

$(I_3^1, I_2^{1,1}), (I_3^1, I_2^{1,2}), (I_3^1, I_2^{1,3})$, where I_3^1 is the union of the open intervals, $(-\infty, 0)$ and $(0, +\infty)$ in x_3 , and $I_2^{1,j}$, for $j = 1, 2, 3$, are shown in Fig. 1 and also defined in the text above. This approach is readily extendable for the $(n - 1)$ -variable nonnegativity test. Since the implementation of the $(n - 1)$ -variable nonnegativity test discussed here depends upon an n -variable positivity test, a detailed discussion of the latter, with emphasis on the singular cases, is contained in [13].

III. CONCLUSIONS

A procedure to perform the multivariable polynomial nonnegativity test is given. The implementation of the test depends upon a relation between $(n - 1)$ -variable nonnegative polynomials and a class of n -variable positive polynomials. As single-variable polynomials can be tested for nonnegativity by other methods [10], the contents of this letter are especially intended for nonnegativity tests on polynomials in more than one variable, for which other procedures are not yet known. The conceptual simplicity and the computational facility provided by symbolic language programs are aids in the application of the result to several system theory problems. A nontrivial illustrative example has been given; many other examples have been satisfactorily completed using the procedure. Attention is now being directed toward further reduction of computational complexities that could be encountered in applying the procedure to test high degree polynomials in a larger number of variables.

ACKNOWLEDGMENT

The authors wish to thank V. Agarwal for his assistance during the course of this research.

REFERENCES

[1] N. K. Bose and R. W. Newcomb, "Tellegen's theorem and multi-variable realizability theory," *Int. J. Electron.*, vol. 36, pp. 417-425, Mar. 1974.
 [2] B. D. O. Anderson, "The inverse problem of optimal control," Stanford Electronics Labs., Stanford, Calif., Rep. SEL-66-038 (TR-6560-3), 1966.
 [3] W. R. Perkins and J. B. Cruz, Jr., "The parameter variation problem in state feedback control systems," *Trans. ASME, J. Basic Eng.*, vol. 87, pp. 25-30, Mar. 1965.
 [4] J. C. Hsu and A. U. Meyer, *Modern Control Principles and Applications*. New York: McGraw-Hill, 1968.
 [5] B. D. O. Anderson, N. K. Bose, and E. I. Jury, "Output feedback stabilization and related problems—Solution via decision methods," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 53-66, Feb. 1975.

[6] N. K. Bose and P. S. Kamat, "Algorithm for stability test of multidimensional filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-22, pp. 307-314, Oct. 1974.
 [7] N. K. Bose, "Test for Liapunov stability by rational operations," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 700-702, Oct. 1975.
 [8] N. K. Bose and E. I. Jury, "Inner algorithm to test for positive definiteness of arbitrary binary forms," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 169-170, Feb. 1975.
 [9] E. I. Jury, *Inners and Stability of Dynamic Systems*. New York: Wiley, 1974.
 [10] —, "The theory and applications of the inners," *Proc. IEEE*, vol. 63, pp. 1044-1068, July 1975.
 [11] A. C. Hearn, "Reduce 2: A system and language of algebraic manipulations," in *ACM Proc., 2nd Symp. Symbolic and Algebraic Manipulations*, (Los Angeles, Calif., Mar. 1971), pp. 128-133.
 [12] F. R. Gantmacher, *The Theory of Matrices*, vol. 2. New York: Chelsea, 1964.
 [13] N. K. Bose and A. R. Modarressi, "General procedure for multi-variable polynomial positivity test with control applications," to be published.

Correction to "Linear Prediction: A Tutorial Review"

JOHN MAKHOUL

In the above paper,¹ on page 567, line 44 should have read: . . . then it can be shown that the polynomials $z^i A_i(z)$, for $i = 0, 1, 2, \dots$, form an orthogonal set over the unit circle [35], [42], [93]:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} P(\omega) A_n(e^{j\omega}) A_m(e^{-j\omega}) e^{j(n-m)\omega} d\omega = E_n \delta_{nm},$$

$$n, m = 0, 1, 2, \dots \quad (48)$$

Manuscript received August 24, 1975.

The author is with Bolt Beranek and Newman, Inc., Cambridge, MA 02138.

¹J. Makhoul, *Proc. IEEE*, vol. 63, pp. 561-580, Apr. 1975.