EIGENSTRUCTURE METHODS FOR DIRECTION-OF-ARRIVAL ESTIMATION OF FREQUENCY HOP (FH) EMITTERS

V.V. Krishna J.V. Avadhanulu K. Giridhar A. Paulraj

Central Research Laboratory
BHARAT ELECTRONICS
BANGALORE - 560 001, India

Abstract

Array signal processing in frequency hop (FH) spread spectrum environments has so far been largely focussed on the signal copy problem in which the desired FH signal arriving from a known direction is extracted in the presence of jammers and interference. The problem of direction-of-arrival (DOA) estimation of FH emitters is equally important since the desired signal DOA may not be known *a priori* and has to be estimated from the array data. An interceptor also can use this spatial information for effective jamming and to launch other countermeasures. In this paper, we describe the application of MUSIC (MUltiple SIgnal Classification) to the wide-band, time varying FH/SS signal environment. Both block as well as adaptive approaches are discussed for multiple FH emitter environment. Simulation results are also presented.

1. Introduction:

Eigenstructure-based DOA estimation techniques such as MUSIC [1], ESPRIT [2] provide high resolution capability. In this paper, we consider the application of MUSIC to localize multiple frequency hop (FH) spread spectrum emitters.

FH systems [3,4] are being increasingly used to provide jam-resistant communication links. The FH signal has a constant narrow instantaneous (signal) bandwidth and a much larger overall (hopped) bandwidth. This large ratio of hopped to signal bandwidth provides the necessary processing gain against any narrow band CW jammers. Even intelligent (repeater) jammers can be rendered ineffective with fast FH (a few Khops/s) systems [5]. The receiver in a FH system requires a knowledge of the carrier frequencies and pattern besides other code parameters in order to acquire and sustain the FH link. Multiple users can share the same operating band by means of Code Division Multiple Access (CDMA) technique.

Array signal processing in a FH environment has so far been mainly considered for improving the SINR at the receiver. The Maximin algorithm of Bakhru and Torrieri [6] is an adaptive technique developed specifically for signal acquisition by a FH receiver. Another recent technique [7], proposes exploiting the "gated nature" of FH signals to separate out the desired signal from CW interference, without requiring explicit DOA estimation. However, this method needs knowledge of the "gating" and does not appear to be appropriate when multiple FH emitters are present. As against this, DOA estimation in a FH environment consisting of multiple emitters is of interest from two viewpoints. First, the DOA information is helpful to a genuine receiver for signal acquisition and nulling strong interferences [8]. At the same time, an interceptor also can utilise this knowledge to initiate suitable countermeasures.

In this paper, we discuss the problem of high resolution, multiple FH emitter DOA estimation using a wide band array (see Fig. 1). In essence, we are addressing a 2-D harmonic retrieval problem and our approach is built upon previous work by different researchers [9-12]. Use of wide band processing is essential as the overall bandwidth involved may be of the order of one or more octaves. Current high resolution techniques (MUSIC, ESPRIT) were originally formulated for narrow band sources. Many extensions available to the wideband case [10,13] are not directly suited to handling FH emitters. While both block and adaptive methods are considered here, it must be pointed out that adaptive implementation of the eigenstructure approach is preferable when there is relative motion between the emitters and the array. Bakhru and Krieger [8] have described an adaptive eigenstructure method for DOA estimation by using only one noise eigenvector which corresponds to the minimum eigenvalue of the array signal covariance matrix. More importantly, they proposed reduction of the wide band array problem to a narrow band model by assuming that the desired signal hop pattern is known and employing a local oscillator for dehopping. In contrast, our approach is to tackle the wide band problem directly and DOA estimation can be carried out without *a priori* knowledge of the FH emitters' characteristics including their individual hop patterns. The receiver proposed here has a simple structure, but has high computational needs. Also, the MUSIC approach used here requires array manifold information. Finally, MUSIC uses all noise eigenvectors for DOA estimation compared to using only one as proposed by Bakhru and Krieger [8].

