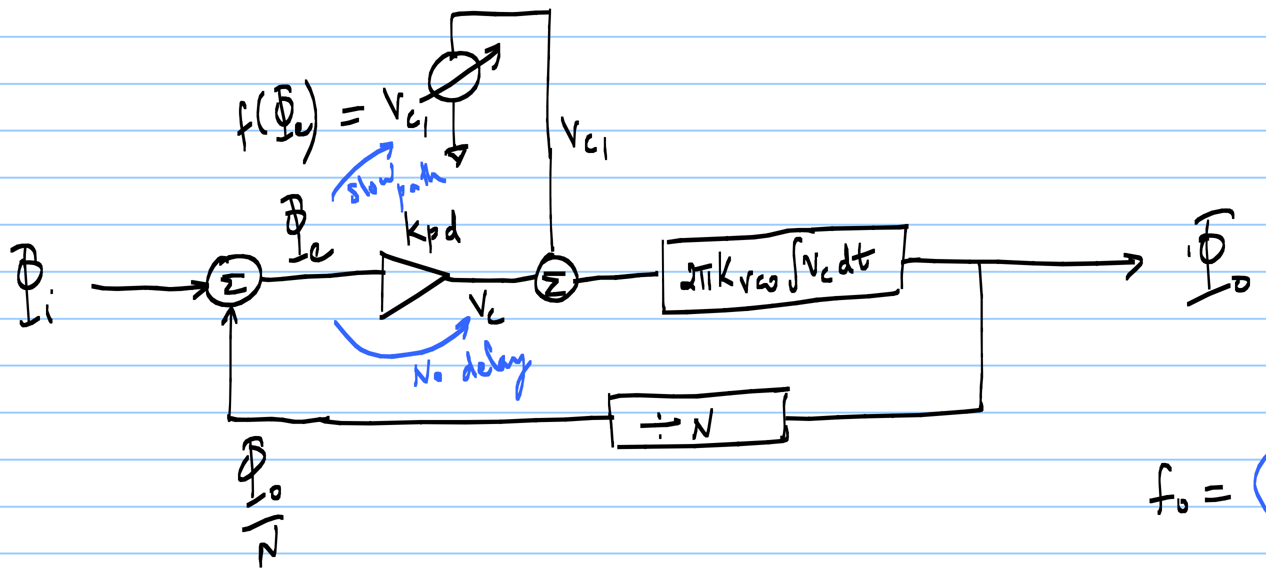
$\Downarrow$

$\Phi_e \neq 0$

$\Downarrow$

$$2\pi f_i t + \Phi_i \neq \frac{2\pi f_o t}{N} + \frac{\Phi_o}{N}$$

$$\Phi_e = \frac{N f_i - f_{free}}{K_{vco}\, K_{pd}} \Rightarrow \Phi_e = 0 \text{ only when } N f_i = f_{free}$$



