EE6323 Feb-May, 2021

# Class #5 (Mar. 09, 2021)

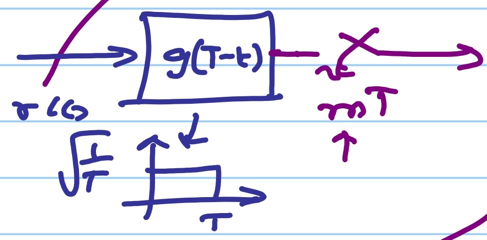
Note Title 27-03-2020

→ Soft capacity of DS-CDMA

→ Noise Rise and Coverage in DS-CDMA

$$\left\| \leftarrow \sum_{j=1}^{4} I_j(k)\, c_j(t) + n(t) \right.$$

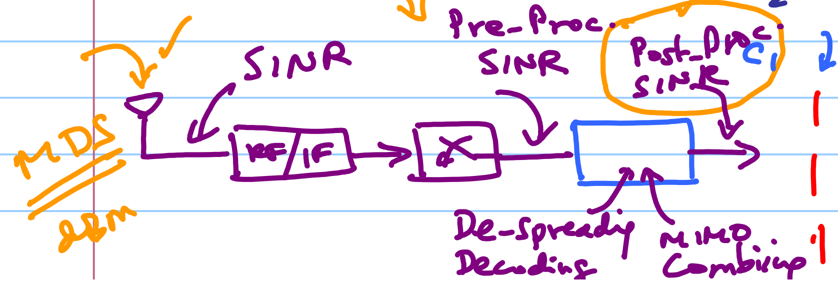

$$\longleftrightarrow \quad \bar{c}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \cancel{\sqrt{T}} \; ;$$





$$\bar{c}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\bar{r}(k) = I_1(k)\,\bar{c}_1 + \sum_{j=2}^{4} I_j(k)\,\bar{c}_j + \bar{n} \leftarrow 4 \times 1$$




$c_1$: −1 −1 −1 −1 1

$c_2$: −1 1 1 −1 −1

$$\left\{ \begin{array}{l} \bar{c}_1^T \bar{r} \\ \bar{c}_2^T \bar{r} \end{array} \right.$$

$$\bar{c}_1^T \bar{r} = 4 I_1(k) + n_1 + \varepsilon_{1j}$$

$W$

$\varepsilon_{12}, \varepsilon_{13} \ldots \varepsilon_{1N}$

"$\tau$" $T_c$

$\bar{c}_i$ $\quad$ $\bar{c}_j$ $\leftarrow$ $W \times 1$ $\Rightarrow$ $T' = W T$ $\qquad$ $W \rightarrow$ Spreading Gain

$\rightarrow T = W T_c$

SINR

(*) orthogonal Codes

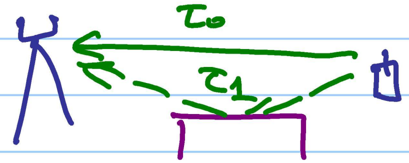
$$\bar{c}_i^T \bar{c}_i = W$$
$$\bar{c}_i^T \bar{c}_j = 0$$
$$\bar{c}_i^T \tilde{c}_j \neq 0 = \epsilon_{ij}$$

$|\epsilon_{ij}| \longleftrightarrow W$ ?