2. Array and Signal Considerations:

Let us consider a wide band array consisting of 'M' sensor elements each of which is followed by 'P' taps in a delay line (see Fig. 1). The delay element 'Δ' must be chosen such that the signal received by any sensor is

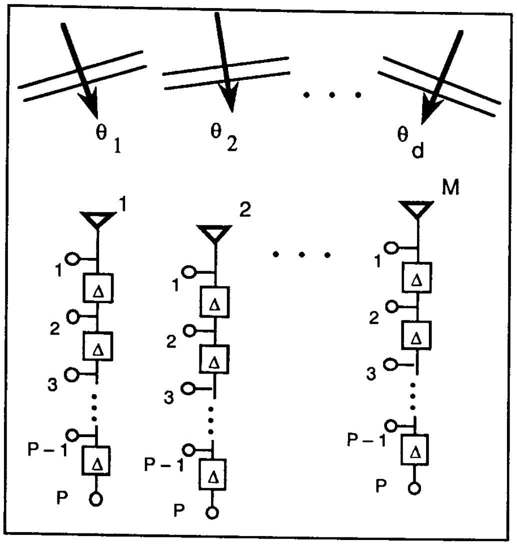

FIG.1 Wideband array with 'M' sensor elements and 'P' taps, in a field due to 'd' plane wavefronts.

sampled higher than or equal to the Nyquist rate. We consider that wavefronts due to 'd' sources, each of which is either FH or CW type, impinge on the array. The signals emitted by the different sources are assumed to be zero mean, stochastic processes. Using complex signal representation, the output after the p^{th} delay unit of the m^{th} sensor at any instant 't' is:

$$r_m(t-p\Delta) = \sum_{i=1}^{d} a_{mi} s_i(t-\tau_{mi}-p\Delta) + n_m(t-p\Delta)$$

$$m=1,2,\ldots,M; \quad p=0,1,\ldots,(P-1) \quad \ldots \quad (1)$$

Here, $s_i(.)$ is the signal due to i^{th} source observed with respect to an arbitrary reference point (usually one of the elements in the array). The center frequency of this source is $\omega_i(t)$ which may or may not vary with time depending on the type of source. 'τ_{mi}' is the propagation delay for the i^{th} source between the m^{th} sensor and the reference point. 'a_{mi}' is the amplitude response of the m^{th} sensor element to the i^{th} wavefront and $n_i(.)$ is additive white noise, considered to be both spatially and temporally uncorrelated across the array.

Stacking all the outputs from the 'P' taps of the first sensor followed by those of other sensor elements, the *snap shot* vector of size (MP X 1) corresponding to the time instant 't' can be obtained. The array signal model that we are developing is identical to the formulation used earlier by Reddy *et al* [9] and Wax *et al* [10]. It should also be noted that other approaches formulated for DOA estimation of wide band (persistent) sources, (such as the focussing technique of of Wang and Kaveh [13] and the filter bank (DFT) method explained by Wax *et al* [10]) are not tailored to the FH environment. Making use of the instantaneously narrow band nature of FH signals, the time delays in Eq.1 above can be replaced by appropriate phase shifts. At the same time, the wide band nature of the composite signal received by the array — either on account of multiple FH sources or, in case of a single FH emitter, due to interference and background noise — allows the collection of a sufficient number of *independent* snapshots within one hop duration (further discussed in §3.1). This is an important requirement that must be satisfied so that each new snapshot will imply extra information so as to make the application of eigenstructure techniques effective.