Set V_{c1} so that

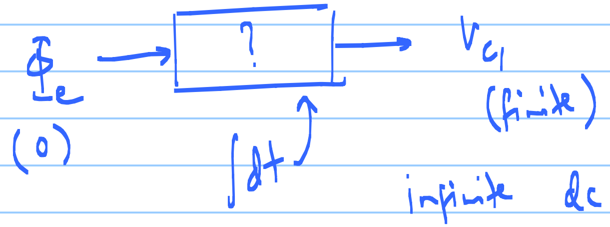
$V_c = 0$ in steady state

$\Downarrow$

$\Phi_e = 0$

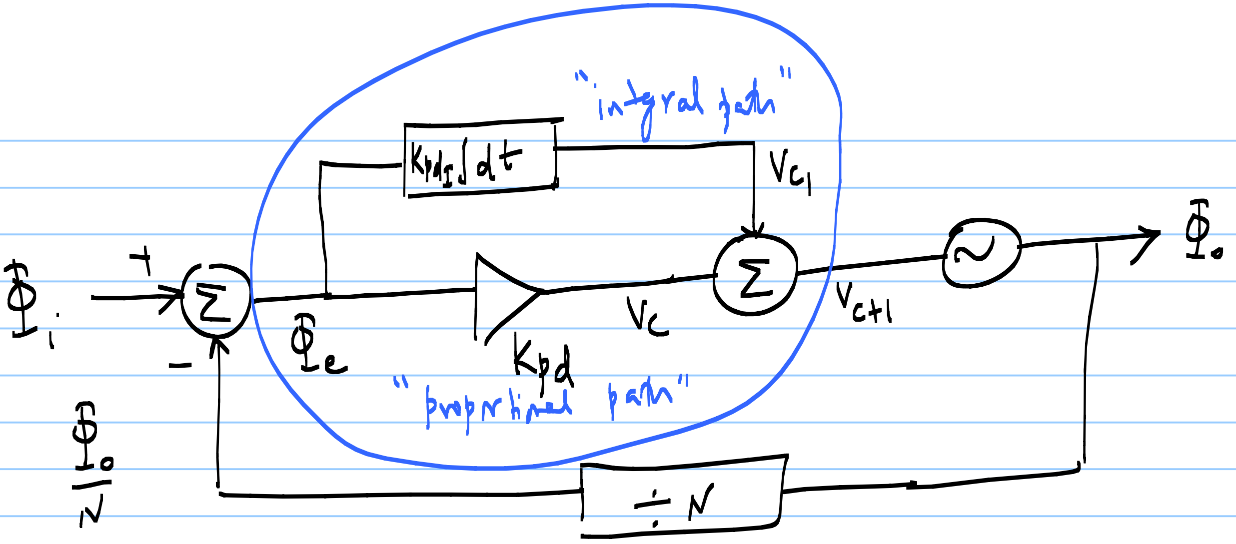
$$f_o = \underbrace{f_{free} + K_{vco}\cdot V_{c1}}_{f'_{free}} + K_{vco}\cdot V_c$$

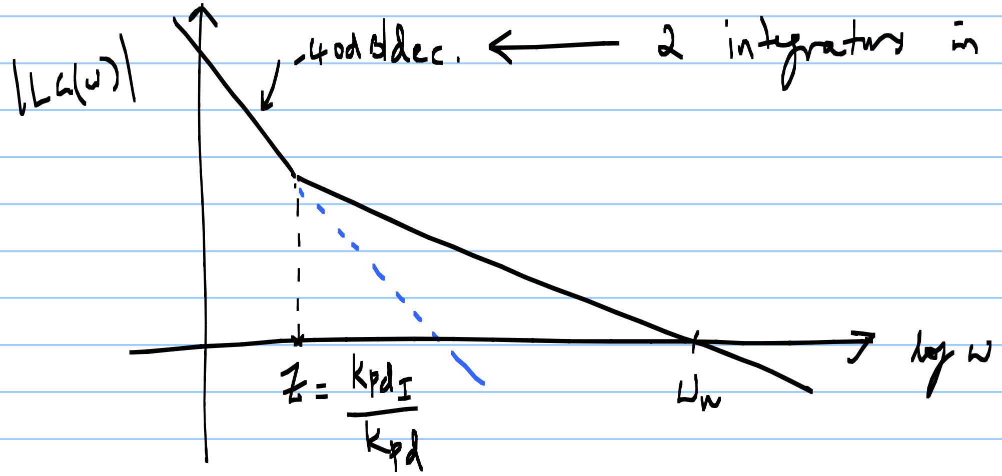
$$V_{c1} = \frac{f_o - f_{free}}{K_{vco}}$$



