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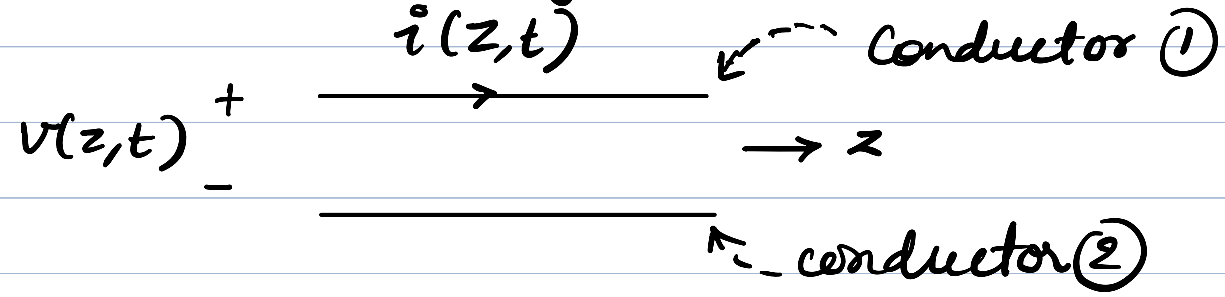
→ Review of transmission line Theory

1. Usual circuit theory assumes physical dimensions of network to be << electrical wavelength (λ). However, when size become comparable we need to move to a distributed element approach, characterized by transmission line (TL) theory.

There are 2 ways to approach TL theory:
① extending circuit theory, or
② field approach.

2. We will start with approach #1.

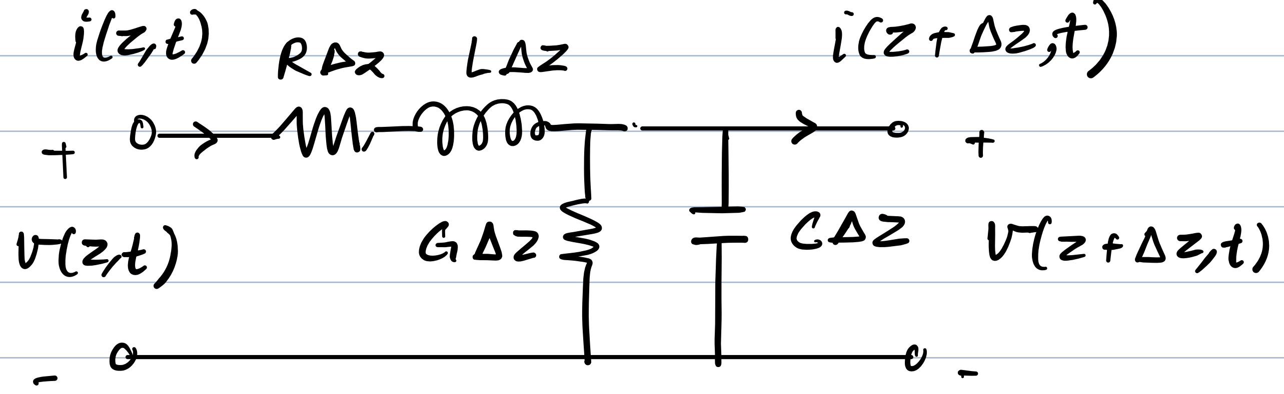
Represent a TL by an abstract picture of 2 conductors along the z direction:



The 4 parameters required to characterize are:
- R → series resistance Ω/m
- L → series inductance H/m
- G → shunt conductance S/m
- C → shunt capacitance F/m.

} all per length.

The equivalent ckt model is:



Applying KVL gives us:

$$v(z,t) - v(z+\Delta z,t) = R\Delta z\, i(z,t) + L\Delta z \frac{\partial}{\partial t} i(z,t)$$

Applying KCL gives us:

$$i(z,t) - G\Delta z\, v(z+\Delta z,t) - C\Delta z \frac{\partial}{\partial t} v(z+\Delta z,t) = i(z+\Delta z,t)$$

Standard manipulations follow, div by Δz, take the limit $\Delta z \to 0$, convert to phasors (Recal: $v(z,t) = \mathrm{Re}\left[V(z)\exp(j\omega t)\right]$), and use $\frac{\partial}{\partial t} \to j\omega$, to get these eqns:

$$\frac{d\,V(z)}{dz} = -(R + j\omega L)\, I(z)$$

$$\frac{d\,I(z)}{dz} = -(G + j\omega C)\, V(z)$$

Called the Telegrapher eqns.

3. Very easy to solve these coupled eqns: (take another d/dz)

$$\frac{d^2 V(z)}{dz^2} = \gamma^2 V(z) , \quad \frac{d^2 I(z)}{dz^2} = \gamma^2 I(z)$$

γ is the complex propogation constant,

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

and this admits two solutions : fwd & bkwd waves , $\exp(\pm \gamma z)$.

