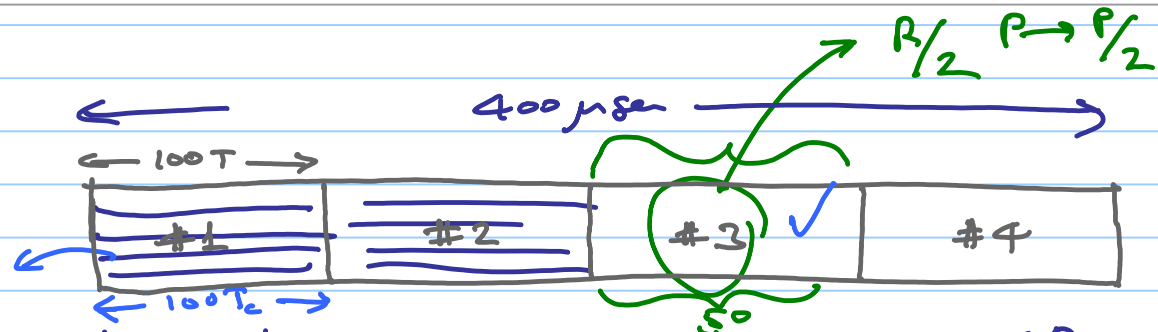
EE6323 Feb-May, 2021

Class #7 (Mar. 30, 2021)

Note Title 27-03-2020

Recall:

$T = 1\,\mu sec$



TDMA

$4T_c = T$

→ (max) per user bit rate : $\frac{100}{400\mu} = 250$ kbps ← R

→ Sum rate → $4 \times R = 1000$ kbps (1 Mbps)

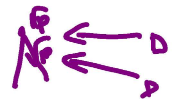
DS-CDMA ~~$4T = T'$~~

→ Every user, in every 100μ will send 25 bits $\frac{25}{100\mu} = 250$ kbps ← R

Sum rate = $4R$ ✓

"Narrow Banding" → TDMA → Repeat the bit

In DS-CDMA



(a) All users @ Rate R, Power P

$$SINR = \frac{W \cdot P}{(N-1)P + \sigma^2} \qquad W = \frac{T}{T_c};$$

target SINR

NR

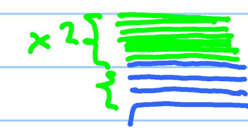
thermal noise variance

(*) N → (A) $N/2$ @ R $\sqrt{P}$ ⟶ $SINR_A$

$\qquad$ → (B) $N/2$ @ $R/2$ $\sqrt{\frac{P}{2}}$ ⟶ $SINR_B$

$SINR_A \vee SINR_B$

$SINR_A, SINR_B > SINR$



(*) we can add $N/2$ @ $R/2$ so that $N' = \overset{R}{N/2} + \overset{R/2}{2(N/2)} = 3/2\, N$

NR

← 50%

(*) $N/2$ @ R ; $N/2$ @ $R/2$; $N/4$ @ R → NR

→ $\frac{SN}{4}$ ← 25%

$$\overset{P}{SINR_{(A)}} = \overset{P'}{SINR_{(B)}}$$

$P' > P$

Uplink Multiple Access

1. TDMA (unspread) → single carrier comm. ✓
   - ✓ → orthogonal support → simple Rx in LoS links
     → no UL power control required
   - ✓ → "narrow banding" possible by repetition coding
     → However, the sum rate ↓
   - ✗ → optimal Rx for non-LoS channels (ISI channels → multitap) is MLSE ⟹ complexity scales exponentially with bandwidth
     M^{L-1} M → mod. size ; L → ISI L↑

2. DS-CDMA
   - → Pseudo orthogonal codes even for LoS link → tight power control required on UL
   - ✓ → Rate ↓ with narrow-banding possible with statistical multiplexing (i.e., sum rate does not ↓)
   - ✓ → Optimal Rx for ISI channel with intra+inter user interference is very complex
     MUD $M^{L_1+L_2+L_3}$ ← exp.

