# Radiation Patterns

We now focus exclusively in the antenna far-field. For the Hertz dipole, the Poynting vector is:

$$\vec{S} = \frac{\eta}{8}\left(\frac{I\Delta z}{\lambda}\right)^2 \frac{\sin^2\theta}{r^2}\,\hat{r} \quad -①$$

From here, the total power is $W = \int_{\text{sphere}, r} \vec{S}\cdot d\vec{A}$

$r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$

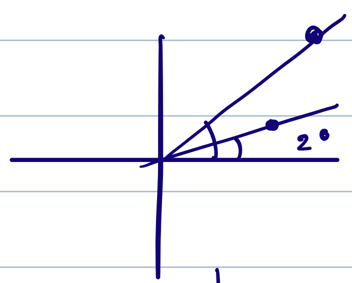
$$W = 40\pi^2\left(\frac{I\Delta z}{\lambda}\right)^2 \quad -③$$

It is useful to characterize rad patterns in terms of normalized quantities.

So, define $F(\theta,\phi) = \left.\frac{S(\theta,\phi)}{S_{max}}\right|_{\text{at the same } r}$ → as normalized radiation intensity.

For the Hertz dipole, $F(\theta,\phi) = \sin^2\theta$

Customary to visualize this as a polar plot. Recall what that is: e.g. given $r = \theta$, plot?




① $r=0 \Rightarrow \theta = 0 \Rightarrow$ origin

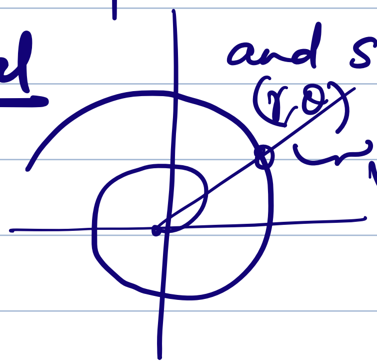
② $\theta = 2^\circ \Rightarrow r = 2\times\pi/180$

③ $\theta = 4^\circ \Rightarrow r = 4\times\pi/180$

and so on, $\theta = 180^\circ \Rightarrow r = \pi$

$\theta = 360^\circ \Rightarrow r = 2\pi$, a spiral.

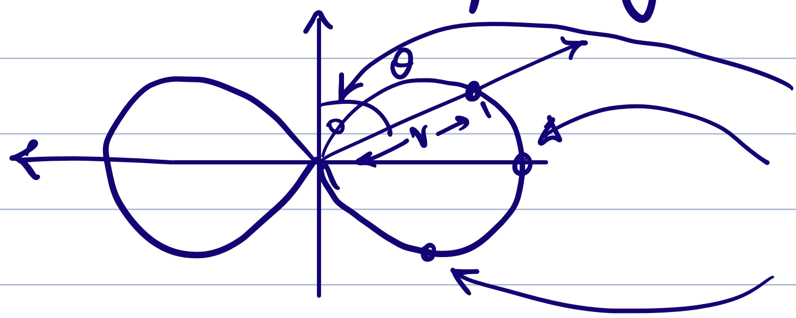
Spiral




[This $r$ is the $r$ for plotting the graph, not $r$ of $S$]

Now let's plot $F(\theta,\phi) = \sin^2\theta$ in a polar plot.

What are we plotting? $r = \sin^2\theta$ in $(r,\theta)$ coord.



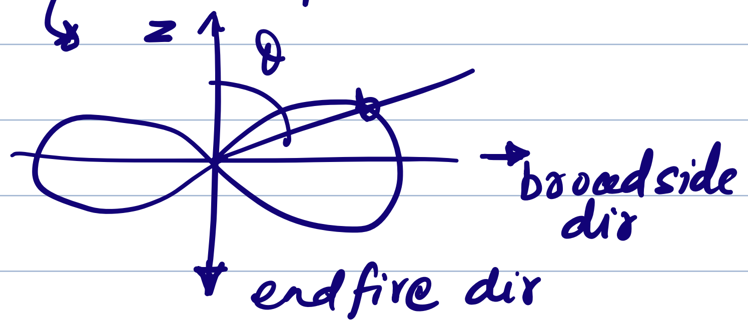

1. $\theta = 0,\ r = 0$
2. $\theta = 1^\circ,\ r = \sin^2(\pi/180)$
3. $\theta = \pi/2,\ r = 1$
4. $\theta = 3\pi/4,\ r = 1/2$
5. sym for $-\theta$

