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→ Back to antenna problems
(after a long detour)

Recall: $\nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J}$ and $\nabla^2 \vec{F} + k^2 \vec{F} = -\varepsilon M$

and Using $g_{3D}(r, r') = \dfrac{1}{4\pi} \dfrac{e^{-jk|r-r'|}}{|r-r'|}$

$$\vec{A}(r) = \int_{-\infty}^{\infty} d^3r' \, \mu \vec{J}(r') g(r, r') \quad \text{and}$$

$$\vec{F}(r) = \int_{-\infty}^{\infty} d^3r' \, \varepsilon \vec{M}(r') g(r, r')$$

From here $\vec{H}_A = \dfrac{1}{\mu} \nabla \times \vec{A} \rightarrow \vec{E}_A = \dfrac{1}{j\omega\varepsilon} \nabla \times \vec{H}_A$

From here, $\vec{E}_F = \dfrac{-1}{\varepsilon} \nabla \times \vec{F} \rightarrow \vec{H}_F = \dfrac{-1}{j\omega\mu} \nabla \times \vec{E}_F$

This gives us the fields at any $\vec{r}$ given $\vec{J}, \vec{M}$. exactly.

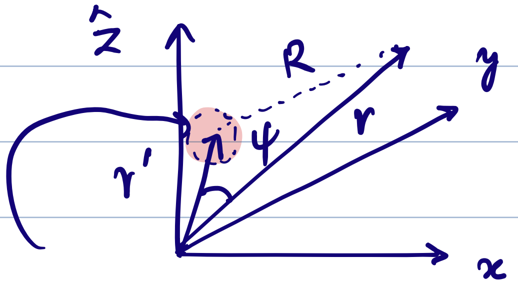
→ If we want near field terms, solve exactly numerically or analytically via 6.8.1. (Balanis)

→ Mostly we are interested in far field radiation where A_θ & $A_\phi \sim \underbrace{\text{fns}(\theta, \phi) \dfrac{e^{-jkr}}{r}}$

Similarly F_θ & F_ϕ also

which gives $E_A \approx -j\omega A$ & $H_F \approx -j\omega F$ for θ, ϕ comps.

Let's specialize this more:



currents are here

$$R^2 = r^2 + r'^2 - 2rr'\cos\psi$$

$$\therefore R \approx \begin{cases} r - r'\cos\psi & \text{, phase} \\ r & \text{, amp} \end{cases}$$

↳ Now let's assume we have surface currents. J_s, M_s. So,

$$\vec{A} = \frac{\mu}{4\pi}\int \frac{\vec{J}_s e^{-jkR}}{R}\,ds' = \frac{\mu e^{-jkr}}{4\pi r}\underbrace{\int \vec{J}_s e^{jkr'\cos\psi}\,ds}_{\vec{N}}$$

and

$$\vec{F} = \frac{\varepsilon}{4\pi}\int \frac{\vec{M}_s e^{-jkR}}{R}\,ds' = \frac{\varepsilon e^{-jkr}}{4\pi r}\underbrace{\int \vec{M}_s e^{jkr'\cos\psi}\,ds}_{\vec{L}}$$

→ Now let's list out all the components.

($\because$ both $\vec{A}$ & $\vec{F}$ lead to both $\vec{E}$ & $\vec{H}$)

a) The straightforward ones:

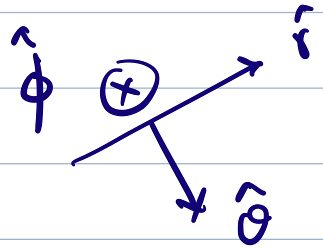
$$(E_A)_\theta = -j\omega A_\theta \quad , \quad (E_A)_\phi = -j\omega A_\phi$$

$$(H_F)_\theta = -j\omega F_\theta \quad , \quad (H_F)_\phi = -j\omega F_\phi$$

b) We also need $\begin{Bmatrix} E_F \\ H_A \end{Bmatrix}$ for θ, ϕ comps.

Recall: $E_F = \dfrac{-1}{j\omega\mu}\nabla \times H_F$ (one way).

Another (smarter way)? In the farfield everything behaves as a (spherical) TEM wave, so:



So $(E_A)_\theta$ must have $(H_A)_\phi$ s.t. $\dfrac{(E_A)_\theta}{(H_A)_\phi} = \eta$

and $(E_A)_\phi$ must have $-(H_A)_\theta$ s.t.

$$\frac{(E_A)_\phi}{(H_A)_\theta} = -\eta$$

So we get: $(H_A)_\theta = -\dfrac{1}{\eta}(E_A)_\phi$,

$$(H_A)_\phi = \frac{1}{\eta}(E_A)_\theta.$$

Similarly we get:

$$(E_F)_\theta = \eta\,(H_F)_\phi$$

$$(E_F)_\phi = -\eta\,(H_F)_\theta$$

RHS know from (a)

(We could have also derived this via duality)

So we can combine all the above:

$E_r \sim 0$	$H_r \sim 0$
$E_\theta = (E_A)_\theta + (E_F)_\theta$	$H_\theta = (H_A)_\theta + (H_F)_\theta$
$= -j\omega A_\theta - j\omega\eta F_\phi$	$= \frac{j\omega}{\eta}(A_\phi - \eta F_\theta)$
$E_\phi = (E_A)_\phi + (E_F)_\phi$	$H_\phi = (H_A)_\phi + (H_F)_\phi$
$= -j\omega A_\phi - j\omega\eta F_\phi$	$= -\frac{j\omega}{\eta}(A_\theta + \eta F_\phi)$

Substituting for A & F in terms of L & N.

$$E_r \sim 0$$

$$E_\theta \sim j\frac{k e^{-jkr}}{4\pi r}(L_\phi + \eta N_\theta)$$

$$E_\phi \sim j\frac{k e^{-jkr}}{4\pi r}(L_\theta - \eta N_\phi)$$

$$H_r \sim 0$$

$$H_\theta \sim j\frac{k e^{-jkr}}{4\pi r}\left(N_\phi - \frac{L_\theta}{\eta}\right)$$

$$H_\phi \sim -j\frac{k e^{-jkr}}{4\pi r}\left(N_\theta - \frac{L_\phi}{\eta}\right)$$

[Summary: input $J_s, M_s \rightarrow$ Calc N, L $\rightarrow$ Get E, H.]