3. OFDM / OFDMA

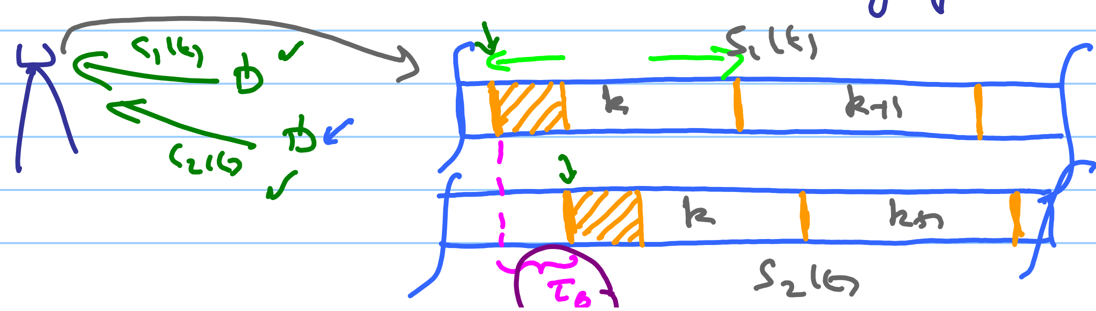
→ Orthogonal support across users

↳ strict power control not reqd.

⇒ → UL ranging errors do not kill orthogonality

→ optimal Rx for ISI channels is simple → complexity scales only linearly with BW

Question: How to preserve orthogonality even when the codes have ranging errors?



$s_1(t) = I_1(k) \cdot c_1(t)$

$s_2(t) = I_2(k)\, c_2(t)$

$r(t) = s_1(t) + s_2(t-\tau_0)$

→ Extend the length of the codes by say T_{cp} so that

cp → Cyclic Prefix

$$\rightarrow T = T_u + T_{cp}$$

"guard time"

"useful time"

→ As long as $0 \le \tau_0 < T_{cp}$, we can show that $s_1(t)$ & $s_2(t-\tau_0)$ can be separated orthogonally.

WT_c $\sum_{i=1}^{W}$

Key Idea: check orthogonality only over T_u (out of T) seconds. (integration)

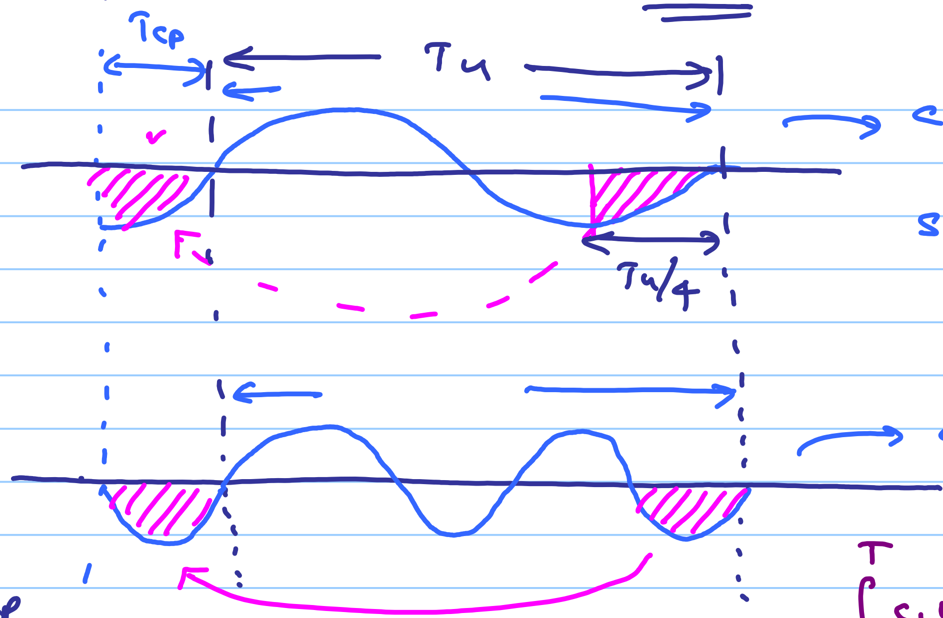
OFDM / OFDMA

⇒ in discrete time, check orthogonality for only N samples out of $N + N_{cp}$ received samples

