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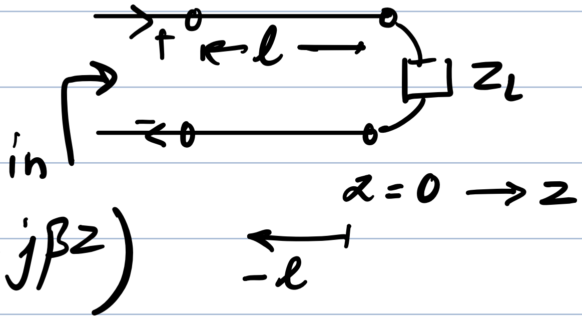
(f) We saw that at the load end $Z_L = \frac{V(0)}{I(0)}$.
But we can define this Z at any point, consider:



$$\therefore Z_{in} = \frac{V_0^+\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right)}{\frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{+j\beta z}\right)}$$

↳ Note: no need to memorize. Go term by term.

↳ Now subs $z = -l$

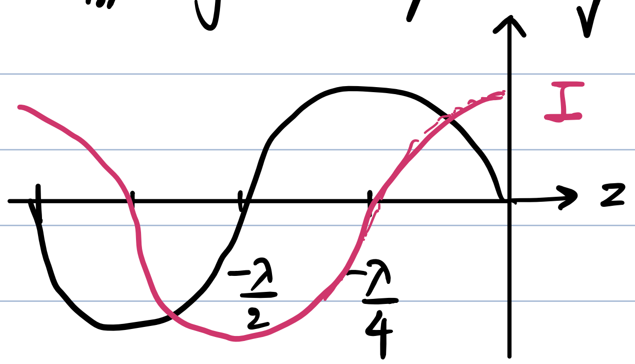
$$\frac{Z_{in}}{Z_0} = \frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l} \iff \bar{Z}_{in} = \frac{\bar{Z}_L + j\tan\beta l}{1 + j\bar{Z}_L\tan\beta l}$$

where, $\bar{Z} = Z/Z_0$.

2 special cases of practical use:

1) short ckt $\Rightarrow Z_L = 0 \Rightarrow Z_{in} = jZ_0\tan\beta l$.

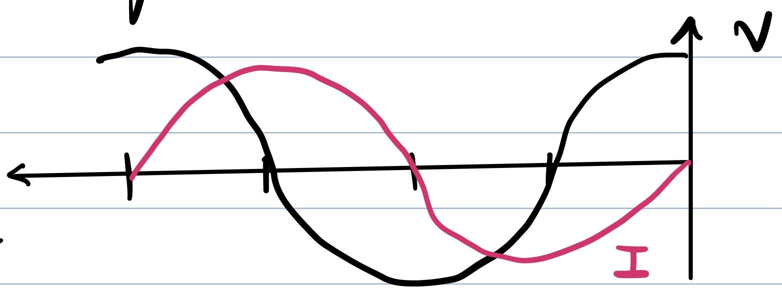
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$



⇒ Inductive element for $0 \le l \le \lambda/4$.

2) Similarly for open ckt., $Z_L \to \infty$, $\Gamma = 1$

$Z_{in} = -jZ_0\cot\beta l$



⇒ Capacitive element for $0 \le l \le \lambda/4$.