infinite dc gain

Type-II PLL



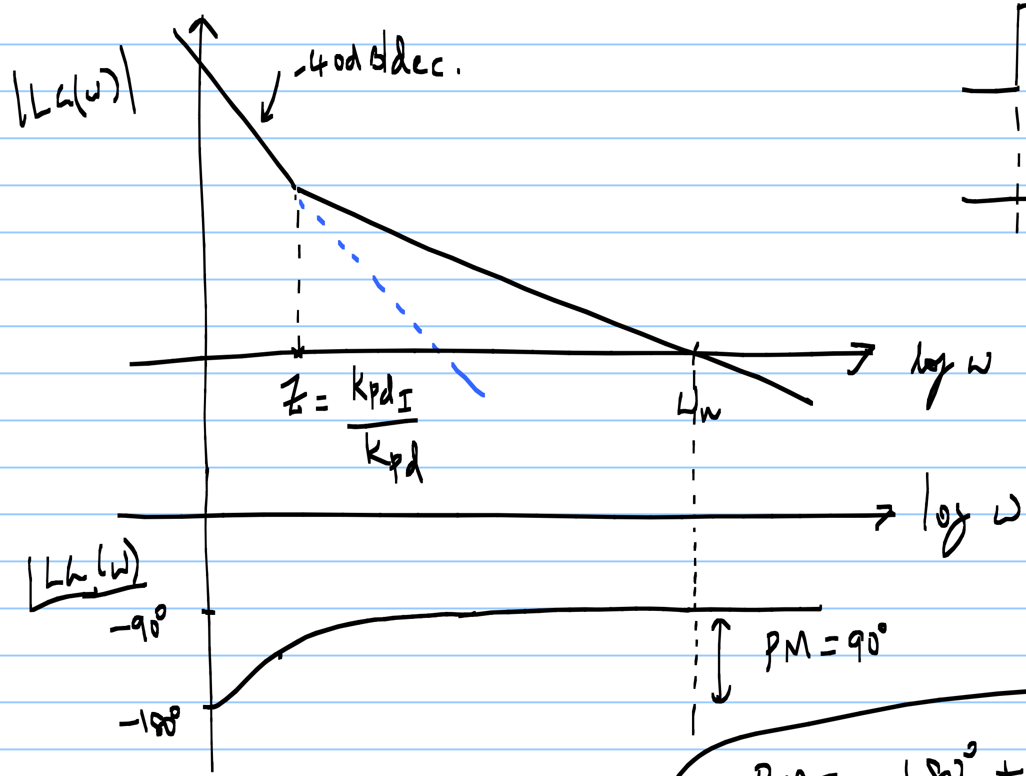


← 2 integrators in loop (K_{pd_I} ; Vco)

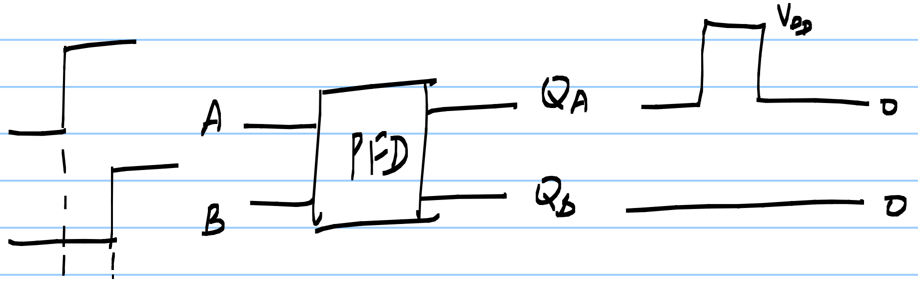
$$\omega_u = \frac{2\pi K_{pd} K_{vco}}{N}$$

