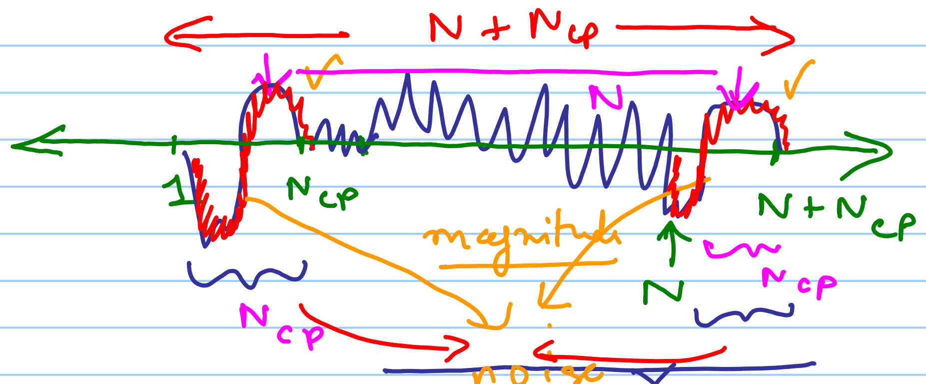
Note Title 4/25/2020

Class #6 April 25, 2020

OFDM Timing Synchronisation
→ Cross corr.
→ CP based autocorr.
→ Schmidl - Cox autocorr.

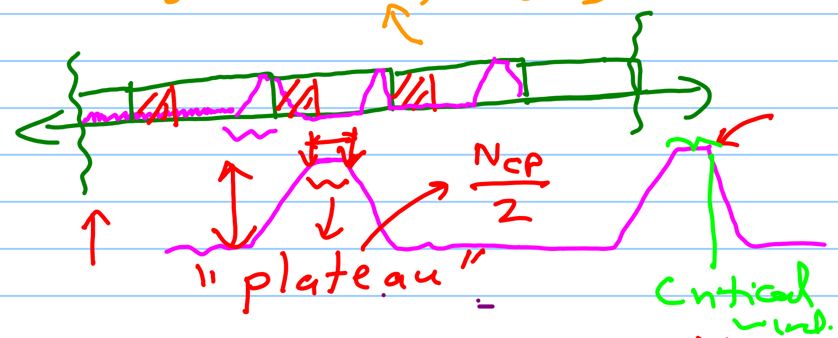
$L = 1 \leftarrow$ memoryless channel

$h[k,m] = \alpha(k,m)\,\delta[m]$



$L = \frac{N_{cp}}{2}$

~~phase noise~~, ~~fading~~, Freq. offset



IBI → $N_{cp}/2$

IBI-free → $N_{cp}/2$

$\frac{N_{cp}}{2}$

$$z(k,m) = \boxed{\frac{\displaystyle\sum_{i=m-\frac{N_{cp}}{2}+1}^{m} \tilde{y}^*(k,i)\,\tilde{y}(k,i+N)}{\displaystyle\sum_{i=m-\frac{N_{cp}}{2}+1}^{m} \left|\tilde{y}(k,i)\right|^2}}$$

→ normalisation factor

$L \to$ Delay spread

Assuming $\boxed{L < \frac{N_{cp}}{2}}$

(*) Note: (i) $L=1$, # of |B| terms $= 0 \Rightarrow N_{cp}$ choices for timing window