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

[Aside : why is $e^{-\gamma z}$ fwd traveling?
Convert from phasor to time domain:

$$v(z,t) = Re[V_0^+ e^{-\gamma z} e^{j\omega t}]$$
$$= Re[V_0^+ e^{-\alpha z} \exp(j(\omega t - \beta z))]$$

$$= |V_0^+| \, e^{-\alpha z} \cos(\omega t - \beta z + \phi_0)$$

To maintain the same phase front in the z-t coordinate space, i.e. $\omega t - \beta z + \phi_0 =$ const, as time $\uparrow$, z must also $\uparrow$.
This implies a right (fwd) travelling wave.]

Further simplifications from Telegraphers eqns give $\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}$ where

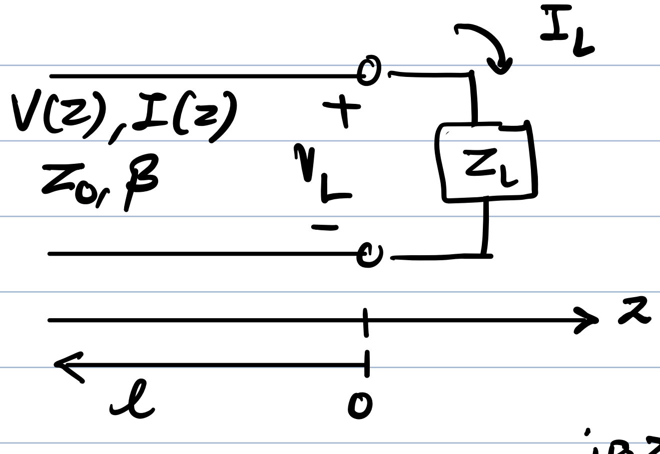
$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$ is called the characteristic impedance of the TL.

The wavelength on the line is $\lambda = \frac{2\pi}{\beta}$

and phase velocity $v_p = \frac{\omega}{\beta} = \lambda f$.

→ self study: Calculate all parameters for a lossless TL, i.e. when $R = G = 0$, including v, i expressions

4. Load Terminated TL. (Assume lossless)



l increases in the −z direction.

To recap: $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$

and $I(z) = \frac{1}{Z_0}\left[V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right]$

∴ At the load end $(z=0)$, $Z_L = \frac{V(0)}{I(0)}$

and we define Γ, voltage reflection coefficient as $\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$,

where V_0^- was eliminated.

Implications:

ⓐ In general when $\Gamma \neq 0$, we have both fwd & bkwd waves leading to standing waves due to interference.

ⓑ For $\Gamma = 0$, $Z_L = Z_0$, called the matched load condition.

ⓒ The time avg power in the line is

$$P_{avg} = \frac{1}{2} \mathrm{Re}\left[V(z) I(z)^* \right] = \frac{1}{2} \frac{|V_0^+|^2}{Z_0} \left(1 - |\Gamma|^2\right)$$

can be seen as power sent by source minus power reflected by load.

Sometimes reported as "Return loss" RL

$RL = 20 \log |\Gamma|$ (dB)

$\Rightarrow |\Gamma| = 0 \Rightarrow RL \to \infty$

$|\Gamma| = 1 \Rightarrow RL \to 0\,dB$

ⓓ In general we will not have $Z_L = Z_0$

So we will have a voltage wave, i.e.

$$|V(z)| = |V_0^+|\,|e^{-j\beta z} + \Gamma e^{j\beta z}|$$

$$= |V_0^+|\,|1 + \Gamma e^{2j\beta z}|$$

$$= |V_0^+|\,|1 + \Gamma e^{-2j\beta l}|,$$

where $z = -l$ measured from load end.

$\therefore$ $V_{max} = |V_0^+|(1 + |\Gamma|)$

$V_{min} = |V_0^+|(1 - |\Gamma|)$

VSWR (voltage standing wave ratio) $= \dfrac{V_{max}}{V_{min}}$

$= \dfrac{1+|\Gamma|}{1-|\Gamma|}$, $\quad 1 \leq \text{VSWR} \leq \infty$.

VSWR $= 1 \Rightarrow$ matched load.

(e) with $\Gamma = |\Gamma| e^{j\theta}$

$$|V(z)| = |V_0^+|\,|1 + |\Gamma| e^{j(\theta - 2\beta l)}|,$$

$\beta = 2\pi/\lambda$. $\Rightarrow$ $l \to l + \lambda/2$ leads to the same value of $|V(z)|$ whereas $l \to l + \lambda/4$ leads to a max value becoming a min value or vice versa.