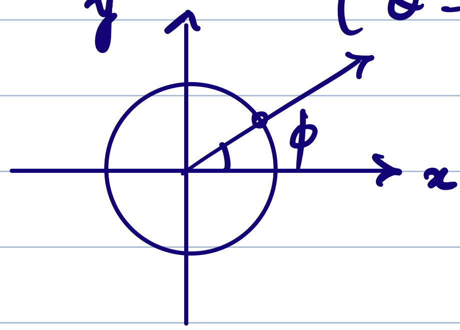
↳ How do we use this graph? Given a value of $\theta_0$, we find the value of $r_0$, i.e. where on the curve the ray from $\theta_0$ cuts. That's $r_0$ which we equate to $F(\theta_0,\phi_0) = \sin^2\theta_0$.

↳ We can make this 3D by rotating about the z axis, each rotation is a different $\phi$.

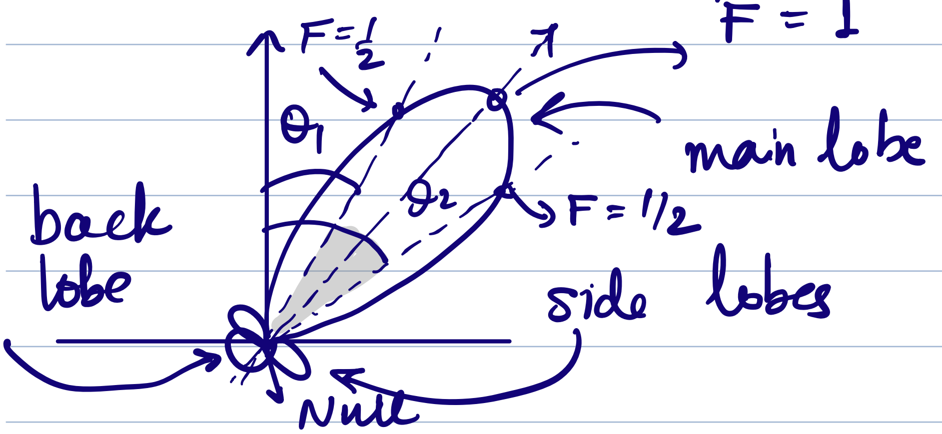
↳ Two common 2D views of the radiation pattern:

<u>Elevation</u> pattern $(\phi = 0)$, <u>Azimuthal</u> pattern $(\theta = 90^\circ)$




↳ Common radiation patterns are like so




Half power beamwidth $= \theta_2 - \theta_1$ (HPBW)

# Characterizing antenna rad patterns

In the farfield, the total power is $W = 40\pi^2 \left(\frac{I\Delta z}{\lambda}\right)^2$

From the point of view of the power source, that power isn't coming back, so we can model it as power lost across a resistor. So, with

$W = \frac{1}{2} I^2 R_{rad}$, we get $R_{rad} = 80\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$

↳ compute, for $\Delta z = 0.1\lambda$, $R_{rad} \approx 8\,\Omega$.

Directivity is a way to characterize the focusing nature of an antenna.
At a fixed r, define as,

$$D = \frac{\text{Max radiation intensity}}{\text{Avg radiation intensity}}$$

→ Averaged over all directions.

$$= \frac{F_{max}}{\frac{\iint F(\theta,\phi)\, d\Omega}{\iint d\Omega}}$$

$d\Omega = \sin^2\theta \, d\theta \, d\phi$

$\iint d\Omega = 4\pi$.

For a Hertz dipole, $F_{max} = 1 \quad \left[\max \sin^2\theta\right]$

$\Rightarrow D = 1 \Big/ \iint \sin^2\theta \sin\theta \, d\theta \, d\phi / 4\pi = 3/2$.

It is common to report this as 10 log.

So $D_{HE} = 10 \log 3/2 = 1.76 \text{ dB}$.

If I had an isotropic antenna? $D = 1$, (0 dB)

A related quantity is <u>antenna gain</u>.

This is similar to directivity but taking into account ohmic losses in the antenna, i.e. not all power gets radiated.

$$\Rightarrow \quad P_i = \underbrace{W}_{\text{radiation}} + \underbrace{P_\ell}_{\text{loss}}$$

power efficiency $\eta_r = \dfrac{W}{P_i} = \dfrac{W}{P_\ell + W}$.

Now Gain = $\dfrac{\text{Max radiated intensity (actual)}}{\text{Avg radiation intensity (lossless)}}$

$$G = \eta_r D$$

Again, better done in log scale

$$G_{dB} = \underbrace{\eta_{r\,dB}}_{< 0} + D_{dB}.$$