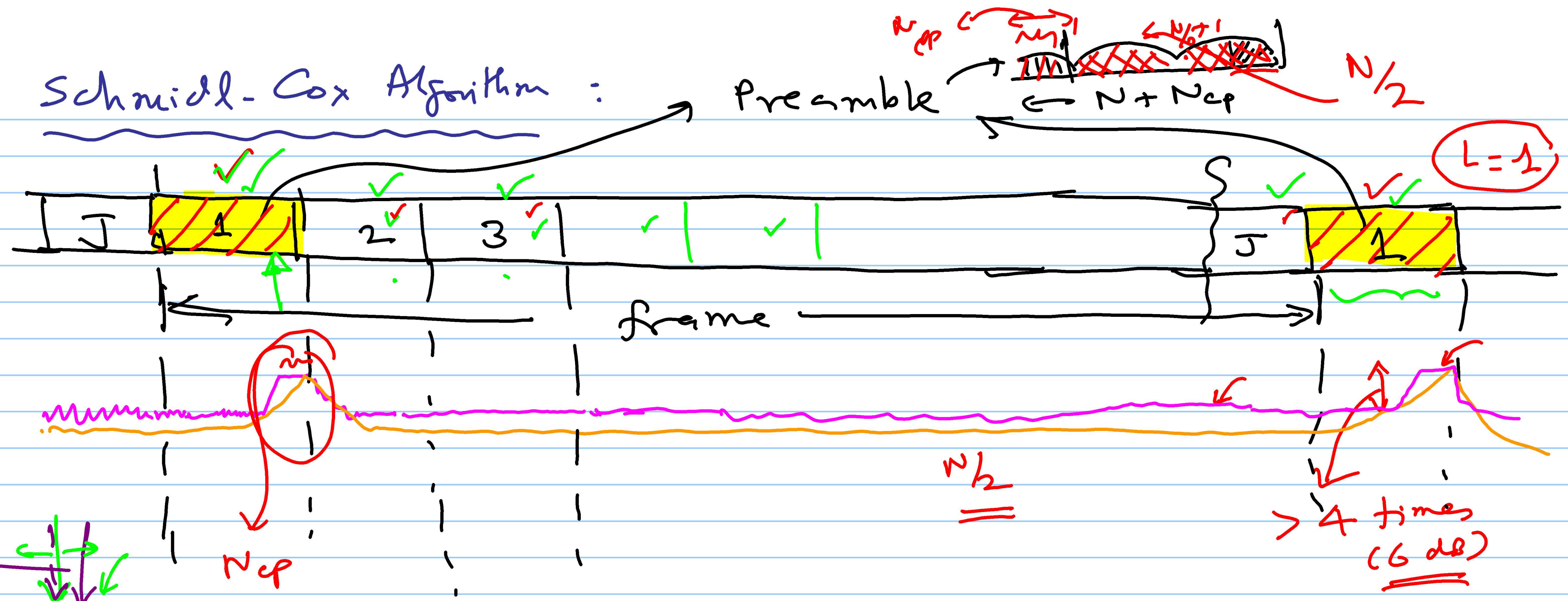
(Recall) (ii) $L=2$, # " $= 1 \Rightarrow N_{cp}-1$ choices (timing)

(iii) $L = N_{cp}+1$, " $= N_{cp} \Rightarrow$ only 1 choice of FFT window → i.e. critical window only window possible!

(*) Pro → simple algo, can avg. over every symbol

Con → higher CP (N_{cp}) than necessary ($CP > 2L$)

Schmidl-Cox Algorithm :



✓ * Symbol boundary → FFT window → N_{cp}

$L = 1$ →

✓ * frame boundary

$\| L = N_{cp}+1$ → 1

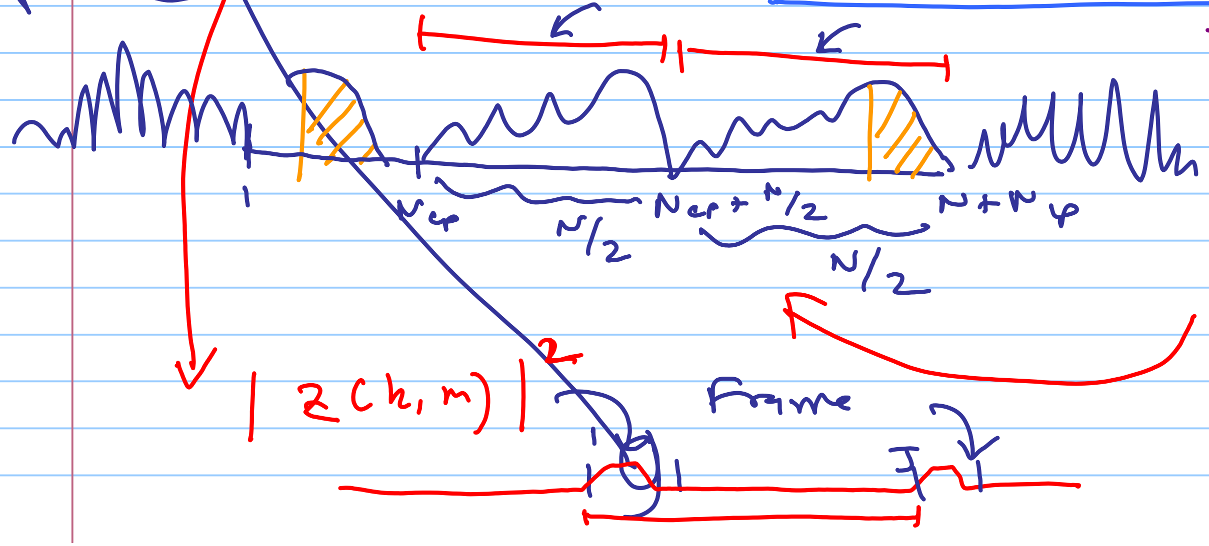
$\arctan(z)$ $\qquad \dfrac{Im\{z\}}{Re\{z\}}$ $\qquad \Delta f$

