Department of Electrical Engineering, IIT Madras

EE6323: Wireless Systems Design

Marks 20 **Simulation Assignment # 3** **June 12, 2020**

Note: *Your pdf report on Assignment#1 to emailed to the TAs Mr. M V Abhay Mohan,* ee17d024@smail.iitm.ac.in*, and Mr. Dibyajyoti Basak,* ee18s014@smail.iitm.ac.in *(and cc to* abhay@tenet.res.in *and* dbasak@tenet.res.in *) on or before* ***5.30pm on Monday, 24th June, 2020****. Name your report* EE6323-SA3-yourrollnumber.pdf *. Access to your running code and/or additional information may be demanded if required.*

For all the problems in this assignment, the following 512-point FFT based OFDM system is to be used.

Sl #	Attribute	Value / Definition
1.	Subcarrier Bandwidth	f_{sub} = 10KHz = $1/T$ where consequently the useful symbol duration T =100 μsec
2.	FFT size	N = 512
3.	OFDM Signal Bandwidth (assumed)	W = 5.12 MHz
4.	Sampling Rate	$1/T_S = W$ = 5.12 Msps
5.	Cyclic Prefix (CP) duration	T_{CP} = 12.5 μsec (i.e., $N_{CP} = 64$)
6.	OFDM Symbol duration	$T_{OFDM} = T + T_{CP}$ = 112.5 μsec i.e., 512 + 64 = 576 samples at 5.12Msps
7.	Frame duration (Q)	Typically, $Q = 11$ OFDM symbols; preamble, if present, will be the 1st symbol in the frame.
8.	Guard Subcarrier (GS) labels	Upper GS: $n \in \{+256 \text{ to } +241\}$ DC subcarrier: $n = 0$ Lower GS: $n \in \{-255 \text{ to } -241\}$
9.	Actual Signal Bandwidth (with used N_U = 480)	W_U = 482x10KHz = 4.82 MHz

The measurement model where the samples are collected every mT_S seconds, is as follows for the k^{th} block:

$$\tilde{z}(k,m) = \tilde{x}(k,m) \otimes h(k,m) + w(k,m) \qquad (1)$$

for $m = 0$ to $N + N_{CP} - 1$ and $k = 1$ *to* Q where $w(.)$ is AWGN at the receiver front-end with variance σ_w^2 and the notation " $\otimes$ " stands for linear convolution. Assuming time-invariant channel, accurate frequency synchronization (i.e., Δf = 0 Hz) and accurate timing synchronization, the "critical" FFT window is used by the receiver to give the below measurement model in the frequency domain:

$$Z(k,n) = H(n)\, I(k,n) + W(k,n) \qquad (2)$$

where $H(n)$ is the channel frequency response (CFR) obtained from the DFT of the channel impulse response (CIR) $h(m)$. The CIR has a length $L \leq N_{CP} + 1$. The non-zero complex coefficients of two example channels are given in the tables below.

*Channel Model #***1 : {** $g_1(m)$ **}**

Tap Delay (in sample number)	0	5	8
Tap Value (complex)	0.8 + *j*0.5	1.0 - *j*0.7	-0.6 + *j*0.3

*Channel Model #***2 : {** $g_2(m)$ **}**

Tap Delay (in samples)	0	2	7	15	27	32
Tap Value (complex)	1.0 + *j*0.9	0.8 + *j*0.5	-0.3 + *j*0.7	0.1 - *j*0.5	-0.6 + *j*0.2	-0.4 - *j*0.4

The signal to noise ratio (SNR) in the measurement model (1) can be taken to be $SNR = 1/\sigma_w^2$ once we normalize the gain of the channel to have unity; i.e., define

$$h(l) = \frac{g_i(l)}{\sqrt{\sum_{l=0}^{L-1} |g_i(l)|^2}}, i = 1,2 \qquad (3)$$

At the transmitter, $x(k,m)$ of length N = 512 samples is formed after the IFFT of the N_U = 480 symbols I (k, n) on the used sub-carriers, and the symbols are drawn from a unit energy QPSK alphabet (i.e., $I(k,n) \in \{\pm\frac{1}{\sqrt{2}}, \pm j\frac{1}{\sqrt{2}}\}$). Finally, $\tilde{x}(k,m)$ in (1) is the transmit samples after CP addition.