$\underline{\Phi_e \text{ to } V_{ctl}}$

$$K_{pd} + \frac{K_{pd_I}}{s} = \frac{sK_{pd} + K_{pd_I}}{s}$$

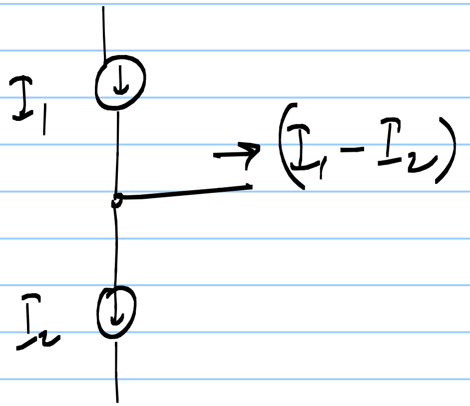


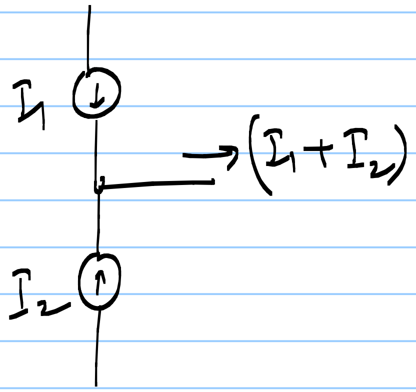
$$\boxed{PM = 180^\circ + \angle L_G(\omega_u)}$$



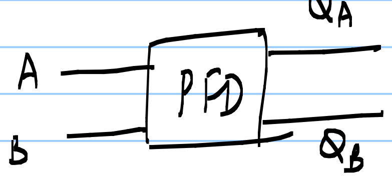
$$\underbrace{(Q_A - Q_B) \rightarrow \text{represents } \Phi_e}$$

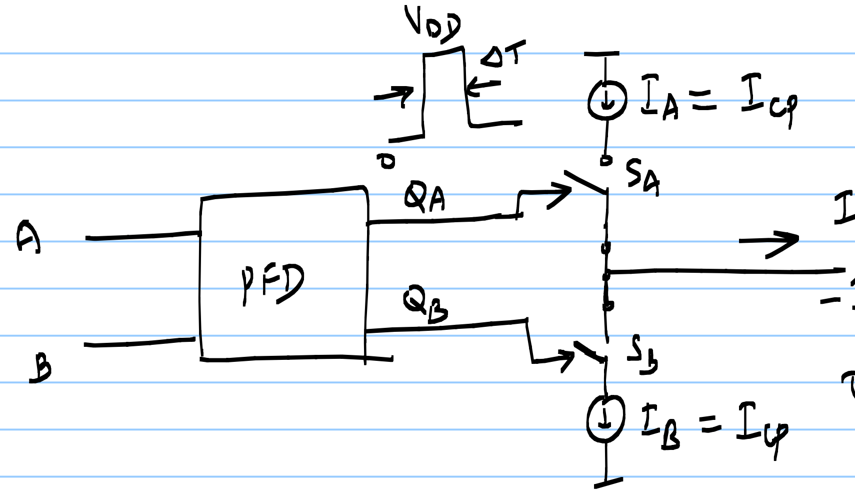
addition & subtraction are most easily done using currents





charge pump

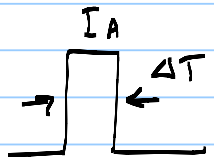




I_A when S_A is ON

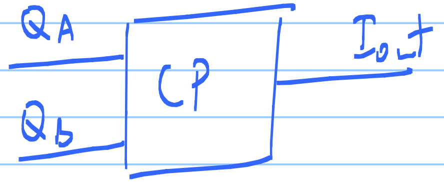
$-I_B$ when S_B is ON

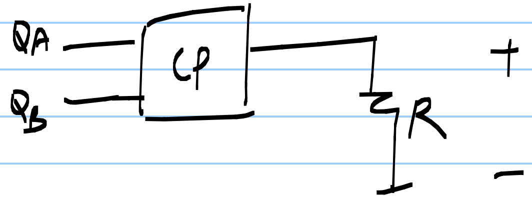
0 when both are OFF



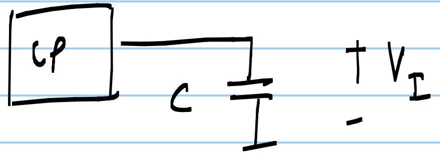
$\Delta Q = I_A \cdot \Delta T$

"charge pump"