We can now write the snapshot vector as

$$\mathbf{r}(t) = \mathbf{A}(t)\,\mathbf{s}(t) + \mathbf{n}(t) \quad \ldots \quad (2)$$

where s(t) is the (d X 1) source vector, **n**(t) is the (MP X 1) noise vector and **A** is the (MP X d) matrix whose columns are the *direction-frequency vectors*, corresponding to the different sources. Ignoring the time dependence of each vector arising due to frequency hopping for the time being, the i^{th} column of **A** will be of the form,

$$\begin{aligned}\mathbf{a}_i = [\, & a_{1i}\exp\{-j\omega_i\tau_{1i}\}, \quad a_{1i}\exp\{-j\omega_i(\tau_{1i}+\Delta)\}, \\ & \cdots a_{1i}\exp\{-j\omega_i(\tau_{1i}+(P-1)\Delta)\}, \\ & a_{2i}\exp\{-j\omega_i\tau_{2i}\}, \quad a_{2i}\exp\{-j\omega_i(\tau_{2i}+\Delta)\}, \\ & \cdots a_{2i}\exp\{-j\omega_i(\tau_{2i}+(P-1)\Delta)\}, \\ & \cdots\cdots a_{Mi}\exp\{-j\omega_i(\tau_{Mi}+(P-1)\Delta)\}\,]^T \quad \ldots \quad (3)\end{aligned}$$

Taking expectation on both sides of Eq. 2, we can now obtain the (MP X MP) signal covariance matrix,

$$\mathbf{R} = \mathbf{A}\,\mathbf{S}\,\mathbf{A}^H + \sigma^2\mathbf{I} \quad \ldots \quad (4)$$

This model leads to the standard eigenstructure problem so that MUSIC or any similar technique could be employed for DOA estimation [10]. MUSIC involves a 2-D search in a selected frequency band and angular sector, over which the array manifold must be known.

3. DOA Estimation:

The application of MUSIC to an FH environment using block and adaptive versions will now be discussed. While the block method is quite popular and has been thoroughly investigated over the past decade [1,14], its adaptive implementation is of a more recent origin [11,12]. We have chosen the least squares adaptive version for noise subspace estimation, as it generally exhibits faster convergence. We have applied Babu *et al's* [12] work to the wide band array required for FH emitters as noted earlier (see § 2). The key steps of this algorithm are outlined in § 3.2 below.

Irrespective of whether block or adaptive approach is used, MUSIC exploits the array signal model embodied in Eq.4 by dividing the MP-dimensional vector space into two orthogonal subspaces termed as signal and noise subspaces of dimension 'd' and (MP – d) respectively. In practice, one needs to estimate these subspaces using finite data (taking cover under *ergodicity*) using time averages rather than ensemble averages. In the present context however, it is possible that some of the FH emitters hop over to different carrier frequencies during the period of observation, effectively leading to a larger number ∂, ($\partial > d$) of sources to be "seen" by the array. Though this number ∂ can be estimated using any one of the information theoretic criteria [15], it is essential that it should be as small as possible. Otherwise, the noise subspace shrinks and causes degradation of DOA estimation performance. In the worst case, $\partial = (MP - 1)$ and the DOA estimates obtained using the single noise eigenvector will exhibit large variance. For larger values of ∂, eigenstructure techniques are no longer applicable. Therefore, the amount of data that can be used is restricted in a FH environment — in the block method, snap shots can only be gathered over a rectangular window of limited extent, whereas in the adaptive method, it becomes necessary to use an appropriate *forgetting factor* (in effect, an exponential window).

3.1 Block Method:

The chief concern here is the selection of an appropriate observation window. Considering that only 'h' of the 'd' sources are FH emitters (the remaining being *narrow band* CW or pulsed jammers/interference), we assume for simplicity that all of these have equal dwell time T_D, and hop without any net-synchronization. If the covariance matrix **R** is estimated using snap shots obtained over a time window of duration T_W, then the probability that at most 'm' of the 'h' emitters have hopped to new carrier frequencies within this time is [16]

$$P_{mh} = \sum_{i=0}^{m} \binom{h}{i} F^i (1-F)^{h-i} \qquad \text{where } 0 \leq (F = T_W / T_D) \leq 1. \qquad \ldots (5)$$

