Department of Electrical Engineering, IIT Madras

EE6141 : Multicarrier Communications (Jan-Apr, 2017)

KG/IITM ***ASSIGNMENT #1*** ***______50 Marks***

Nearly all these problems involve the uplink (MAC) channel, implicitly assuming purely Line of Sight (Los) links. The assignment is due on or before **5.00pm, Monday, March 13, 2017**. *The hand-written assignment answers (with your name and roll # clearly marked on the first sheet) should be handed over to* Mr. S. Abhijith Nambi *(*ee15s047@ee.iitm.ac.in), *at the Intel Wireless Lab.*

1. TDMA MAC Channel [10 marks]: A cellular TDMA base-station is required to serve users who are up to 9km away on the uplink. The RF bandwidth used by this system is W=2MHz, and SRRC pulse shaping with excess bandwidth factor β=0.5 is used to produce the 8-PSK encoded waveforms. The frame duration is 6msec with 5 slots per frame of 1.2msec duration each.

(a) Find the symbol duration, and the number of symbols per slot.

(b) What is the number of guard symbols required on the uplink MAC channel to compensate for the worst-case time of flight difference?

(c) What is the effective (useful) bit-rate per user, if 10% of the slot duration is given for pilot symbols and other control overheads? What is the useful spectral efficiency in bits/sec/Hz ?

(d) Now, the RF bandwidth is increased to 20MHz and 50 slots are planned in every 6msec frame duration (with all other parameters as before). What is the new value of useful spectral efficiency?

(e) Recall also that the received power P_R in dBm at a distance d meters from the transmitter can be determined from the below formula in the log-scale:

$$P_R(d) = P_T - L - 10log_{10}(d^{\alpha})$$

where P_T is the transmit power in the dBm scale (note that 1mwatt = 0dBm), α is the path loss exponent, and L (in dB) accounts for all other fixed losses. Also recall that the receiver sensitivity or minimum detectable signal strength (MDS) required at the receiver is given by

$$MDS\ (dBm) = -174dBm + 10log_{10}(W) + NF(dB) + SNR(dB)$$

where NF is the receiver noise figure, and SNR is the signal to noise ratio required to achieve the target bit error rate for the given modulation.

> (e1) Given P_T = 1watt, L=47dB, α=2, NF=6dB, and required SNR=8dB, find the link distance d for W=2MHz.
>
> (e2) Repeat the above, but for W=20MHz.
>
> (e3) If it is required to serve users up to 9km link distance on the uplink for W=20MHz, that should be the new P_T ? Compare with (e2) and comment.

2. Orthogonal Codes in DS-CDMA [10 marks]: With an aim to provide orthogonal support even in the presence of time-of-flight differences on the uplink, a DS-CDMA system uses symbol-repetition where a symbol T=4T_c long, in repeated twice to make it 2T long, as follows:

$$\boldsymbol{s_1}(n) = I_1(n2T)\begin{bmatrix}\boldsymbol{c_1}\\ \boldsymbol{c_1}\end{bmatrix}$$

where the first user uses a code $\boldsymbol{c_1}$=[1 1 1 1], and the user's data symbol $I_1(n2T)$ is picked from a BPSK alphabet. This discrete-time sequence is sent to a DAC to convert it into a waveform $s_1(t)$

which is $2T$ long. In a similar manner, the second uplink user uses the code $\boldsymbol{c}_2$=[1 -1 1 -1] to obtain $s_2(t)$. The received waveform from the users is given by $r(t)= \alpha_1 s_1(t) + \alpha_2 s_2(t-\tau_2)$, where the relative time-of-flight delay is (assumed to be) $\tau_2=jT_c$, where integer j=0,1,2,….,. The receiver at the base-station, however, "de-spreads" only a T long waveform corresponding to the second part of the $2T$ long waveform corresponding to each bit.
(a) For what values of τ_2 can the two signals be orthogonally separated?
(b) Now, a third user is added to the network using a code $\boldsymbol{c}_3$=[1 1 -1 -1] and the received waveform at the base-station is now modified to $r(t)= \alpha_1 s_1(t) + \alpha_2 s_2(t-\tau_2) + \alpha_3 s_3(t-\tau_3)$. In this case, for what values of τ_2 and τ_3 can all the 3 signals be orthogonally separated?
(c) Finally, a fourth user is to be added to the network using the code $\boldsymbol{c}_4$=[1 -1 -1 1], resulting in the measurement model $r(t)= \alpha_1 s_1(t) + \alpha_2 s_2(t-\tau_2) + \alpha_3 s_3(t-\tau_3) + \alpha_4 s_4(t-\tau_4)$. Now, for what choices of τ_2, τ_3, and τ_4 can all the 4 signals be orthogonally separated? Comment.

