IMPROVED CHANNEL ESTIMATION FOR OFDM BASED WLAN SYSTEMS

G.V.Rangaraj *M.R.Raghavendra* *K.Giridhar*

Telecommunication and Networking (TeNeT) Group
Department of Electrical Engineering
Indian Institute of Technology, Madras
ranga, raghumr, giri@tenet.res.in

ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) has recently been proposed to used in Wireless Local Area Networks (WLAN) standards like IEEE 802.11 and ETSI HIPERLAN/2. For these systems the receiver requires channel state information for decoding. In this paper, we propose a novel improved channel estimation algorithm for OFDM based WLAN systems. The performance of the proposed channel estimation is demonstrated by computer simulations.

1. INTRODUCTION

OFDM (Orthogonal Frequency Division Multiplexing) is a multi-carrier block modulation scheme which is highly efficient since it allows for spectral overlap. OFDM transforms a frequency selective fading channel into multiple narrow flat fading parallel sub-channels. This increases the symbol duration and mitigates inter-symbol interference (ISI) caused due to multipath [1, 2]. OFDM has been incorporated in high-bit rate wireless LANs like IEEE 802.16a and HiperLAN/2. It is also being strongly considered for the emerging IEEE 802.11a and IEEE-ISTO BWIF.

In the receiver, channel estimation is required for equalization and decoding. Typically channel estimation in OFDM is done by using a simple operation in the frequency domain independently in all the sub-carriers. This method does not exploit the correlation among the channel estimates in the various sub-channels. Exploiting this correlation we can get better channel estimates by transforming the frequency domain channel estimates to time domain. In the time domain a windowing operation along with a correction operation is required. Finally by transforming back to frequency domain we can get channel estimates which can directly be used in equalization and decoding. This improves the receiver performance and we have demonstrated the improvement in Normalized Mean Square Error (NMSE) of the channel estimates using computer simulations.

2. OFDM SYSTEM

A block diagram of a conventional OFDM system is as shown in Fig. 1. In the transmitter, the serial-to-parallel converter collects blocks of K serial data symbols to be modulated by the Inverse Discrete Fourier Transform (IDFT). The serial-to-parallel converter can also be viewed as a time-to-frequency mapper. K is usually a power of 2 to facilitate the use of the Fast Fourier Transforms (FFT). Denoting $s[n,k]$ as the signal modulating the k^{th} sub-channel during the n^{th} block, the IDFT generates the required time domain OFDM

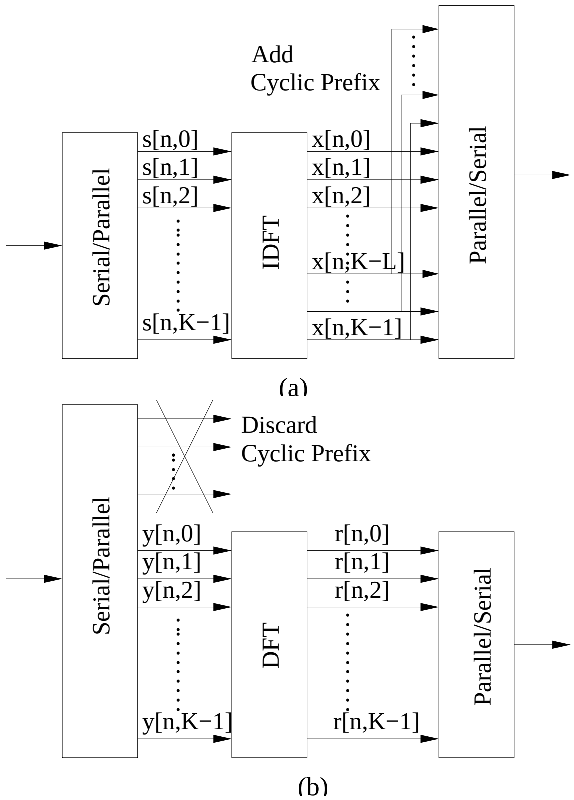

Figure 1: (a) Generic OFDM Transmitter (b) Generic OFDM Receiver

symbol as,

$$x[n,l] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} s[n,k] \exp\left(\frac{j2\pi lk}{K}\right), \quad l = 0,1,..,K-1 \qquad (1)$$