As an example, if h = 8 (FH emitters), we need to choose $F \leq 0.34$ to ensure that the probability for a maximum of m = 4 sources to hop within the observation window, is better than 90%. If the window size is selected accordingly, then on an average, $\partial \leq (d + 4)$ at least nine out of ten times. The point to be noted here is that in a multiple FH emitter environment, the observation window will have to be much smaller than the dwell time so that the dimension of the estimated noise subspace would usually be marginally smaller than (MP – d). As the performance of MUSIC has been related to both the data length and the signal-to-noise ratios (SNR) available [14], the question now is *can we collect sufficient data in a FH environment to ensure that block MUSIC can resolve closely spaced emitters?*

Even though a single FH signal is instantaneously narrow-band, it is possible to obtain two independent snapshots after a time delay much smaller than the coherence time of the narrowband waveform as the signal received by the array would also consist of background interference. Furthermore, if there are 'h' FH emitters operating at an equal hop rate and identical bandwidth, it can be shown that the instantaneous normalised bandwidth of the composite signal 'x', is by itself a random variable having a probability density function given by:

$$f_{BW}(x) = h(h-1)(1-x)\,x^{(h-2)} \qquad \text{subject to } (h \geq 2) \qquad \ldots (6)$$

For h = 4, there is a 95% probability that the instantaneous bandwidth is larger than $(B_o/5)$. Therefore, in a HF communication environment with four FH emitters (as specified in Table-I), one can easily expect upwards of 200 snapshots even after selecting 'F' such that the probability for a maximum of two emitters to hop within the observation window T_W is better than 90% (Eq.5). Considering the typical SNR levels of FH signals (around 20 dB), this amount of data should be adequate to handle the high resolution DOA estimation problem.

Table-I: FH environment and corresponding snapshot characteristics – an example:

Feature	Value
Operating Bandwidth: (B_0)	5 - 25 MHz
Channel Bandwidth:	20 KHz
Hop Rate: ($1/T_D$)	500 / s (T_D = 2 m sec)
Number of FH transmitters:	Four, each with 20dB SNR
Nominal Bandwidth of the Composite Signal:	1 MHz ($= B_0 / 20$)
Array Tap-Tap Delay: (Δ)	$\leq$ 20 nano seconds
Snapshot-Snapshot Delay:	$\geq 1\ \mu$ seconds
Observation Time: (T_W)	0.5 m seconds (F = 0.25)
Number of Snapshots	$\approx$ 250 (@ 2 μ s delay)

3.2 Adaptive Noise Subspace Estimation:

While we have established the basis for block MUSIC to localize FH emitters, adaptive estimation of the noise subspace offers two benefits. First of all, it allows estimation of the noise subspace directly from the data, without the intermediate step of having to estimate the covariance matrix **R** as well. Secondly, in case there is any relative motion between the array and any of the emitters (FH or otherwise), the adaptive approach is better suited to track these changes. One difficulty, however, is that estimation of the effective number of sources '∂' is no longer straight forward as in the case of block method where the eigenvalues obtained from eigendecomposition of **R** could be used for the purpose; the problem is compounded by the fact that '∂' is also time varying. A practical solution to this problem involves choice of a suitable *forgetting factor* to keep $(\partial - d)$ small. This ensures that the noise subspace is as large as possible which allows sufficient safety margin to use an overestimate ∂° of ∂. Obviously, some performance degradation can be expected as the entire noise subspace will not be used for DOA estimation. This aspect deserves more detailed investigations.

