Department of Electrical Engineering, IIT Madras
EE6323: Wireless Systems Design

Marks 20 **Simulation Assignment # 1** **Updated on : Mar. 29 2020**

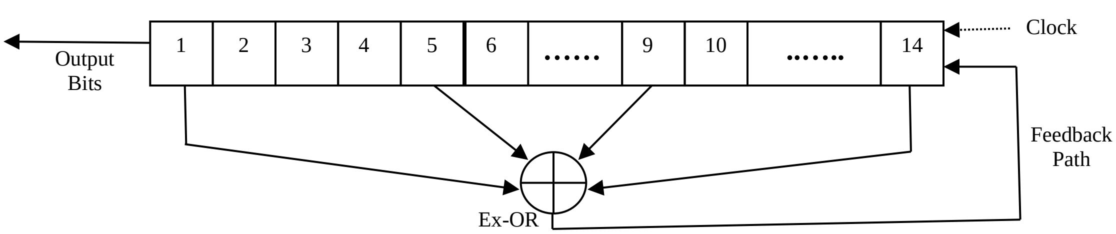
Note: *Your pdf report on Assignment#1 to emailed to the TAs Mr. M V Abhay Mohan,* ee17d024@smail.iitm.ac.in, *and Mr. Dibyajyoti Basak,* ee18s014@smail.iitm.ac.in *(and cc to* abhay@tenet.res.in *and* dbasak@tenet.res.in *) on or before* ***4.30pm on Tuesday, April 07, 2020****. Name your report* EE6323-SA1-yourrollnumber.pdf. *Your* Matlab *or* C *code must be included as an appendix to your report. Independent work is expected from each student, and access to your running code, and additional information, may be demanded if required.*

1. [10 marks] Coherent vs Semi-coherent Despreading: A DS-CDMA link uses T/T_c=512 chips per bit in order to provide a "low probability of intercept" which is an important aspect of covert communications. At the output of the ADC at the receiver, the following measurements are obtained for the k^{th} symbol at every chip-time of mT_c as follows:

$$r(kT + mT_c) = \sqrt{P}\ I(kT)\ c(kT + mT_c)\ Re\{e^{j2\pi\Delta f(kT+mT_c)}\} + w(kT + mT_c), \text{for } m = 0 \text{ to } 511 \quad (1)$$

where $w(.)$ is AWGN with variance $\sigma_w^2 = 1$ and P is the received signal power. Given that the (real) data symbol $I(kT)$ is bipolar and **differentially encoded**[1], the spreading chips $c(mT_c)$ are bi-polar and zero-mean, the signal to noise ratio (SNR) before de-spreading is given by $P/\sigma_w^2 = P$. Finally, Δf is the residual frequency offset in Hz, seen by the receiver which models the impact of frequency estimation error and fade Doppler-induced errors. For simplicity, we have taken only a real base-band measurement model and therefore the real-part $Re\{.\}$ of the exponential is considered in (1). Also, assume T_c=0.5µsec and T=256µsec.

The chip sequence is derived by mapping the bit-sequence at the output of a linear feedback shift register (LFSR) to bipolar values (i.e., map {0,1} to {-1, +1}). This 14-bit LFSR structure with tap-positions as given below (see pp. 667-668 of Proakis and Salehi, "Communication Systems" for more details) generates a pseudo-random sequence of length 2^{14}-1, one output for every clock which will be at chip-rate. Use the initial state of the shift-register to be "100….0". The first 512 chips are used to spread the first message bit, and the next 512 chips spread the second bit, and so on.



For each SNR, generate k = 1 to 10,000 "random" message bits[2] $I(kT)$ and the corresponding 512,000 chips. After multiplying this with $\sqrt{P}$ (which is fixed by the given SNR) and the $Cos(.)$ term of the exponential in (1), add AWGN samples with unit variance to each of them. Accurate timing synchronization to the exact bit-boundary is assumed at the receiver. The receiver de-spreads the samples given by (1), and every T=256µsec, and uses **coherent differential decoding**[3] on the de-spread output samples $y(k) = y(kT)$. The bit error rate (BER) for each SNR can then be computed by using the knowledge of the transmitted bits. Vary the SNR from -30dB to -6dB in 3dB steps and compute the BER for each of the questions below.

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[1] In differential bipolar encoding, set the 1st transmit bit as "0", represented by symbol I(1) = -1. Let this be called the anchor bit. Now, if the 2nd bit (generated randomly) is a "0", then the symbol I(2) will have the same sign (i.e., no change of phase); else, the sign will be reversed (i.e., 180^O phase change). Generate 10,000 bits randomly, *excluding* the anchor bit, and measure the BER only over the 10,000 phase-changes which signify the 10,000 information bits.

[2] *Hint*: For generating the message bits, use the uniform pdf between (0,1) and map 0 to 0.5 to -1, and 0.5 to 1 to +1.

