Marks 20   **Simulation Assignment # 2**   **May 15 2020**

Note: *Your pdf report on Assignment#1 to emailed to the TAs Mr. M V Abhay Mohan,* ee17d024@smail.iitm.ac.in, *and Mr. Dibyajyoti Basak,* ee18s014@smail.iitm.ac.in *(and cc to* abhay@tenet.res.in *and* dbasak@tenet.res.in *) on or before* **5.30pm** *on* **Monday 1ˢᵗ June, 2020**. *Name your report* EE6323-SA2-yourrollnumber.pdf. *Your Matlab or C code must be included as an appendix to your report. Independent work is expected from each student, and access to your running code, and additional information, may be demanded if required.*

For all the problems in this assignment, the following 512-point FFT based OFDM system is to be used.

Sl #	Attribute	Value / Definition
1.	Subcarrier Bandwidth	$f_{sub} = 10\text{KHz} = 1/T$ where consequently the useful symbol duration $T = 100$ μsec
2.	FFT size	$N = 512$
3.	OFDM Signal Bandwidth (assumed)	$W = 5.12$ MHz
4.	Sampling Rate	$1/T_S = W = 5.12$ Msps
5.	Cyclic Prefix (CP) duration	$T_{CP} = 12.5$ μsec (i.e., $N_{CP} = 64$)
6.	OFDM Symbol duration	$T_{OFDM} = T + T_{CP} = 112.5$ μsec i.e., $512 + 64 = $ **576** samples at 5.12Msps
7.	Frame duration (Q)	Typically, $Q = 11$ OFDM symbols; preamble, if present, will be the 1ˢᵗ symbol in the frame.
8.	Guard Subcarrier (GS) labels	Upper GS: $n \in \{+256 \text{ to } +241\}$ DC subcarrier: $n = 0$ Lower GS: $n \in \{-255 \text{ to } -241\}$
9.	Actual Signal Bandwidth (with used $N_U = 480$)	$W_U = 482 \times 10\text{KHz} = 4.82$ MHz

The measurement model where the samples are collected every mT_S seconds, is as follows for the kth block:

$$\tilde{z}(k,m) = \tilde{y}(k,m)\, e^{j2\pi\Delta f[(k-1)(N+N_{CP})+m]T_S} + w(k,m) \qquad (1)$$

for $m = 0$ to $N + N_{CP} - 1$ and $k = 1\ to\ Q$ where $w(.)$ is AWGN at the receiver front-end with variance σ_w^2 and the residual frequency error $\Delta f = 8.75\text{KHz}$. Further,

$$\tilde{y}(k,m) = \tilde{x}(k,m) \otimes h(m) \qquad (2)$$

where $\otimes$ represents linear convolution and the multitap (samplespaced) channel impulse response $h(m)$ has a length $L \le N_{CP} + 1$. The non-zero complex coefficients of two example channels are given in the table below.

*Channel Model #**1** : { $g_1(m)$ }*

Tap Delay (in sample number)	0	5	9
Tap Value (complex)	0.8 + j0.5	1.0 - j0.7	-0.6 + j0.3

*Channel Model #**2** : { $g_2(m)$ }*

Tap Delay (in samples)	0	2	7	15	27	31
Tap Value (complex)	1.0 + j0.9	0.8 + j0.5	-0.3 + j0.7	0.1 - j0.5	-0.6 + j0.2	-0.4 - j0.4

The signal to noise ratio (SNR) in the measurement model (1) can be taken to be SNR $= 1/\sigma_w^2$ once we normalize the channel coefficients to have unity gain; i.e., define

$$h(l) = \frac{g_i(l)}{\sqrt{\sum_{l=0}^{L-1}|g_i(l)|^2}}, i = 1,2 \qquad (3)$$

Also, to with the Nx1 data vector given by $\bar{x}(k) = F^H \bar{d}(k)$ where F is the NxN full FFT matrix (and F^H is the IFFT), both with scaling factor $1/\sqrt{N}$ to ensure that statistically the average gain of each $\tilde{x}(k,m)$ is unity. At the transmitter, $x(k,m)$ of length $N = 512$ samples is formed after the IFFT of the $N_U = 480$ symbols $I(k,n)$ on the used sub-carriers, and the symbols are drawn from a unit energy QPSK alphabet (i.e., $I(k,n) \in \{\pm \frac{1}{\sqrt{2}}, \pm j \frac{1}{\sqrt{2}}\}$). Finally, $\tilde{x}(k,m)$ in (2) is the transmit samples after CP addition.