For the present study, we have selected the least squares adaptive technique [9,12] and extended it to the wide band array to estimate the required weight vectors. Note that we are concerned only with the array output minimization problem as we wish to obtain (some of) the noise eigenvectors alone. The algorithm can be briefly described in the following steps (a more detailed description can be found in the references cited above):

Step 1: Select an orthonormal set of initial weight vectors $\mathbf{w}_i(0)$, $\{i = 1, 2, \ldots MP\text{-}\partial^\circ\}$ (for example, these may simply be selected from the standard basis for an MP-dimensional vector space).

Step 2: Given a snapshot $\mathbf{x}(k)$ and one of the current weight vectors $\mathbf{w}_i(k-1)$, we obtain the updated weight vector as

$$\mathbf{w}_i(k) = \mathbf{w}_i(k-1) - e_i(k)\, \mathbf{P}_i(k)\, \psi_i(k) \quad \ldots (7)$$

where

$$e_i(k) = \mathbf{w}_i^T(k-1).\mathbf{x}(k),$$

$$\psi_i(k) = \mathbf{x}^*(k) - \mathbf{w}_i(k-1)\ e_i^*(k)$$

and

$$\mathbf{P}_i(k) = (1/\lambda)\,[\,\mathbf{P}_i(k-1) - \{\mathbf{P}_i(k-1)\,\psi_i(k)\,\psi_i^H(k)\mathbf{P}_i(k-1)\}/\{\lambda + \psi_i^H(k)\mathbf{P}_i(k-1)\psi_i(k)\}]$$

The parameter 'λ' is the *forgetting factor* $(0 < \lambda \leq 1)$. The upper limit corresponds to the case of no forgetting factor at all — i.e. all the past data is used for the present adaptation.

Step 3: Normalise the updated weight vector.

Step 4: Repeat steps 2 and 3 for the required number of different weight vectors {maximum of (MP-∂°)}.

Step 5: Apply Gram-Schmidt orthogonalisation to the updated weight vectors to render them mutually orthonormal.

Step 6: Repeat steps 2 - 5 with a new snapshot $\mathbf{x}(k+1)$.

4. Simulation Results:

Some simulation studies of the block and adaptive versions of MUSIC were undertaken to examine their resolution performance. The sources were considered to be stationary with respect to the array and for this case the two methods performed equally well. Owing to the space restrictions, we will present here some results for the adaptive method alone. Also, in our simulation trials, we chose to derive only two weight vectors to reduce the computational load.

A linear equi-spaced (LES) array with identical and isotropic sensor elements was considered with an inter-element spacing of half the wavelength corresponding to the center frequency of the overall bandwidth. In the first of our experiments, the received signals (snapshots) were modelled as arising due to two FH/SS emitters whose frequency transitions occur independent of one another - this is more realistic in CDMA environment rather than forcing the sources to hop in step. The background noise was considered to be both spatially and temporally uncorrelated. A small array consisting of four elements and five taps was employed for a source separation of 11°, which is less than half the beam width of the array. The aim of this experiment is to highlight the need for appropriate selection of the forgetting factor (FF). As the adaptive algorithm is initiated, the two sources (SNR = 25 dB each and DOA of 6° and -5°) have initial frequencies (normalised) equal to 0.34 and 0.18. These frequencies change to 0.26 and 0.42 at the 80th and 130th snapshots respectively. While the different weight vectors are built up snapshot by snapshot, the DOA estimation is done once every 25 snapshots. In Table-II the variations with time (expressed in terms of the snapshots) of the estimated peak heights (in dB) in the pseudo power spectrum at the four different frequencies is tabulated along with the DOA estimates where relevant; a 'dash' indicates that the peak at the particular frequency is not discernable from the background level and does not indicate the presence of any emitter. It is observed (Table-IIa) that as the two sources hop over to new frequencies, new peaks are obtained in the very next iteration near the correct bearing values. In this case the FF is equal to 0.995. However, the important point here is that the peaks corresponding to the old frequency values persist even after a few more iterations. This problem is corrected by choosing a value of 0.85 for FF (Table-IIb).