The OFDM symbol duration is KT_s where T_s is the incoming data symbol duration. A cyclic prefix (CP) of length L which is the repetition of the last L samples of the IDFT is prepended for each block. This causes the overall symbol duration of the OFDM block to be PT_s, where, $P = K + L$. The CP acts like a guard interval between successive OFDM symbols and prevents intersymbol interference (ISI) if the channel impulse response length is less than or equal to the length of the CP [2]. The received time domain signal $y[n,l]$ is a linear convolution of $x[n,l]$ and the channel impulse response $h[n,l]$. The CP transforms this linear convolution into cyclic convolution. Thus,

$$y[n,l] = x[n,l](*)h[n,l] + w_t[n,l] \qquad (2)$$

where, $(*)$ denotes cyclic convolution and $w_t[n,k]$ is Additive White Gaussian Noise.

In the receiver, the CP is first discarded and then a K-point DFT is applied. The DFT demodulates the time domain OFDM signal generating $r[n,k]$ the received signal in the k^{th} sub-channel and n^{th} block as follows,

$$r[n,k] = \frac{1}{\sqrt{K}} \sum_{l=0}^{K-1} y[n,l] \exp\left(-\frac{j2\pi lk}{K}\right), \quad k = 0,1,..,K-1 \qquad (3)$$

The parallel-to-serial converter transforms blocks into serial output symbol stream. It also can be viewed as a frequency-to-time mapper. From (1), (2) and (3), we get,

$$r[n,k] = H[n,k]s[n,k] + w[n,k], \quad k = 0,1,..,K-1 \qquad (4)$$

where,

$$H[n,k] = \frac{1}{\sqrt{K}} \sum_{l=0}^{K-1} h[n,l] \exp\left(-\frac{j2\pi lk}{K}\right),$$

$$w[n,k] = \frac{1}{\sqrt{K}} \sum_{l=0}^{K-1} w_t[n,l] \exp\left(-\frac{j2\pi lk}{K}\right), \quad k = 0,1,..,K-1$$

This implies the frequency selective channel is transformed into K parallel flat sub-channels with gains given by the DFT of $h[n,k]$. This makes frequency domain equalization simple at the receiver.

3. CHANNEL MODELING AND ESTIMATION

3.1. Wireless Channel Model

The complex baseband channel representation of the wireless channel impulse response can be described as,

$$h(t,\tau) = \sum_k \nu_k(t)\delta(\tau - \tau_k)$$

where, τ_k is the delay of the k^{th} path and $\nu_k(t)$ is the corresponding complex amplitude. Discretising the above model, i.e., $h[n,l] = h(nT_f, lT_s)$ and applying DFT yields

$$H[n,k] = \frac{1}{\sqrt{K}} \sum_{l=0}^{K_0-1} h[n,l] \exp\left(-\frac{j2\pi kl}{K}\right)$$

where, $H[n,k] = H(nT_f, k\Delta f)$, K is the number of sub-channels of an OFDM block, T_f and Δf are the block duration and sub-channel spacing of the OFDM system, respectively, and T_s is the sample interval of the system that relates to Δf by $Ts = \frac{1}{K\Delta f}$. In the above expression, K_0 is the channel delay spread in samples or channel impulse response length which is usually much less than K ($K_0 << K$).

3.2. Channel Estimation

A simple technique to do channel estimation is to send pilot signal $t[n,k]$ during training in all the sub-channels.

$$r[n,k] = H[n,k]t[n,k] + w[n,k]$$

$$k = 0, 1, .., K-1. \qquad (5)$$

The channel estimation is done in the frequency domain independently in all the sub-channels. The channel estimates $H_{FDE}[n,k]$ are obtained by a simple division of the received signal $r[n,k]$ and the training signal $t[n,k]$ and we refer this as Frequency Domain Estimation (FDE),i.e,

$$H_{FDE}[n,k] = r[n,k]/t[n,k]$$

$$k = 0, 1, .., K-1. \qquad (6)$$

This technique is simple to implement however fails to take into account the correlation in the channel estimates. In the proposed channel estimation we exploit the correlation of channel estimates in the frequency domain by transforming to time domain. We know that the channel estimates in the time domain is typically limited to delay spread length (K_0) which is less than cyclic prefix length (L). Hence windowing only the required first K_0 channel estimates in the time domain helps to zero out the noise which would otherwise be present and results in better channel estimates. Then transforming back to frequency domain gives the required improved channel estimates. Expressing in equations, we get,