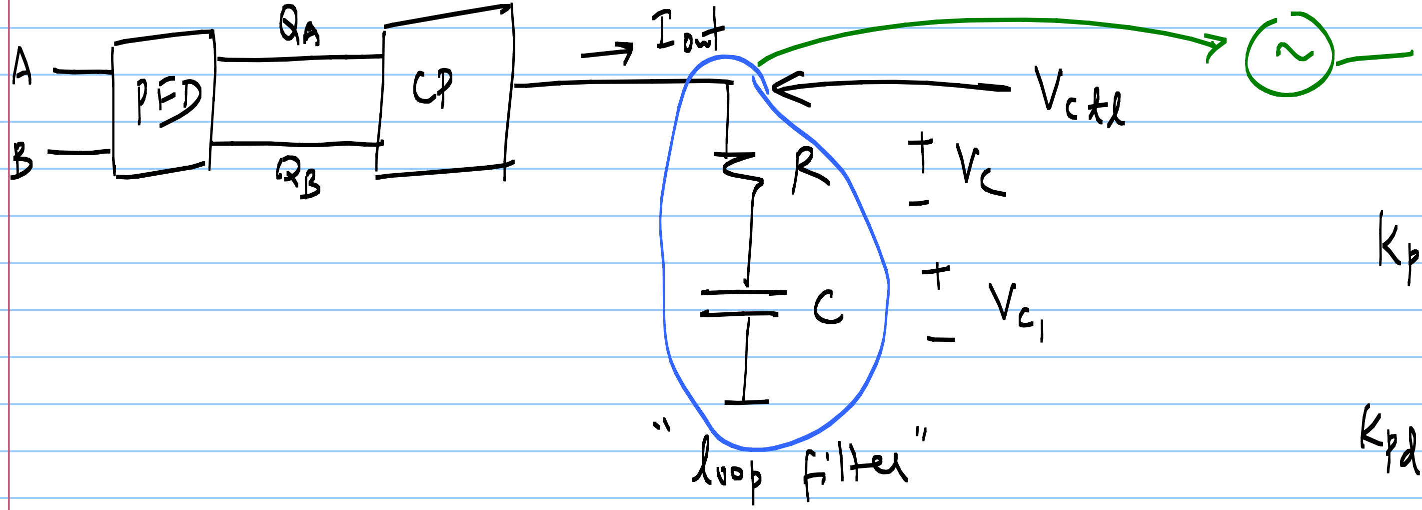




"proportional path"

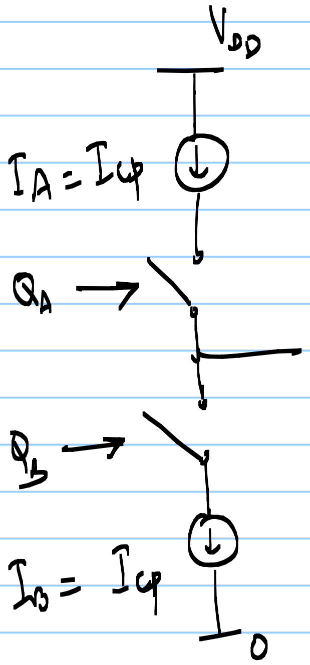


"Integral path"

