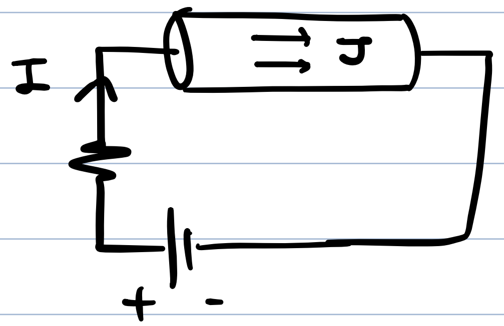
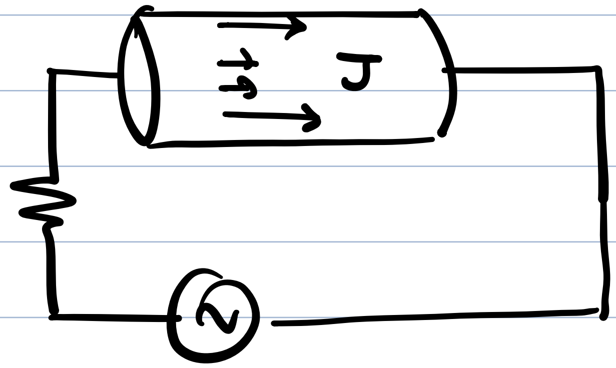
Current flow in a good conductor.

In a DC case current flows uniformly across the wire cross-section.



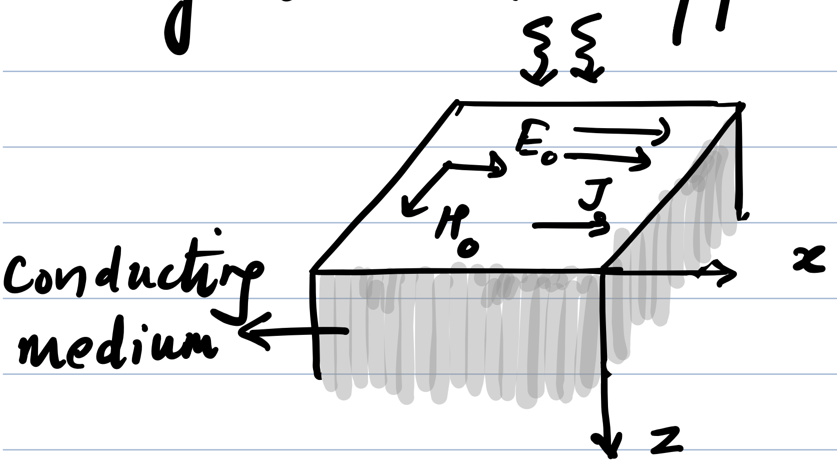
However, in the AC case the story changes.



Most current flows near the surface.

↳ Why does this happen?

(or an applied voltage from L to R)



Imagine a wave impinging from top and going into the conducting medium.

Say that the $\bar{E}$ field at the interface $z=0$ is $E_0\hat{x}$. Then the field at a $z>0$ is given as:

$$\vec{E}(z) = E_0 e^{-\alpha z} e^{-j\beta z}\,\hat{x}.$$

What is the magnetic field now?

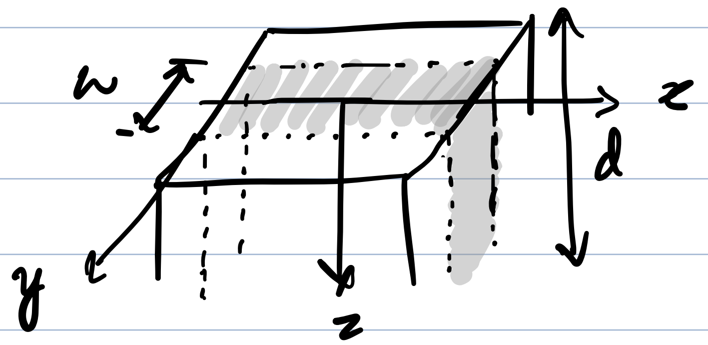
$$\vec{H}(z) = \frac{E_0}{\eta_c} e^{-\alpha z} e^{-j\beta z} \hat{y}$$

↳ From Ohm's law what is the induced current? $\bar{J} = \sigma \bar{E}$

$$\Rightarrow \bar{J}(z) = \underbrace{\sigma E_0 e^{-\alpha z} e^{-j\beta z}}_{J_x} \hat{x}$$

↳ Can this be simplified in a good conductor?