Using this value of FF we next considered the following scenario: an array of four sensors and four taps (M=4, P=4), three sources at (−8°, 1°, 9°) having an SNR of 25 dB each and initial frequencies of (0.11, 0.33, 0.24) respectively. The first source hops to a frequency of 0.38 at the 45th snapshot and the third source hops over to 0.16 at the 68th snapshot. The second source remains all the time at its initial frequency. Contour plots of the 2-D pseudo power spectrum estimated at the 30th and 120th snapshots show (Fig.s 2a, 2b) that the adaptive technique performs satisfactorily.

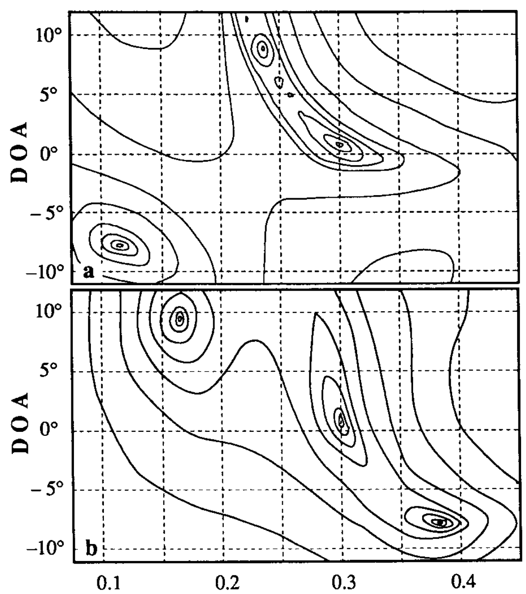

FIG.2 Contour plots of 2-D parameter spectrum

Table - II: Example demonstrating the effect of forgetting factor λ on DOA estimation
The peak heights (top row) are in dB.

a) $\lambda = 0.995$

Frequency \ Time:	25	50	75	100	125	150	175	200	225
0.18	41.4 -5.1°	43.0 -5.0°	45.6 -5.0°	45.6 -5.1°	45.2 -5.0°	44.3 -5.0°	44.4 -5.0°	43.5 -5.0°	43.8 -5.0°
0.26	5.8 —	6.2 —	4.4 —	45.4 5.9°	39.6 6.0°	43.5 5.9°	41.4 6.0°	40.5 6.1°	51.9 6.1°
0.34	33.1 6.1°	40.1 6.2°	41.3 6.1°	41.4 6.0°	41.1 6.1°	40.9 6.2°	41.3 6.2°	41.2 6.2°	41.0 6.2°
0.42	6.5 —	6.1 —	7.1 —	14.4 —	13.2 —	27.8 -5.2°	40.6 -5.2°	40.2 -5.0°	42.8 -5.0°

b) $\lambda = 0.85$

Frequency \ Time:	25	50	75	100	125	150	175	200	225
0.18	38.4 -5.1°	41.2 -4.8°	34.7 -5.3°	37.9 -5.3°	46.4 -4.9°	44.4 -4.9°	23.5 -4.3°	8.20 —	2.96 —
0.26	12.0 —	6.9 —	7.4 —	41.4 6.1°	38.9 5.9°	39.4 6.0°	35.4 6.0°	37.1 5.9°	35.7 6.2°
0.34	34.6 6.3°	35.9 6.0°	42.1 5.9°	46.0 5.8°	28.8 6.5°	4.6 —	9.5 —	8.40 —	6.90 —
0.42	4.3 —	4.3 —	-1.0 —	11.1 —	15.2 —	37.1 -5.0°	39.2 -4.9°	38.7 -5.0°	41.0 -5.0°

5. Discussion:

The utility of MUSIC for high resolution DOA estimation of FH emitters has been demonstrated. A more detailed investigation of the block and adaptive approaches is needed in order to appreciate their relative performance levels. Finally, it may be noted that sources which are closely spaced in bearing can be resolved due to their likely large separation in frequency.

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