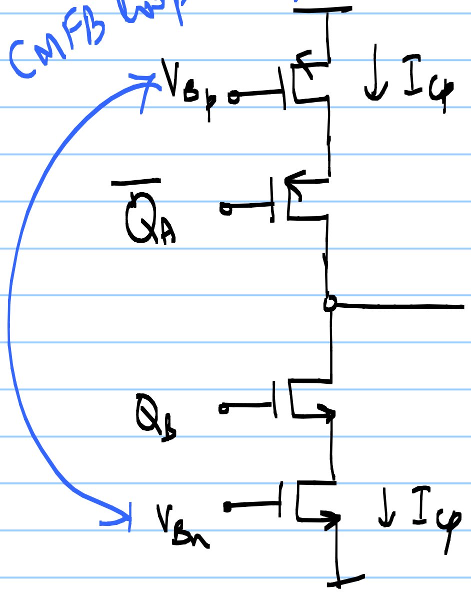


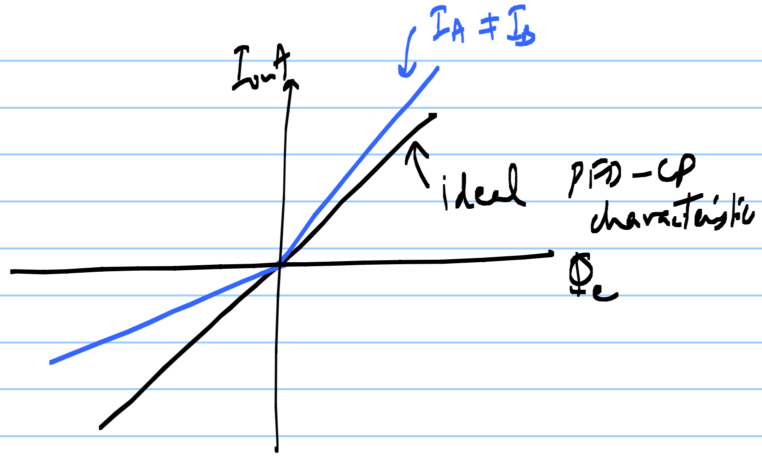
$$K_{pd} = \frac{V_{DD}}{2\pi} \times \frac{I_{cp}}{V_{DD}} \times R = \frac{I_{cp} \cdot R}{2\pi}$$