Yes! $\alpha \simeq \beta \left(= 1/\delta_s\right) \Rightarrow J_x = \sigma E_0 e^{-(1+j)\frac{z}{\delta_s}}$ A/m^2



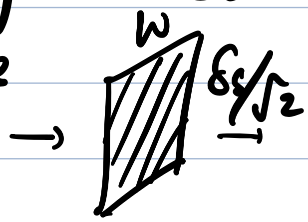
Now in a strip in the y-z plane, what is the total current flowing?

$$I = \iint dy\,dz\; J_x = w \int_0^\infty J_x\, dz$$

$$I = w \int_0^\infty \sigma E_0\, e^{-(1+j)z/\delta_s}\, dz = \frac{J_0 w \delta_s}{1+j} \ (A)$$

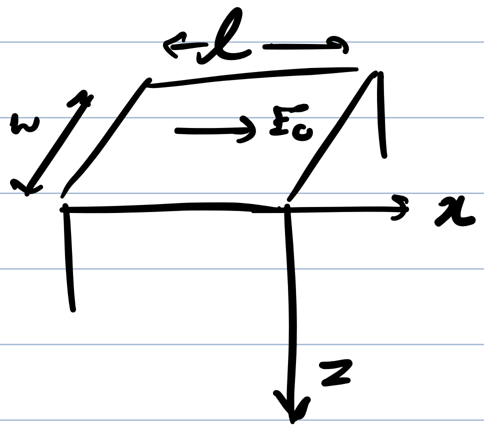
Infact if we do $I_{3\delta} = \iint_0^{3\delta_s} J_x\, dy\,dz$ the error in $I_{3\delta}$ is < 5% from the value of I.

The expression $\left| \frac{J_0 w \delta_s}{1+j} \right| = (J_0)\left(\frac{w\delta_s}{\sqrt{2}}\right)$

looks like a uniform current J_0 flowing in a box of size 

. So, as long as $d > 3\delta_s$, the entire current seems to flow like a const. qty in a skin depth! No matter what the value of d is.

$\Rightarrow$ I can save weight by making my conductors hollow. At 1 MHz, $\delta_s = 66\ \mu m$ for Copper.



Voltage drop at the surface?

$V \approx E_0 (V/m) \times l . (m)$

$V = \frac{J_0}{\sigma} l .$

So, the impedance of the slab of width w, length l, depth ∞, is:

$$Z \approx \frac{V}{I} = \frac{J_0 l/\sigma}{J_0 w \delta_s / 1+j} = \frac{1+j}{\sigma \delta_s} \frac{l}{W}$$

This is usually written as $Z = Z_s \frac{l}{w}$

(idepn of dim) surface/internal impedance of the conductor

$Z_s = R_s + j\omega L_s \Rightarrow R_s = \frac{1}{\sigma \delta_s}, \; L_s = \frac{1}{\omega \sigma \delta_s}$

Surface/~~intrinsic~~ resistance.

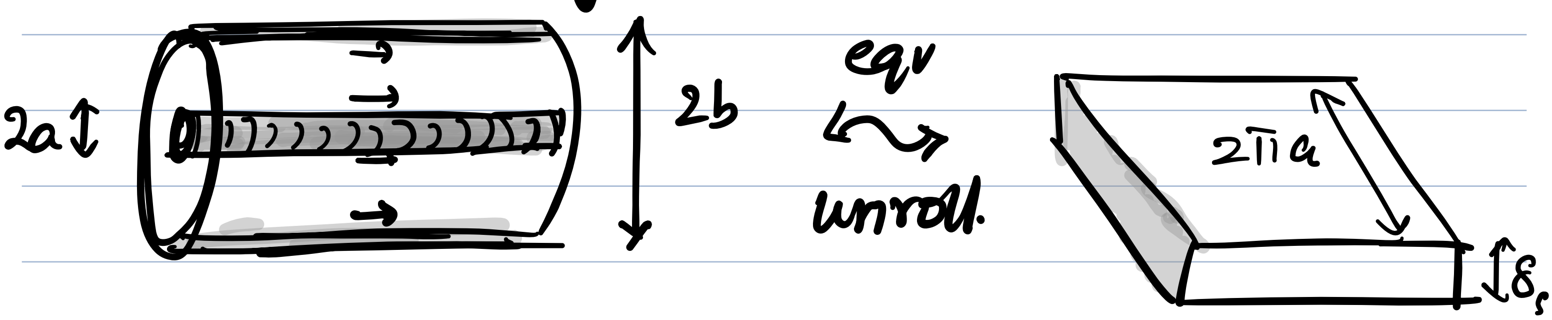
$\Rightarrow$ AC resistance is $R = R_s \frac{l}{w} = \frac{1}{\sigma \delta_s} \frac{l}{W}$

This reminds us of DC resistance

$R = \rho \frac{l}{A}$ or $\frac{1}{\sigma} \frac{l}{A}$ where $A = \delta_s W$.

What changed? Area $\rightarrow$ $\underline{\delta_s W}$

↳ Application of this? Consider coax cable.



$\therefore$ Resistance per length $R' = \frac{R}{l} = \frac{R_s \, l}{l \, W}$

$= \frac{R_s}{2\pi a}$. Similarly, $R_s/2\pi b$. Net $R = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$