$$N \left\{ \sum_{N_0}^{N_0+N} \right.$$

$s_2(t-\tau_0)$

eg : Real sinusoids $\longrightarrow$ codes



$c_1(t)$

$s_1(t) = I_1(k)\, c_1(t)$, $\in \{+1, -1\}$

$c_2(t)$

$s_2(t) = I_2(k)\, c_2(t)$

$T = T_u + T_{cp}$

$$\int_0^T c_1(t)\, c_2(t)\, dt = 0$$

$$\int_{t_1}^{t_1+T_u} c_1(t)\, c_2(t-\tau_0)\, dt = 0, \quad \text{provided} \left\{ \begin{array}{l} t_1 > \tau_0 \\ 0 \le \tau_0 < T_{cp} \end{array} \right.$$

no ISI

T_u

which T_u

Recall : Trigonometric Identities $f_0 = \frac{1}{T}$;

(*) $\int_0^T \sin 2\pi f_0 t \, \sin 4\pi f_0 t \, dt \xrightarrow{\theta_0} \frac{1}{2}\left[\cancel{\cos(6\,)}^{\,0} + \cancel{\cos(2\,)}^{\,0}\right] = 0$

(*) $\int_0^T \sin 2\pi f_0 t \, \sin(4\pi f_0 t - \underbrace{4\pi f_0 \tau_0}) \, dt \longrightarrow = 0$;

<u>codes</u>

From Real Sinusoids (harmonically related) $\longrightarrow$

(*) Discrete-time ✓ — DTFT

(*) Complex symbols ✓ — DFT

(*) "Block of Samples" ← DFT

"code-set" ✓

$\left\{-\frac{N}{2}+1 \cdots\cdots 0 \cdots\cdots +N/2\right\}$ ← N →

$\left\{0 \cdots\cdots N-1\right\}$ ← N →

DFT basis set $\left\{ e^{-j\frac{2\pi}{N} nm} \right\}$, $n, m \in \{0, \cdots N-1\}$

$$\bar{q}_n = \begin{bmatrix} e^{j\frac{2\pi}{N}n\cdot 0} \\ e^{j\frac{2\pi}{N}n\cdot 1} \\ \vdots \\ e^{j\frac{2\pi}{N}n\cdot (N-1)} \end{bmatrix} \rightarrow m$$

IDFT $\rightarrow$ $F = \frac{1}{\sqrt{N}} \begin{bmatrix} | & | & & | \\ \bar{q}_0 & \bar{q}_1 & \cdots & \bar{q}_{N-1} \\ | & | & & | \end{bmatrix}_{N\times N}$; $\underset{\text{DFT}}{\overset{\text{DFT}}{F^H}} = \frac{1}{\sqrt{N}} \begin{bmatrix} - & \bar{q}_0^H & - \\ - & \bar{q}_1^H & - \\ & \vdots & \\ - & \bar{q}_{N-1}^H & - \end{bmatrix}_{N\times N}$;

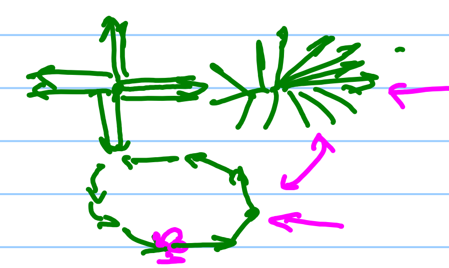
$k \rightarrow$ Block #

$n \rightarrow$ Sub-carrier (code) index

$m \rightarrow$ Time sample.

Prop #0 :

$$\frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} e^{j\frac{2\pi}{N}nm} = \begin{cases} 0, & \text{otherwise } (n = 1 \cdots N-1) \\ \sqrt{N}, & n = 0 \end{cases}$$



Prop #1 :

Delay $l=0$

$$\langle \bar{q}_n, \bar{q}_n \rangle = \bar{q}_n^H \bar{q}_n$$

$$= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} e^{-j\frac{2\pi}{N}nm} e^{j\frac{2\pi}{N}nm} = \sqrt{N} ;$$

Prop # 2 :

delay $l=0$

$$\langle \bar{q}_n , \bar{q}_{n'} \rangle \qquad n' \neq n , \quad n, n' \in \{0, 1, \dots N-1\}$$

$$= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} e^{j \frac{2\pi}{N} \underbrace{(n'-n)}_{} m}$$