The first symbol (preamble) in every frame of Q=11 OFDM block, carries the pilot information in every 4th sub-carrier location, starting from n = +239 until n = -237 (i.e., pilots are in n = 239, 235, 231,…-233, -237). Pilot information is conveyed using pseudo-random QPSK symbols, with the mapping of pilot QPSK symbol to sub-carrier number fully known to the receiver. The remaining subcarriers in the preamble symbol convey control information. Channel estimation (CE) based on the schemes described in the following questions is to be performed using these pilot subcarriers in the preamble symbol, and using this CE, the remaining 10 OFDM blocks carrying information bits (encoded as QPSK symbols) are to be decoded. The Symbol Error Rate (SER) is also measured only on the QPSK mapped information symbols present in these last 10 OFDM blocks.

In this assignment, we study three different CE schemes, namely Zero-Forcing-with-Interpolation ($\hat{H}_{ZFI}$), FFT based method ($\hat{H}_{FFT}$) , and Modified Least Squares method ($\hat{H}_{MLS}$). The performance of the estimator is measured using Mean Square Error, measured using P independent Monte-Carlo trials as follows:

$$MSE = \frac{1}{PN_U}\sum_{i=1}^{P}\sum_{n=1}^{N_U}\left|H_i(n) - \hat{H}_i(n)\right|^2 \qquad (4)$$

Plot $10\log_{10}(MSE)$ versus $10\log_{10}(SNR)$ for different values of SNR on the x-axis. Plot from 0dB to 30dB, in 3dB steps, for each of these CE schemes.

(a) Zero Forcing CE: Here, the following 2 steps are followed:

(a1) On the pilot subcarriers,

$$\begin{aligned} Z(k,n)I^*(k,n) &= H(n)\,|I(k,n)|^2 + W(k,n) \\ &= \hat{H}_{ZF}(n), \text{for } n \in \text{pilot subcarriers} \end{aligned}$$

(a2) For the remaining subcarriers, use a linear interpolator to obtain $\hat{H}_{ZFI}$ from the above $\hat{H}_{ZF}$. Note that the value of the CE at the pilot locations will remain unaltered in this methodfrom the value obtained in (a1). What is the order (number of coefficients) of the linear interpolator that you used?

(a3) Repeat (a2) but using a cubic-spline interpolator. Again, what order did you use for the interpolator?

(b) FFT based CE: Here, the following 4 steps are followed:

(b1) On the pilot subcarriers,

$$\begin{aligned} Z(k,n)I^*(k,n) &= H(n)\,|I(k,n)|^2 + W(k,n) \\ &= \hat{H}_{ZF}(n), \text{for } n \in \text{pilot subcarriers} \end{aligned}$$

(b2) Make all other (non-pilot) subcarrier locations zero; i.e.,

$$Z(k,n) = 0, \text{for } n \in \text{nonpilot subcarriers}$$

(b3) Take N=512-point IFFT; retain only the first N_{CP} = 64 samples, and replace rest of the samples with zeroes. Call this vector $\hat{h}_{FFT}$.

(b4) Finally, a N-point FFT yields $\hat{h}_{FFT} \leftrightarrow \hat{H}_{FFT}$.

(c) MLS based CE: Follow the procedure explained in the class discussion to estimate $\hat{H}_{MLS}$. This class was recorded on May 2, 2020 and is available in the URL. Assume the CIR length to be N_{CP} when developing the MLS estimator. *Bonus Question:* If the actual CIR length L is used instead (for the 2 channel models), what will be new performance of the MLS CE? Plot your result and comment.