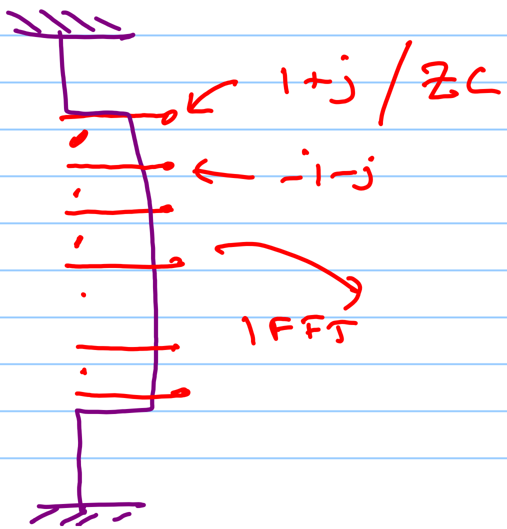
$$\underline{z(k,m)} = \boxed{\dfrac{\sum_{i=m-\frac{N}{2}+1}^{m} \tilde{y}^*(k,i)\,\tilde{y}(k,i+N/2)}{\sum_{i=m-\frac{N}{2}+1}^{m} \left|\tilde{y}(k,i)\right|^2}}$$

$N/2$

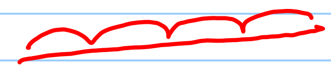
for $\underline{\underline{2}}$ replicas within $\underline{\underline{N}}$

i.e., $D = 2$





3 zeros $\quad D=4$



$N/2 \longrightarrow$?

1

k → block
m → time

Frequency Sync. (using Schmidl. Cox)

$\boxed{\text{For } D = 2}$

Δf Hz

$$y(m) = \tilde{y}(m) \cdot e^{j 2\pi \Delta f \breve{m}} \quad \text{where} \quad \breve{m} = m T_s \; ; \quad T_s = \frac{1}{W} = \frac{1}{N \Delta f_{sub}} \; ;$$

$$= \boxed{\frac{m}{N \Delta f_{sub}}}$$

Rewriting fully,

$$y(mT_s) = \tilde{y}(mT_s)\, e^{j \frac{2\pi \Delta f\, m}{N \Delta f_{sub}}} \; ;$$

$\hat{\Delta f}$

$N/2$ later

$D = 2$

$$y(m + N/2) = \tilde{y}(m + N/2)\, e^{j 2\pi \Delta f \frac{(m + N/2)}{N \Delta f_{sub}}} \; ;$$

D

self-similarity

$$\Rightarrow \overbrace{y^*(m)\, y(m+N/2)} = \overbrace{\tilde{y}^*(m)\, \tilde{y}(m+N/2)}\, e^{j 2\pi \Delta f \left(\frac{N/2}{N \Delta f_{sub}}\right)}$$

$$= \underbrace{|\tilde{y}(m)|^2}_{\text{positive}}\, e^{j\pi \underbrace{(\Delta f / \Delta f_{sub})}}$$

D = 2

$\therefore$ for un-ambiguous estimation $\pi \left(\frac{\Delta f}{\Delta f_{sub}}\right) < \pm \pi$ radians ✓

LTE 15 KHz

$$\Rightarrow \boxed{\Delta f < \pm \Delta f_{sub}} ;$$

i.e., in general, for "D" ✓ replicas

$$\boxed{\Delta f < \pm \frac{D}{2} \Delta f_{sub}} ;$$

D − 1 zeros

$D = 2^k$

D = 3 || D = 5 ×