$n'-n \in \{\pm 1, \pm 2 \dots \pm N-1\}$

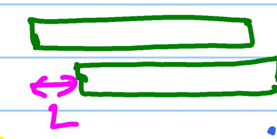
$$= 0 \, ;$$

Prop # 3 :

delay $0 < l \leq L-1$

$L-1 \leq N_{cp}$

$$\langle \bar{q}_n , \bar{q}_n(l) \rangle$$



$$= \frac{1}{\sqrt{N}} \sum e^{-j \frac{2\pi}{N} n m} \; e^{j \frac{2\pi}{N} n (m-l)} = \sqrt{N} \cdot e^{-j \frac{2\pi}{N} n l} \, ;$$

for each "n", we get a different exp.

Prop # 4 :

$$\langle \bar{q}_n , \bar{q}_{n'}(l) \rangle$$

$$= \frac{1}{\sqrt{N}} e^{-j \frac{2\pi}{N} n l} \cdot \cancel{\sum_m e^{j \frac{2\pi}{N} m (n'-n)}}^{\,0} = 0 \, ;$$

$d(1) \quad d(N)$

$[1 \cdots N] \longrightarrow \boxed{h[n]} \longrightarrow \oplus \longrightarrow y[n]$

AWGN channel

$h[n] = \delta[n-\ell]$, $w(n)$

$$(*) \quad \underset{N\times 1}{\bar{x}} = \frac{1}{\sqrt{N}} \underbrace{\begin{bmatrix} | & | & | & & | \\ \bar{q}_0 & \bar{q}_1 & \bar{q}_2 & \cdots & \bar{q}_{N-1} \\ | & | & | & & | \end{bmatrix}}_{N\times N} \underbrace{\begin{bmatrix} | \\ \bar{d}(k) \\ | \end{bmatrix}}_{N\times 1} \qquad \begin{bmatrix} d(k,1) \\ d(k,2) \\ \vdots \\ d(k,N) \end{bmatrix}$$

IDFT

$$\bar{x} = F\,\bar{d}$$

(*) Adding CP

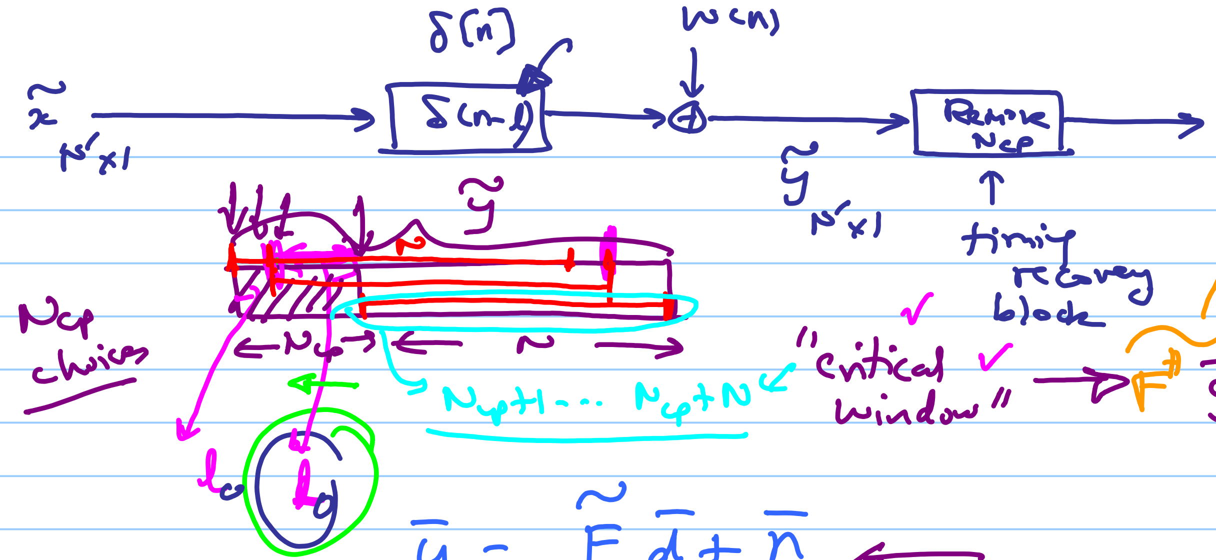
$$\underset{N\times 1}{\bar{x}} \longrightarrow \underset{N+N_{cp}\times 1}{\tilde{x}}$$