3. Pseudo-Orthogonal DS-CDMA and Uplink Power Control [10 marks]: With an aim to be relatively insensitive to "time-of-flight" delays, we now consider pseudo-noise (PN) codes $\mathbf{c}_i$ on the uplink where the symbol duration is given by $T=NT_c$, with T_c being the chip duration and N being the spreading factor. Each chip is either +1 or -1, and let $\varepsilon_{i,j}= \mathbf{c}_i^T \mathbf{c}_j$.
(a) Prove that $E[\varepsilon_{i,j}^2] = N^2$ if i=j, and equals N if $i\neq j$. State your assumptions clearly.
(b) For the first user, let $s_1(t)$ be the transmitted uplink signal over $kT \leq t < (k+1)T$, corresponding to the DAC output for $I_1(kT)\,\mathbf{c}_1$, where data symbols I_1 are either +1 or -1. Given the measurement model $r(t) = \alpha_1 s_1(t) + n$, where n in AWGN with variance σ_n^2, find the expression for the Signal to Interference plus Noise ratio (SINR) after dispreading the signal appropriately.
(c) Now the second user is added on the uplink to give the new measurement model as below: $r(t) = \alpha_1 s_1(t) + \alpha_2 s_2(t-\tau_2) + n$. The base-station now uses two de-correlators (de-spreading operations) in parallel, to detect the two streams. Find the SINR expressions involved in the recovery of both I_1 and I_2. Does τ_2 play any role in defining the de-correlation operation or in the SINR expression? Please explain clearly.
(d) Assume now that σ_n^2 is very small compared to the signal powers and that it can be neglected. If N=32 and $\alpha_1 = 10\,\alpha_2$, what will be the values of the two SINRs in part (c) above?
(e) Repeat (d) for N=256. Compare with (d) and comment.
(f) For N=256, what would be the SINRs if a third user is added to the uplink having $\alpha_3 = 4\alpha_2$?

4. OFDM Pilot Design and Overheads [10 marks]: Consider a 20.48 MHZ OFDM system with FFT size of N=1024. Assuming that the sub-carriers are indexed from 512 to -511, the guard sub-carriers are indexed by (512 to 472) and (-472 to -511). The sampling rate is taken to be 20.48 MHZ. Answer now the following:
(a) What is the sub-carrier band-width in kHz?
(b) If the Cyclic Prefix (CP) length Ncp = N/4 samples of the useful symbol duration, what is the duration (in μsecs) of the full OFDM symbol?

(c) If QPSK modulation is employed on all the useful subcarriers, what is the "gross" spectral efficiency of this OFDM system (in bits/sec/ Hz)?
(d) If 5ppm clocks are used on both the Tx and Rx nodes, how often should the preamble symbol be repeated in time so that between preambles, there is no more than half-a-sample of slip? (*Hint*: First find the worst-case slip-rate in number of sample slips per second.)
(e) Putting (d) in perspective, make a neat sketch of the OFDM blocks, say drawn over two preamble intervals. The preamble can be assumed to also mark the beginning of a new frame. What is the frame rate in frames/sec?
(f) Let in every alternate symbol, every 9^{th} subcarrier be a pilot subcarrier. What is the "nett" spectral efficiency, after accounting for preamble and pilots?

5. Link Distance in Uplink OFDMA [10 marks]: We consider now the uplink of an OFDMA system where links are to established *without* any timing-advancement (ranging) support. Critical sampling is employed and the 20.48MHz bandwidth is divided into N=1024 subcarriers, and a cyclic prefix of $1/8^{th}$ the useful symbol duration is used (i.e., $T_{CP}=T/8$). The maximum transmit power from a mobile station is P_T=36dBm. The loss at 1m is L_{1m}=33dB, and the path-loss exponent α=3. The ambient noise PSD can be taken as -174dBm. The receiver noise-figure N_F=6dB, and for QPSK detection at the target BER, we require the received SNR to be SNR=8dB.

Recall also that the received power P_R in dBm at a distance d meters from the transmitter can be determined from the below formula in the log-scale:

$$P_R(d) = P_T - L_{1m} - 10log_{10}(d^{\alpha})$$

Also recall that the required P_R at the receiver (i.e., sensitivity or the minimum detectable signal strength) is a function of uplink band-width used, SNR, and N_F .
(a) If a mobile station wants to use all the N=1024 subcarriers on the uplink, what is the maximum range (link distance in meters) that it can have for P_T=36dBm?
(b) Since no uplink ranging is provisioned here, what is the maximum link possible given the above T_{CP} if there was no limitation on choosing P_T?
(c) If only 100KHz of uplink bandwidth is used by the mobile station, what will be the maximum link distance possible? Will this be admissible from the view-point of T_{CP} provided? Explain.
(d) Given the limitation imposed by the T_{CP} as in (b), for that link-distance in (b), what is the maximum uplink bandwidth possible?