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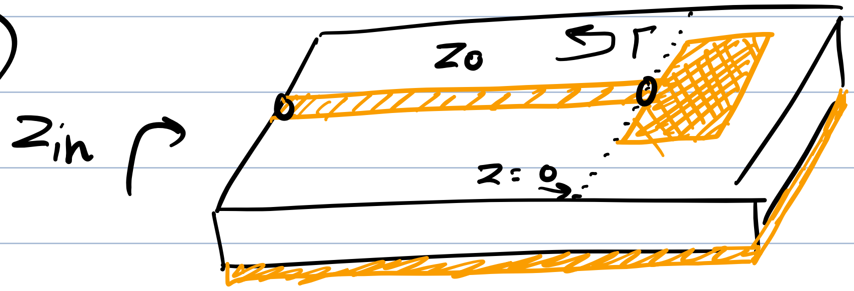
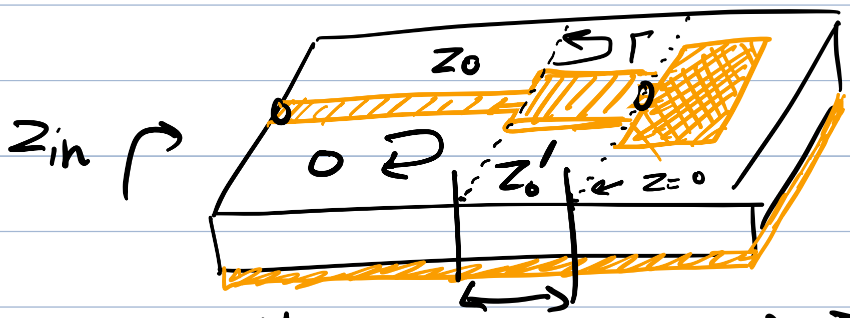


e.g. unmatched antenna & TL.

At $z=0$ we have a load Z_L. eg. $100\,\Omega$
We want $\Gamma = 0$. How? & $Z_0 = 50\,\Omega$.

Consider a new TL, $l = \lambda/4 \rightarrow \frac{Z_{in}}{Z_0'} = \frac{Z_0'}{Z_L}$, i.e. Z_L is transformed to $Z_{in} = \frac{Z_0'^2}{Z_L}$

We want $Z_{in} = Z_0$. $\therefore Z_0 = Z_0'^2/Z_L$
$\Rightarrow Z_0' = \sqrt{Z_0 Z_L}$



No refl.

$\lambda'/4$ → Imp to keep in mind that $\lambda' \neq \lambda$

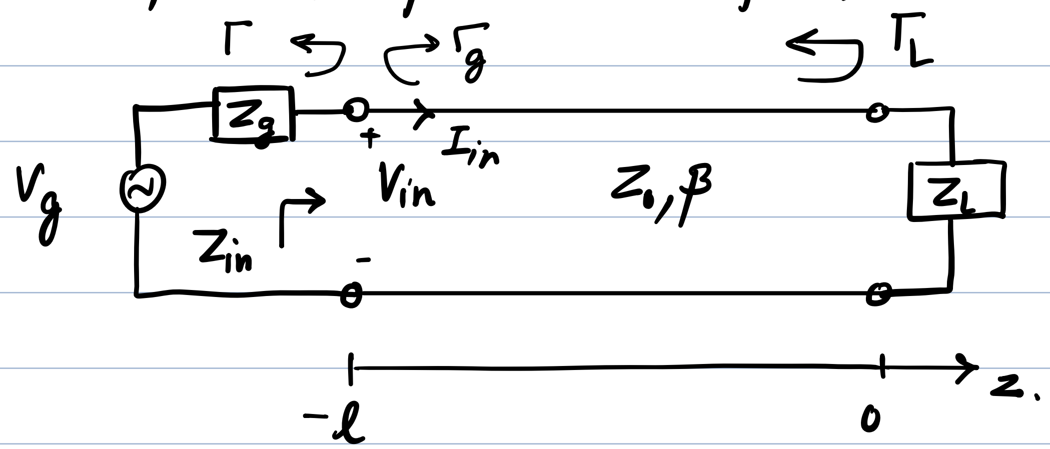
standing waves

Use new TL params. → $\lambda' = 2\pi/\beta'$

Called a quarter wave transformer.

→ Complication: Z_L is complex! Put a stub in series/parallel s.t. Z_L' is purely real. Then do QWT. → Single stub matching.

⑥ Optimal power transfer, the full picture.



Q: Is it possible that under max power transfer to the load, there can be standing waves on the line?