1. [4+4 = 8 marks] Plotting MSE versus SNR for Static Channel

Compute the MSE for P=10 Monte Carlo trials for the following cases:

(a) Use channel model **#1** to compute the MSE using (4) for all the three CE schemes in **(a), (b),** and **(c)**. Plot all the three curves on the same graph, in 3dB steps.

(b) Use channel model **#2** and repeat **(a)**.

2. [5 marks] Plotting MSE versus Average SNR for Block Fading Channel (Q=1)

The OFDM signal is now transmitted over a block fading multi-path model $h(k,m) = \sum_{i=0}^{L-1} a_i\,(k)\,\delta(m-\tau_i)$, defined by the following power delay profile (PDP). Here, the path gains vary from frame to frame (but are assumed to be constant within a block or a frame). In this question, we assume the block length to be equal to the frame length by having only one (block) symbol in every frame i.e., Q=1. This symbol will be the preamble, with pilots in every 4th sub-carrier.

*Channel Model #***3**

Path Gain σ_i^2(in dB)	-3	0	-1	-6	-9	-16
Tap Delay τ_i (in samples)	0	3	6	10	15	22

Hint: To normalize average channel gain to unity, rescale the (linear value of) the path variance σ_i^2 to ensure that over the L=6 paths, $\sum_{i=0}^{L-1} \sigma_i^2 = 1$. Each zero-mean path gain $a_i(k)$ where $E[|a_i|^2] = \sigma_i^2$, is a complex circular Gaussian random variable with each dimension having a variance of $\sigma_i^2/2$. The impulse-response snap-shot $h(k,m)$ is obtained by calling a circular Gaussian function L times, and scaling the instantaneous values with the corresponding gain from the power profile. The "short-term" frequency response snap-shot $H(k,n)$ is obtained by appropriate zero-padding followed by the 512-point FFT. Note that this way of generating the CIR implies that the instantaneous receiver SNR can vary, but the (statistical) average SNR will remain $ASNR = 1/\sigma_w^2$ as in question **1.**

We study the MSE by averaging over 100 different blocks (frames), i.e., k = 1,2, …,100, where there is a different realization of $h(k,m)$ for each k; i.e.,

$$MSE' = \frac{1}{100N_U}\sum_{k=1}^{100}\sum_{n=1}^{N_U}\left|H(k,n) - \widehat{H}(k,n)\right|^2 \qquad (4)$$

where $\widehat{H}(k,n)$ is estimated using **(a)**, **(b)**, and **(c)**. Plot $10\log_{10}(MSE')$ versus $10\log_{10}(ASNR)$ for different values of *ASNR* on the x-axis. Plot from 0dB to 30dB, in 3dB steps, for each of these CE schemes. Plot all the 3 curves in the same graph sheet.

3. [7 marks] Plotting the SER Performance for Block Fading Channel (Q=11)

For this part, we measure the Symbol Error Rate (SER) over a frame of Q =11 OFDM symbols (blocks). A block fading model is assumed over each frame. This implies that $h(k,m)$ is constant over a frame, and changes independently from frame to frame. The first block is each frame is the preamble symbol carrying the pilots. After this is used for CE (using **(a)** or **(b)** or **(c)**), the SER is measured over the remaining 10 OFDM blocks carrying QPSK symbols. In other words, for each channel realisation, the symbol errors are counted over $10\text{x}N_U$ QPSK symbols in that frame. Like this, at each ASNR, we use 100 channel realisations to measure the SER.

Plot $\log_{10}$(SER) versus $10\log_{10}(ASNR)$ for different values of *ASNR* on the x-axis. Plot from 0dB to 30dB, in 3dB steps, for each of these CE schemes. Plot all the 3 curves in the same graph sheet.

Compare the MSE performance in **2.** with the SER performance here, and comment.