N'

$$\tilde{x} = \underbrace{\begin{bmatrix} F' \\ \cdots \\ F \end{bmatrix}}_{N'\times N} \underset{N\times 1}{\begin{bmatrix} \bar{d} \end{bmatrix}}$$

N_{cp}, N, N, $\tilde{F}$

(*)



"no channel estimation is required"

$$\boxed{\bar{z} = \bar{d} + \tilde{n}}$$

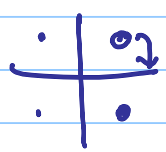
$$F^{H} \bar{y}_{N\times 1} = F^{H} F \bar{d} + F^{H} \bar{n}_{N \times 1}$$

$$[\ddots] \, \bar{d}$$

$$F^{H} F = I$$

$$\bar{y} = \tilde{F} \bar{d} + \bar{n}$$

$$F^{H} \bar{y} = F^{H} \tilde{F} \bar{d} + F^{H} \bar{n}$$

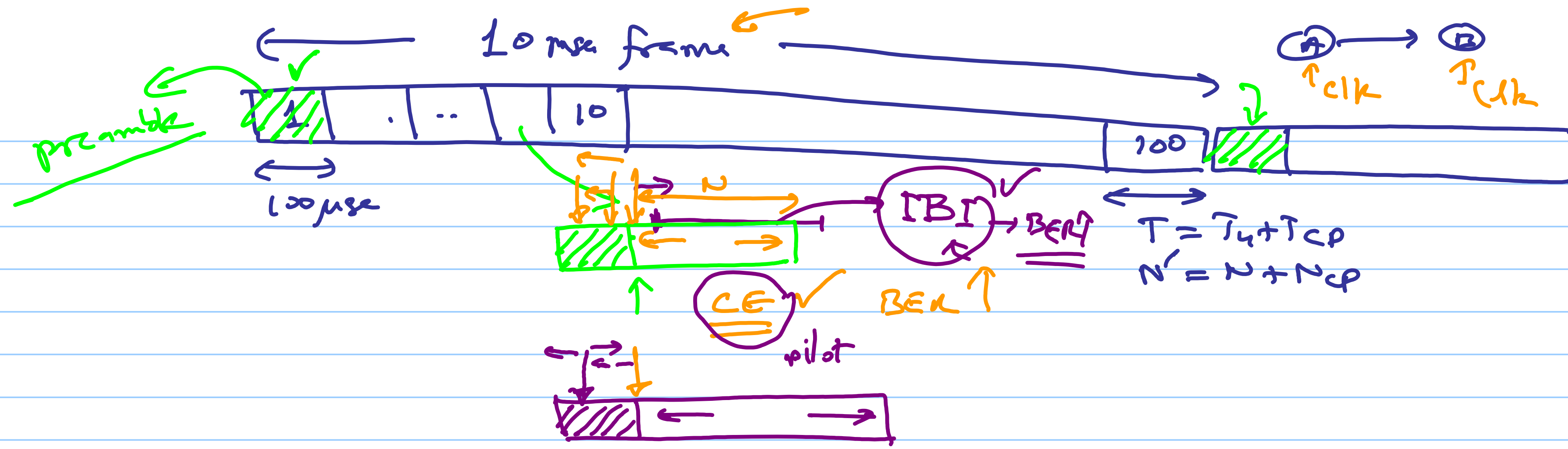


"channel"

"phase ramp"

$$\begin{bmatrix} e^{-j\frac{2\pi}{N} 0 \cdot l_0} & & 0 \\ & e^{-j\frac{2\pi}{N} 1 \cdot l_0} & \\ 0 & & \ddots \end{bmatrix} \quad e^{-j\frac{2\pi}{N}(N-1) l_0}$$

$$\bar{z} = \begin{bmatrix} \vdots \end{bmatrix} \circ \begin{bmatrix} \vdots \\ \bar{d} \end{bmatrix}$$



$$\tilde{y} = \tilde{x} * \bar{h} \longrightarrow ?$$