[3] In coherently differential decoding, the product $z(k) = y(k+1)y(k)$ is computed between adjacent de-spread output samples, and hard-decision decoding is done on $z(k)$ to map it back to bit "1" (for +1) or bit "0" (for -1), and BER is then computed.

1a. For $\Delta f = 0\ Hz$, compute and plot the log(BER) in the y-axis versus SNR (in dB scale) in the x-axis when coherent de-spreading is deployed by the receiver.
1b. For $\Delta f = 100\ Hz$, repeat the effort in 1a. Plot the log(BER) on the same graph.
1c. For $\Delta f = 250\ Hz$, repeat the effort in 1a. Plot the log(BER) on the same graph. Comment on your result.
1d. Instead of the above coherent approach, we do a semi-coherent dispreading where only every 32 chips are used to decide on the data. In 512 chips, we can get 16 distinct "tentative" $y_i(k)$s, $i = 1,2,\ldots 16$, for the same data symbol. Coherent differential decoding is done on the corresponding 16 tentative $y_i(k+1)$s from the next symbol duration. Now, a majority logic rule among the 16 tentative differential decisions of $z_i(k) = y_i(k+1)y_i(k)$ is used to make the k^{th} final bit decision (over the T=256μsec bit period). Use this approach and repeat the experiment for both $\Delta f = 100\ Hz$, and $\Delta f = 250\ Hz$. Compare your results with **1b.** and **1c**., respectively, and comment.
1e. Instead of 32 chips, can there be a (single) better choice for the de-spreading window in the semi-coherent approach to improve the performance when compared to both **1b.** and **1c.**? Discuss with evidence over the range of SNRs specified.

2. [10 marks] Orthogonal versus Quasi-orthogonal Codes: Consider an uplink DS-CDMA measurement model where signals from two distinct users arrive at the base station with delays τ_1 and τ_2 as follows:

$$r(t) = h_1\sqrt{P_1}\, s_1(t-\tau_1) + h_2\sqrt{P_2}\, s_2(t-\tau_2) + w(t) \tag{2}$$

where for simplicity we assume the short-term fade variables $h_1 = h_2 = 1$, and $w(t)$ is AWGN with variance σ_w^2. The signal $s_i(t)$, i=1,2, is given by

$$s_i(kT + mT_c) = I_i(kT)\, c_i(kT + mT_c), \text{for } m = 0 \text{ to } 127, \text{where spreading factor} \frac{T}{T_c} = 128. \tag{3}$$

We assume perfect knowledge of the delays τ_1 and τ_2 and the corresponding codes at the base-station receiver. The (real) data symbol $I_i(kT)$ and the spreading chips $c_i(mT_c)$ are bi-polar, mutually uncorrelated, and zero-mean. In this example, no differential modulation is required over the data symbols. This problem aims to compare the performance of orthogonal versus quasi-orthogonal codes and the impact of uplink power control on the receiver performance. The receiver will have two de-spreading arms, time-aligned with the (known) delays of the two incoming signals.

2a. Consider first orthogonal codes; let c_1 = [1 -1 1 -1 1 -1 1 -1... 1 -1 1 -1] and c_2 = [1 1 -1 -1 1 1 -1 -1 …1 1 -1 -1] where both are 128x1 vectors. The same code is used by each user, to spread each bit. For each of the below cases, compute the individual BERs for user-1 and user-2 over 100,000 transmitted bits each, and present the results in tabular form. The 2 transmit bit streams can be derived as explained in Pbm #1.
(i) P_1=100; P_2=45; $\tau_1 = 0$; $\tau_2 = 0$ and AWGN with $\sigma_w^2 = 1600$; BER_1 = ? BER_2 = ?
(ii) P_1=100; P_2=45; $\tau_1 = 0$; $\tau_2 = 7T_c$ and AWGN with $\sigma_w^2 = 1600$; BER_1 = ? BER_2 = ?
(iii) P_1=100; P_2=45; $\tau_1 = 0$; $\tau_2 = 75T_c$ and AWGN with $\sigma_w^2 = 1600$; BER_1 = ? BER_2 = ?
(iv) P_1=60; P_2=60; $\tau_1 = 0$; $\tau_2 = 75T_c$ and AWGN with $\sigma_w^2 = 1600$; BER_1 = ? BER_2 = ?

2b. Now consider using instead pseudo-orthogonal codes as described in Pbm #1. Define c_1 by taking successive 128 chips starting with the initial state of the 14-bit shift-register being "1000….00". In a similar fashion, define c_2 by taking successive 128 chips starting with the initial state of the shift-register being alternating "0101….01" pattern. Also, note here that successive bits get spread by different chips (unlike in the case of orthogonal codes), and although both c_1 by c_2 are drawn from the same LFSR sequence, they are in "different arcs of the 2^{14}-1 long code circle" and hence are quasi-orthogonal.

With these two codes in place, repeat (i) thro (iv) as in **2a.**, and tabulate the measured BER values. Comment on your results and justify.