↳ 2 sources of reflections:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ (load)}, \quad \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} \text{ (gen)}$$

↳ Power to load $= P = \frac{1}{2} \mathrm{Re}\left[V_{in} I_{in}^* \right] \rightarrow$ ∵ TL is lossless, all power goes to load

with $V_{in} = Z_{in} I_{in}$, $P = \frac{1}{2} |V_{in}|^2 \mathrm{Re}\left[\frac{1}{Z_{in}^*} \right]$

$$V_{in} = \frac{V_g Z_{in}}{Z_g + Z_{in}}, \quad Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

$$= R_{in} + jX_{in}.$$

$$\mathrm{Re}\left[\frac{1}{R_{in} + jX_{in}} \right] = \frac{R_{in}}{R_{in}^2 + X_{in}^2}.$$

$$\Rightarrow P = \frac{1}{2}|V_g|^2 \left|\frac{Z_{in}}{Z_g + Z_{in}}\right|^2 \mathrm{Re}\left[\frac{1}{Z_{in}}\right]$$

$$\boxed{P = \frac{1}{2}|V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}.}$$

→ with Z_g as considered fixed, look at 3 cases. [Z_0 is a lossless TL]

① Match the load to the line, i.e. $Z_L = Z_0$

$$\Rightarrow Z_{in} = Z_0 \Rightarrow P = \frac{1}{2}|V_g|^2 \frac{Z_0}{(Z_0 + R_g)^2 + X_g^2}$$

Here $\Gamma_L = 0$.

② Match $\underbrace{\text{generator}}_{Z_g}$ to $\underbrace{\text{loaded line}}_{Z_{in}}$

i.e. $Z_{in} = Z_g$. An overall coeff $\Gamma = \frac{Z_{in} - Z_g}{Z_{in} + Z_g} = 0$.

$$P = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g^2 + X_g^2)}$$

Possible that ② is less than ①.

③ Conjugate Matching.

What if we vary Z_{in} for optimal power tx? Then we can back calculate Z_L.

So, set $\frac{\partial P}{\partial R_{in}} = 0$ & $\frac{\partial P}{\partial X_{in}} = 0$

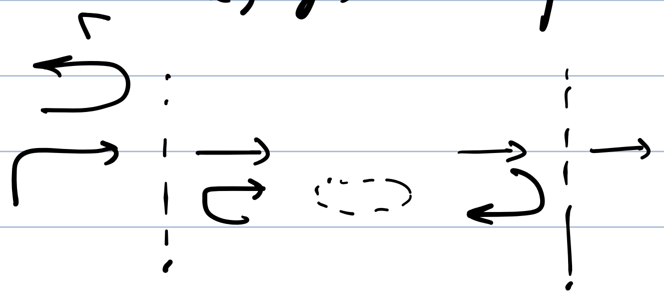
Simple math will give you $R_{in} = R_g$, $X_{in} = -X_g$.

i.e. $\boxed{Z_{in} = Z_g^*}$ and now $P = \frac{1}{2}|V_g|^2 \frac{1}{4R_g}$.

greater than ① or ②.

This gives the max power tx to the load for a fixed Z_g.

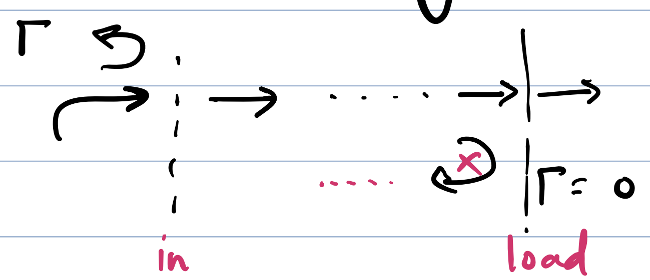
Here $\Gamma_e, \Gamma_g, \Gamma$ may all be nonzero.



Multiple reflections adding back in phase to give opt power.

Notes:

a) In ① will we have reflections? $\Gamma_L = 0$, $\Gamma_g \neq 0$. But, there is no source of a bkwd wave since $\Gamma_L = 0$. Recall the more fundamental def: $\Gamma = \frac{V_{ref}}{V_{inc}}$.



② If $X_g = 0$ then last 2 cases give same result.