$\bar{c}_i^T \bar{c}_i = W$

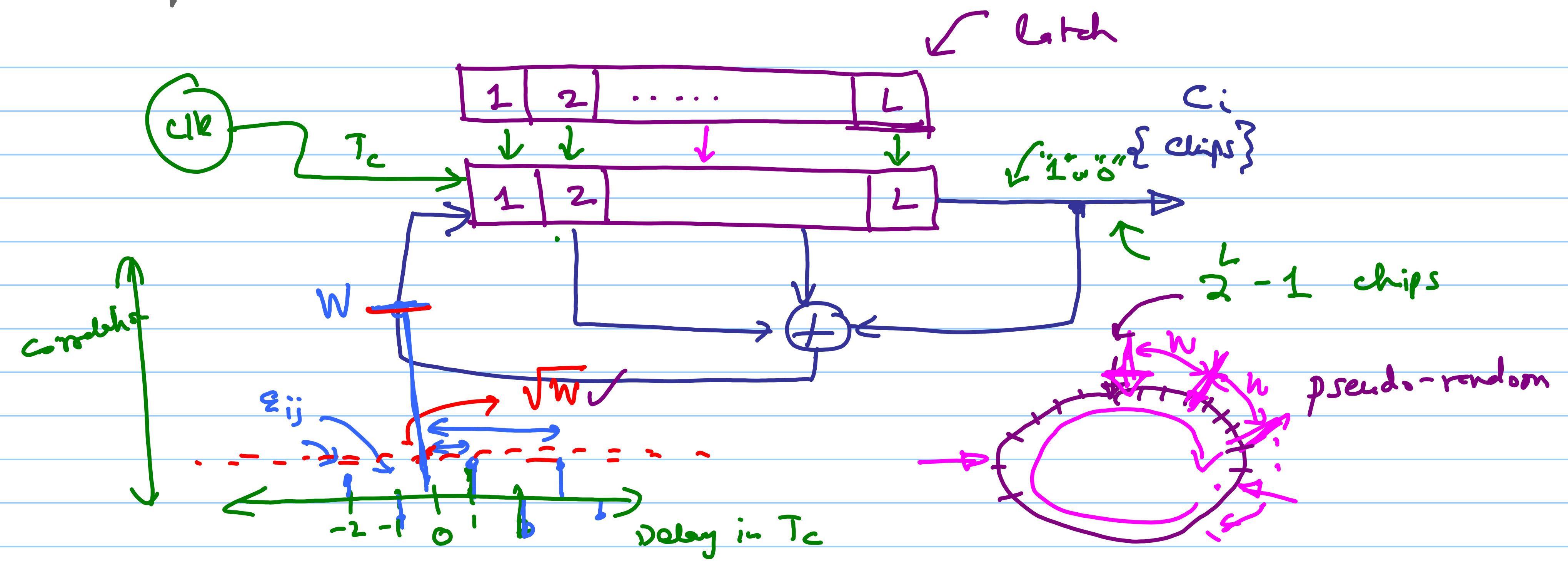
(*) Pseudo-orthogonal Codes.

$$\bar{c}_i^T \bar{c}_i = W$$
$$\bar{c}_i^T \bar{c}_{i,D} = \epsilon \,?$$
$$r(t) = \alpha_0 s(t-\tau_0) + \alpha_1 s(t-\tau_1)$$



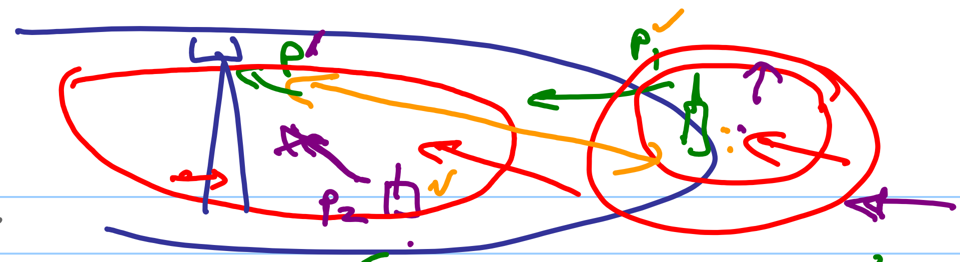

$$\bar{c}_i^T \bar{c}_j = \epsilon_{ij}$$
$$\bar{c}_i^T \bar{c}_i = W$$
$$\bar{c}_i^T \bar{c}_{j,D} = \epsilon_{ij}'$$

m-sequence $\longrightarrow$ LFSR




$\bar{c}_1^T \rightarrow [1 \quad -1 \quad 1 \quad 1 \quad -1 \quad \cdot \quad \cdot \quad \cdots ]$ ; $\bar{c}_i \rightarrow W \times 1$

Bernoulli r.v. 1 or -1 $p(1) = p(-1) = 1/2$ ;




# Noise Rise on the Uplink

Time "0" → first user arrives.