$$h_{FDE}[n,l] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} H_{FDE}[n,k] \exp\left(\frac{j2\pi kl}{K}\right),$$

$$l = 0, 1, .., K-1$$

$$h_{PRO}[n,l] = h_{FDE}[n,l]\gamma[n,l], \; l = 0, 1, .., K-1$$

$$\gamma[n,l] = \left\{ \begin{array}{l} 1, \; l = 0, 1, .., K_0 - 1 \\ 0, \; l = K_0, K_0+1, .., K-1 \end{array} \right\}$$

$$H_{PRO}[n,k] = \frac{1}{\sqrt{K}} \sum_{l=0}^{K-1} h_{PRO}[n,l] \exp\left(-\frac{j2\pi lk}{K}\right),$$

$$k = 0, 1, .., K-1$$

where, $h_{FDE}[n,l]$ is IDFT of $H_{FDE}[n,k]$, $\gamma[n,k]$ is the time domain window, $h_{PRO}[n,l]$ is windowed channel estimates in the time domain and $H_{PRO}[n,k]$ is frequency domain channel estimates obtained using the proposed technique.

4. CHANNEL ESTIMATION IN IEEE 802.11A

The proposed channel estimation approach described in the previous section which exploits frequency correlation has an inherent assumption that the channel estimates H_{FDE} are available for all the sub-channels. We refer this approach as ideal but impractical with reference to IEEE 802.11a systems since it is not possible to use the DC sub-channel and eleven sub-channels in the center. This is because these sub-channels falls in the guard band of the frequency spectrum and hence not usable due to the possibility of interference from a neighboring IEEE 802.11a system. The approach discussed in the previous section is actually a special case of Least Squares (LS) formulation. Hence, the generalized LS approach is a more pragmatic method for channel estimation in IEEE 802.11a systems. In order to formulate the LS approach mathematically as in [3], we first represent (5) and (6) describing the FDE approach in matrix notation (all bold alphabets are vectors or matrices), i.e.,

$$\mathbf{H_{FDE}} = \mathbf{H} + \mathbf{w} \tag{7}$$

where, $\mathbf{H_{FDE}}$ is vector of frequency domain channel estimates obtained by using the FDE approach at sub-channels where channel sounding is possible and hence it is a 52x1 vector for IEEE 802.11a, $\mathbf{H}$ is vector of the actual channel coefficients in these sub-channels and $\mathbf{w}$ is an i.i.d complex zero mean white Gaussian noise vector. Substituting the product of $\mathbf{F}$ Fourier Transform matrix (52xK_0) and time domain channel coefficient vector $\mathbf{h}$(K_0x1) for $\mathbf{H}$,we get,

$$\mathbf{H_{FDE}} = \mathbf{Fh} + \mathbf{w} \tag{8}$$

where,

$$\mathbf{F} = \begin{bmatrix} 1 & e^{-i2\pi/K} & .. & e^{-i2\pi(K_0-1)/K} \\ 1 & e^{-i4\pi/K} & .. & e^{-i4\pi(K_0-1)/K} \\ . & . & . & . \\ . & . & . & . \\ 1 & e^{-i52\pi/K} & .. & e^{-i52\pi(K_0-1)/K} \\ 1 & e^{-i76\pi/K} & .. & e^{-i76\pi(K_0-1)/K} \\ 1 & e^{-i78\pi/K} & .. & e^{-i78\pi(K_0-1)/K} \\ . & . & . & . \\ . & . & . & . \\ 1 & e^{-i126\pi/K} & .. & e^{-i126\pi(K_0-1)/K} \end{bmatrix}$$

Using the standard LS solution [5], we get the LS channel estimates in time domain $\mathbf{h_{LS}}$ (K_0x1) as,

$$\mathbf{h_{LS}} = (\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F^H}\mathbf{H_{FDE}} \tag{9}$$

The above expression can be thought as a two step process, where the first $\mathbf{F}^H\mathbf{H_{FDE}}$ is very similar to previously discussed channel estimation and is essentially a transformation (using a 52 point DFT) to time domain and windowing only the first K_0 channel estimates and the second step $(\mathbf{F}^H\mathbf{F})^{-1}$ is the correction term due to fact that $\mathbf{F}$ is not a complete KxK_0 Fourier transform matrix. Finally the frequency domain LS channel estimates $\mathbf{H_{LS}}$ are obtained by transforming the time domain LS channel estimates $\mathbf{h_{LS}}$ using the $\mathbf{F}$ operation, i.e.,

$$\mathbf{H_{LS}} = \mathbf{Fh_{LS}} \tag{10}$$

Moreover, $\mathbf{F^H}\mathbf{F}$ (K_0xK_0) happens to be a toeplitz matrix and hence its inverse could be evaluated using Levinson Durbin recursion with much lower complexity $O(K_0^2)$ rather than the usual techniques like Gauss elimination with complexity $O(K_0^3)$ [4].