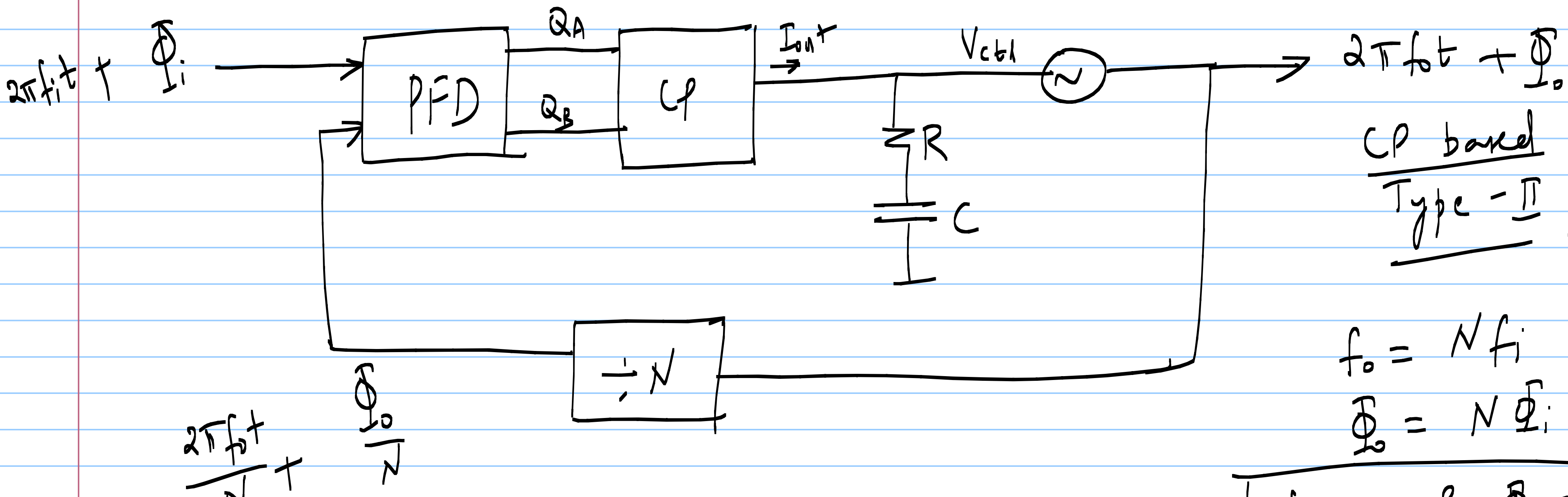
$$K_{pd_I} = \frac{I_{cp}}{2\pi C}$$



CMFB loop may be necessary to set V_{Bn} & V_{Bp} properly







CP based
Type-II PLL

$f_o = N f_i$

$\Phi_o = N \Phi_i$

$$\boxed{f_e = 0 \ \& \ \Phi_e = 0}$$

Type-II PLL design example

- $f_i = 10\,kHz$) $N = 100$
- $f_o = 1\,MHz$)
- Loop BW = 500 Hz ← f_{-3dB}
- $K_{vco} = 500\,kHz/V$
- $\zeta = \frac{1}{\sqrt{2}} = 0.707$ ← ? PM, settling response

$$LG(s) = \frac{I_{cp}}{2\pi}\left[R + \frac{1}{sC}\right] \times \frac{2\pi K_{vco}}{s} \times \frac{1}{N}$$

$$= \frac{I_{cp} \cdot K_{vco}(1+sCR)}{s^2 NC}$$

$$CLG(s) = N \times \frac{LG(s)}{1+LG(s)} = N \cdot \frac{I_{cp} \cdot K_{vco}(1+sCR) / s^2 NC}{1 + I_{cp} \cdot K_{vco}(1+sCR) / s^2 NC}$$

$$= N \cdot \frac{I_{cp} \cdot K_{vco}(1+sCR)}{s^2 NC + s \cdot I_{cp} K_{vco} \cdot CR + I_{cp} \cdot K_{vco}}$$

$$CLG(s) = N \cdot \frac{\frac{I_{cp} \cdot K_{VCO}}{NC}(1+sCR)}{\boxed{s^2 + s \cdot \frac{I_{cp} \cdot K_{VCO} \cdot R}{N} + \frac{I_{cp} \cdot K_{VCO}}{NC}}}$$

$$D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = \sqrt{\frac{I_{cp} \cdot K_{VCO}}{NC}} \quad \longleftarrow \quad ?$$

$$\zeta = \frac{R}{2}\sqrt{\frac{I_{cp} \cdot K_{VCO} \cdot C}{N}}$$

$$\omega_{-3dB} = 2\pi \times 500 \quad rad/s$$

for a second order system:

$$\omega_{-3dB}^2 = \underset{\uparrow}{\omega_n^2}\left[(2\zeta^2+1) + \sqrt{(2\zeta^2+1)^2 + 1}\right]$$

find out wn from here

$$(2\pi \times 500)^2 = \omega_n^2 \times \left(2+\sqrt{5}\right)$$

$$\omega_n = 1.527 \text{ k rad/s}$$

$$\omega_n = \underbrace{\sqrt{\frac{I_{cp}\cdot K_{vco}}{NC}}}$$

K_{vco}, N are known

$\rightarrow$ sets $\dfrac{I_{cp}}{C} = 466.1$ A/F

set I_{cp} via noise constraint

here set $I_{cp} = 10\mu A$ $\Rightarrow$ $\boxed{C = 21.5 \text{ nF}}$

$$\zeta = \frac{1}{\sqrt{2}}$$

$$\zeta = \frac{R}{2}\sqrt{\frac{I_{cp} \cdot K_{vco} \cdot C}{N}} = \frac{1}{\sqrt{2}}$$

$$R = \sqrt{\frac{2N}{I_{cp} \cdot K_{vco} \cdot C}} = 43.1 k\Omega$$