(*) $SINR = 6\,dB \Rightarrow 4 = \dfrac{W \cdot P}{\sigma^2}$

$$P = \frac{4\sigma^2}{W} = \frac{4\sigma^2}{128}$$

$$\boxed{P = \sigma^2/32}$$

$N = 33$

$N' < 33 \rightarrow N' = 16/17$

(*) 2nd user added ~~on~~ on UL

$$4 = \frac{WP}{P + \sigma^2} \Rightarrow 4P + 4\sigma^2 = 128P$$

$$\Rightarrow 4\sigma^2 = 124P$$

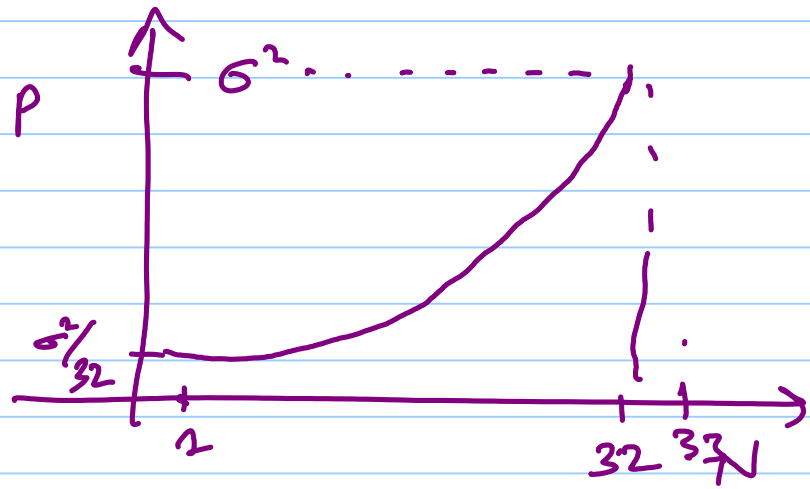
$$\Rightarrow P' = \boxed{\sigma^2/31}$$

$$4 = \frac{\overset{128}{NP}}{\underbrace{(N-1)}_{31}P + \sigma^2} \Rightarrow N = 32$$

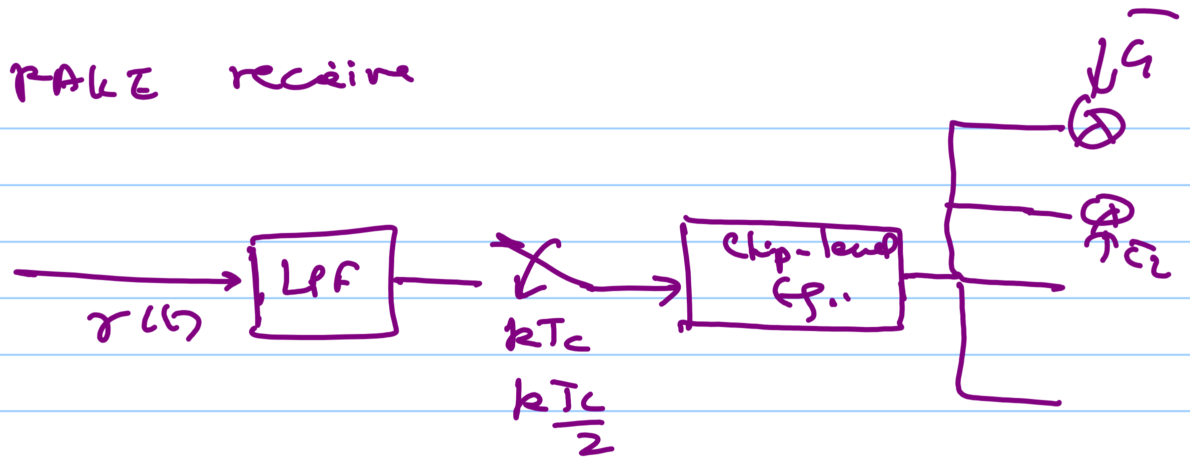
$32\sigma^2 + \sigma^2$

$$\underset{128P}{124P} + 4\sigma^2 = 128P$$

$$\Rightarrow 4P = 4\sigma^2 \Rightarrow \boxed{P = \sigma^2}$$



RAKE receive



user #1 $\longrightarrow \bar{c}_1 \leftarrow w \times 1$

$\bar{c}_1^T \bar{r}_{w\times 1} = I_1 \bar{c}_1 + I_2 \tilde{\bar{c}}_2 + \bar{n}_{w\times 1} \rightarrow N(0, \sigma^2)$ ; $kT\Delta f$ ; $\frac{1}{T_c}$

$= I_1 \cdot \underset{\sim}{W} + I_2 \overbrace{\bar{c}_1^T \bar{c}_2} + \overbrace{\bar{c}_1^T \bar{n}}$