5. SIMULATION AND RESULTS

An OFDM based IEEE 802.11a system was simulated in a delay spread channel with $K_0 = 16$ having a exponential power delay profile (pdp). The parameters of the OFDM are as per IEEE 802.11a standard with a bandwidth of 20 MHz divided into $K = 64$ sub-channels yielding a sub-channel spacing $\Delta f = 312.5$ kHz. To make the sub-channels orthogonal, the OFDM symbol duration is $\frac{1}{\Delta f} = 3.2\mu s$. An additional $0.8\mu s$ is used as guard interval i.e. CP of $L = 16$. The channel estimation was done using the FDE, ideal (impractical in IEEE 802.11a) and LS based methods and their performance are compared in terms of Normalized Mean Square Error (NMSE) defined as

$$NMSE = \frac{\sum_k |H_{est}[n,k] - H[n,k]|^2}{\sum_k |H[n,k]|^2}$$

The NMSE of the proposed LS based method is 5.0 dB better than FDE method as shown in Fig. 2. Also the ideal channel estimation is 6.0 dB better than FDE but it is impractical for IEEE 802.11a systems.

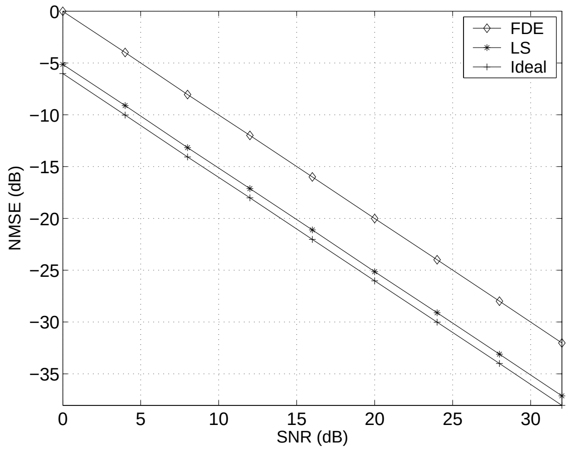

Figure 2: Performance comparison of various channel estimation methods

6. CONCLUSIONS

In this paper we have presented a novel improved channel estimation algorithm based on LS technique for OFDM based IEEE 802.11a. The performance of the proposed scheme is better than FDE method. However it is slightly more complex as it involves matrix inversion and two DFT operations. It is possible to extend this algorithm to transmitter diversity and Multiple Input Multiple Output (MIMO) OFDM systems using techniques proposed in [6] or similar techniques.

REFERENCES

[1] S.B.Weinstein and P.M.Ebert, "Data transmission by frequency- division multiplexing using the discrete Fourier transform," in *IEEE Trans. Commun. Technol.*, vol. 19, pp. 628–634, Oct 1971.

[2] L.J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," in *IEEE Trans. Commun. Technol.*, vol. 33, pp. 665–675, July 1985.

[3] Jan-Jaap van de Beek, Ove Edfors, Magnus Sandell, Sarah Kate Wilson and Per Ola Borjesson, "On Channel Estimation in OFDM Systems," in *Proceedings of IEEE Vehicular Tehnol. Conference (VTC'95).*, vol. 2, pp. 815-819, Chicago, USA, July 1995.

[4] Monson H. Hayes, Statistical Digital Signal Processing and Modeling, John Wiley Sons, Inc., 1996.

[5] Steven Kay, Fundamentals of Statistical Signal Processing, Vol. I - Estimation Theory, Prentice Hall, 1993

[6] G.V.Rangaraj and K.Giridhar, "Low-complexity channel estimation for transmitter diversity OFDM System," in *National Comm. Conf. NCC-2002*, Mumbai, Jan 2002.