1. [1.5+1+1.5 = 4 marks] PAPR: Assume a (preamble-less) super-frame of 1000 OFDM blocks, with i.i.d. QPSK symbols imposed on all the useful subcarriers. The Peak to Average Power Ratio (PAPR) is defined on the base-band samples $\tilde{x}(k,m)$ at the input of the DAC in the transmitter as follows:

$$\mathrm{PAPR}(k) \cong \frac{\max \; |\tilde{x}(k,m)|^2}{\frac{1}{(N+N_{CP})} \sum_{m=1}^{N+N_{CP}} |\tilde{x}(k,m)|^2}$$

(a) Calculate the PAPR(k) for each of the $k =1$ to 1000 blocks and plot the histogram of these values in the Decibel (dB) scale. Use a 0.2dB "class-interval" size for the histogram.
(b) Read Ch-7 Section 7.1 in Cho's book, understand, and then plot the Complementary CDF (CCDF) curve using the histogram obtained in (a) as in Fig. 7.3 in the book.
(c) Interpolate (up-sample) the transmit samples by a factor of 4 and then find the PAPR. One convenient way of doing it is to use a 2048- point IFFT on the zero-padded vector formed by adding zeros above and below the current 480 used sub-carriers. What is the CCDF curve of this up-sampled sequence (where each block k will now have 576x4 = 2304 samples)? Comment on your PAPR results in (b) and (c).

2. [2.5+2.5+3.5+1.5 = 10 marks] Schmidl-Cox based Frequency Sync: A preamble symbol is to be designed to ensure that the *entire* frequency offset can be estimated by the (normalized) Schmidl-Cox (SC) algorithm. Assume that the non-zero subcarriers in the preamble use unit energy i.i.d QPSK symbols. Each frame of Q=11 OFDM symbols has the preamble as the first OFDM symbol, followed by 10 OFDM data symbols.
(a) Given that the maximum expected frequency offset seen on the received samples cannot exceed $\Delta f = 8.75\text{KHz}$, the preamble symbol is defined as follows: in the frequency domain, starting with $n = 239$, every odd sub-carrier up to $n = -239$ uses a (random) QPSK symbol. All the remaining sub-carriers are unused (zeroed). Now answer the following:

　　(i) What is the maximum frequency offset this design can detect using the SC algorithm?
　　(ii) For AWGN channel (i.e., $h(m) = \delta(m)$) and for SNR = 30dB, plot the absolute value of the output of the SC algorithm which is working on the samples of $\tilde{z}(k,m)$ as given by (1). Make this plot over a duration of 3 frames (i.e., 33 contiguous OFDM symbols). What is the width of the "plateau" (at the output of the SC algorithm) in number of sample durations?
　　(iii) Repeat (ii) for { $h(m)$ } formed from { $g_1(m)$ }.
　　(iv) Repeat (ii) for { $h(m)$ } formed from { $g_2(m)$ }. Comment on your results.

(b) We define the ergodic Mean Square Error in the frequency offset estimate by MSE $\triangleq \frac{1}{J} \sum_{j=1}^{J} (\Delta f_j - \Delta \hat{f}_j)^2$, where $\Delta \hat{f}_j$ is the estimate from the "jth" trial of the SC algorithm with independent signal and noise samples in each trial. Use J =10 for your MSE simulations. Using the channel { $h(m)$ } formed from { $g_2(m)$, vary the SNR between 0dB and 30dB in steps of 3dB and measure the MSE in each case. Plot the MSE in the dB scale on the y-axis, and SNR (also in the dB scale) on the x-axis.

(c) Now consider a case where $\Delta f = 25.50 \text{ KHz}$. Re-design the preamble symbol if necessary, to estimate this frequency offset accurately. Carefully explain and justify your design for this case.

(d) Repeat (b) for this choice of your preamble. Compare with (b) and comment.

3. [2+3+1=6 marks] OFDM Timing Sync: Timing sync implies two aspects in the receiver. Firstly, this involves finding the IBI-free FFT window. Secondly, it also reveals the preamble symbol boundary, and hence the frame (and block) boundary. For the channel { $h(m)$ } formed from { $g_2(m)$ and for $\Delta f = 25.50$KHz, use the preamble in the following ways to estimate timing:

(a) Do a cross-correlation with samples of the preamble (assumed known at the Rx). Plot this for SNRs 0dB, 10dB, and 30dB, over 3 consecutive frames of 33 symbols. Comment on your result.

(b) Now, use the Schmidl-Cox approach to do a normalized auto-correlation for the same case. Then:
    (i) First estimate the frequency offset.
    (ii) Using this estimate, "de-rotate" the received samples and therefore correct for the frequency error.
    (iii) Then, do a cross-correlation at the Rx using the stored (time-domain) samples of the preamble symbol. Plot this SNRs 0dB, 10dB, and 30dB, over 3 consecutive frames of 33 symbols. Comment on your result.

(c) Is it possible to get accurate timing estimation purely from the S-C algorithm? If so, explain how.