$\sigma_1^2 \quad W^2 \qquad \sigma_2^2 \underbrace{\sum_{i=1}^{W} \underbrace{c_{1,i} c_{2,i}}}_{\mathrm{var}(\cdot) = W} \qquad \mathrm{var}(\cdot) = W\sigma^2 \qquad W \quad \sum_{i=1} c_{1,i} n_i$

$\in \{+1,-1\}$ $z_i$

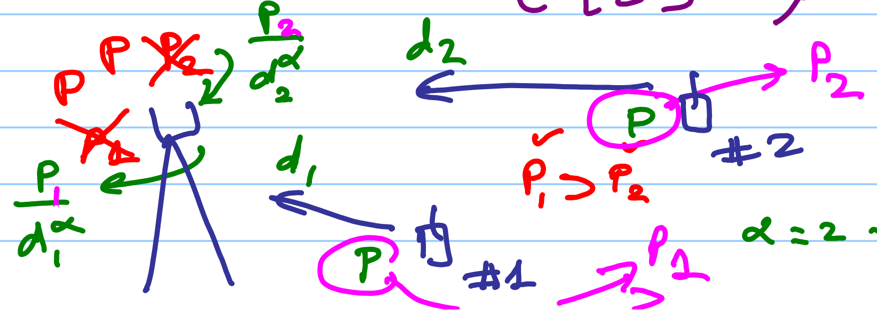
$E[z] = 0 \; 1$

$E[z^2] = E[\cancel{c_i^2}] = E[n_i^2]$ $\sigma^2$

$\sigma_1^2 > \sigma_2^2$

$\text{SINR}_{(1)} \triangleq \dfrac{\sigma_1^2 W^2}{\sigma_2^2 \cdot W + W \cdot \sigma^2}$

$\text{SINR}_{(2)} \triangleq \dfrac{\sigma_2^2 W^2 \cdot}{\sigma_1^2 \cdot W + W\sigma^2} \underset{=}{<} 1$




$\alpha = 2 \dots 5$

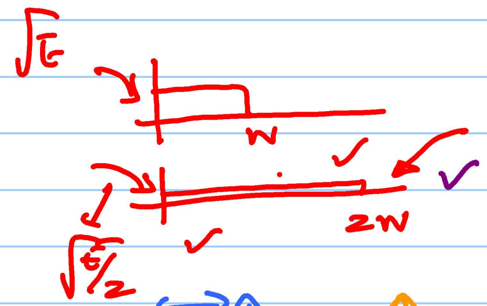
# "Perfect" uplink Power Control

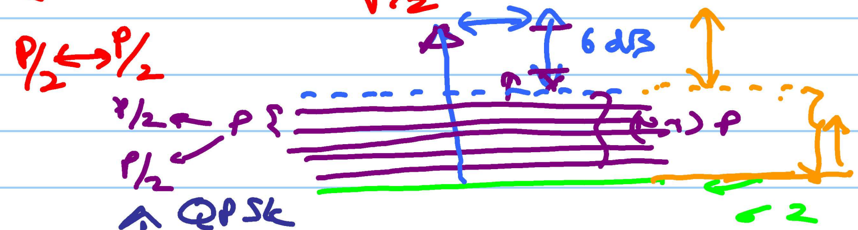
**2 user Case**

$$SINR = \frac{PW^2}{PW + W\sigma^2} = \frac{PW}{P + \sigma^2}$$

**N user Case**

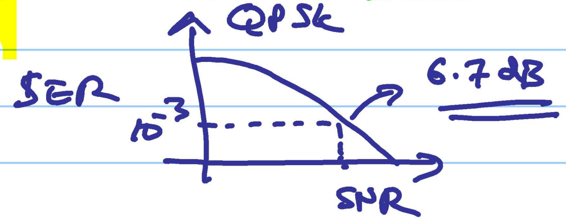
$$SINR = \frac{PW}{\underbrace{(N-1)\cdot P}_{\text{"}P\text{"}} + \sigma^2}$$




**Example** $W = 128$; $\longrightarrow$ $N = 128$

$SINR = 6\ dB \rightarrow 4$ $\longrightarrow$ PN Codes $N = ?$



$SINR = 10dB$

$$\underset{\uparrow 10}{4} = \frac{P \cdot 128}{\underbrace{(N-1) \cdot P}_{} + \cancel{\sigma^2}}$$

Pole Capacity

$$N - 1 = \frac{128}{4} = 32$$

$$\Rightarrow N = 33$$

Noise Rise & Coverage