

Why is Copper Red?

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I mentioned in class that the colour of metals has to do with the frequency dependence of their reflection constant. Let us work this out.

We start with a metal, and hence a conductivity σ . We assume that the material is homogeneous and there is no magnetic field present (else additional effects will appear).

Maxwell's Equations now become

$$\nabla \times (\nabla \times \vec{E}) = -j\omega\mu\vec{J} + \omega^2\mu\epsilon\vec{E}$$

We are discussing the response to very high frequency waves ($\sim 10^{14}$ Hz) and assume that only the electrons can respond at this frequency. The electron momentum equation is given by

$$m\mathbf{n}(j\omega\vec{v}) + m\mathbf{n}\vec{v} \cdot \nabla\vec{v} = qn\vec{E} - m\mathbf{n}v\vec{v}$$

Keeping terms to lowest order (since the wave fields are assumed to be weak compared to the binding fields) the only terms that survive are those that involve \vec{v} and \vec{E} both of which are first order. So their coefficients are kept to zeroth order, and we get

$$mn_0\vec{v}(j\omega + \nu) = qn_0\vec{E}$$

Thus, we have an equation that connects the current to the applied field:

$$\vec{J} = qn_0\vec{v} = \frac{n_0q^2}{m} \frac{\vec{E}}{j\omega + \nu}$$

The wave equation now becomes, assuming transverse waves

$$\begin{aligned} -\nabla^2\vec{E} &= -j\omega\mu \frac{n_0q^2}{m} \frac{\vec{E}}{j\omega + \nu} + \omega^2\mu\epsilon\vec{E} \\ &= \omega^2\mu\epsilon \left(1 - \frac{n_0q^2}{m\epsilon} \frac{1}{\omega(\omega - j\nu)} \right) \vec{E} \end{aligned}$$

In these metals, the collisionality is low (which is why they are excellent conductors). So let us look at the limit of zero ν . We obtain

$$k^2\vec{E} = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2} \right) \vec{E}$$

where $\omega_p^2 = n_0q^2/m\epsilon$ is the "plasma frequency" of the conduction electrons in the metal. As long as this equation yields $k^2 > 0$, there will be atleast some incident angles

at which transmission occurs. If $k^2 < 0$, i.e., if the term in brackets is negative, then the wave cannot propagate in the metal and all of the energy must reflect.

Clearly there is a cutoff frequency above which propagation is possible. That frequency is $\omega = \omega_p$. For copper this frequency is near the red portion of the spectrum. For gold, this frequency is near yellow, while for silver, this frequency is into the blue region. This is why these three metals are the colour they are.

What will the reflection coefficient be? The impedance of the medium is given by E/H . And Faraday's Law has not changed its form. So

$$\frac{E}{H} = \frac{\mu\omega}{k} = \mu \frac{c}{1 - \omega_p^2/\omega^2} = \eta_1 \frac{1}{1 - \omega_p^2/\omega^2} = \eta_2$$

So,

$$R = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{1 - 1/(1 - \omega_p^2/\omega^2)}{1 + 1/(1 - \omega_p^2/\omega^2)} = -\frac{\omega_p^2/\omega^2}{2 - \omega_p^2/\omega^2}$$

If we look into the internet, <http://www.philiplaven.com/p19.html> tells us that the three primary colours on a display are given by

Red	Green	Blue
650 nm	510 nm	435 nm

Here is what Scilab gives if I use the above RGB frequencies and adjust the the plasma frequency:

First I define a function that takes a cut-off wavelength λ_0 and uses the above wavelengths to compute the intensities of R, G and B. The only thing to take care of is that when $\omega^2 < \omega_p^2$ the reflection coefficient remains 100%.

```
2a (* 2a)≡
function []=metal(lambda0)
    l=[650,510,435];
    alpha=l.^2/lambda0^2;
    alpha(find(alpha>1))=1;
    c=alpha ./ (2-alpha);
```

We add this colour to the palette and select that as the current foreground colour.

```
2b (* 2a)+≡
    c0=addcolor(c);
    xset("color",c0);
endfunction
```

Now just call this function with different cutoff wavelengths and see what colour is obtained.

```
2c (* 2a)+≡
plot2d(0,0,rect=[0 0 1 1],axesflag=0);
metal(520);
xfarc(0.1,.7,.4,.4,0,360*64);
xset("font size",4)
xstring(.2,.5,"520 nm");
metal(810);
xfarc(0.6,.7,.4,.4,0,360*64);
xset("font size",4)
xstring(.7,.5,"810 nm");
```



The problem of getting a good match lies in the fact that the primary colours on the display are themselves not pure, and our eyes do not see colours quite as a display screen shows them. So the colour of gold, for example is a combination of a large number of nearly similar display colours. A fixed colour can't reproduce this colour.