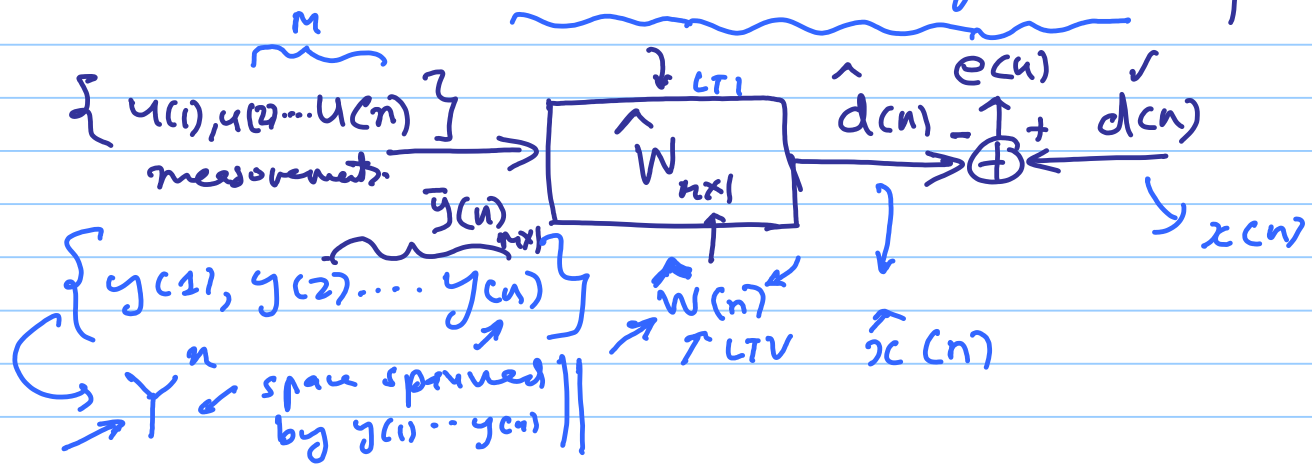
## Class #25 (Nov. 02, 2021)

Note Title 27-03-2020

$\bar{u}(n)$ Kalman filter Algorithm | Sridhar filter ✓




L, G

(*) LMMSE estimate at time $n-1$ is given by

$$\rightarrow \hat{x}(n-1|Y^{n-1}) = E[x(n-1)|Y^{n-1}]$$

$\bar{x} \rightarrow$ state

(*) Updated estimate at time $n$ $\rightarrow$ $\hat{x}(n|Y^n)$

using $y(n)$ ✓

**Question** How to iteratively find $\hat{x}(n|Y^n)$ from $\hat{x}(n-1|Y^{n-1})$?

(*) Forward Prediction Error

$$\alpha(n) = y(n) - \underbrace{\hat{y}(n|Y^{n-1})}_{\text{one-step prediction got from } \hat{x}(n-1|Y^{n-1})}$$

"innovation" "new information in $y(n)$ which is <u>not</u> present in $Y^{n-1}$

(*) Can we transform

"correlated" "uncorrelated"

$$\{y(n), y(n-1), \ldots y(1)\} \longleftrightarrow \{\alpha(n), \alpha(n-1), \ldots \alpha(1)\}$$

$\mathbb{R}^n$ $\underline{\underline{n\times 1}}$

*"Whitening"

Gram Schmidt

$$\{\bar{v}_1, \bar{v}_2, \ldots \bar{v}_n\}_{n\times 1} \rightleftharpoons \{\bar{q}_1, \bar{q}_2 \cdots \bar{q}_n\}$$

$$\to \bar{q}_i^{*} \bar{q}_j = \begin{cases} 1, & i = j \\ 0, & i \neq 0 \end{cases}$$

Properties of $\alpha(n)$:

(*) __Prop 1__: $E[\alpha(n)\, y^*(k)] = 0$, $1 \le k \le n-1$

↳ Principle of orthogonality. $\to y^{n-1}$

__Prop 2__: $E[\alpha(n)\, \alpha^*(k)] = 0$, $1 \le k \le n-1$

Prop 3: $\{y(1) \cdots y(n)\} \rightleftharpoons \{\alpha(1) \cdots \alpha(n)\}$

Causal & Invertible filter exists for this
(without any loss of information)

Exercise : Revise Gram-Schmidt orthogonalisation.

Finally, we can write.

$$\begin{bmatrix} \alpha(1) \\ \alpha(2) \\ \vdots \\ \alpha(n) \end{bmatrix} = \begin{bmatrix} 1 & & & & \\ a_{11} & 1 & & 0 & \\ a_{22} & a_{2,1} & 1 & & \\ \vdots & & & \ddots & \\ a_{n-1,n-1} & a_{n-1,n-2} & \cdots & \cdots & 1 \end{bmatrix} \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(n) \end{bmatrix}$$

$$a_{11} = \frac{E[y(2)y^*(1)]}{E[y(1)\,y^*(1)]};$$

LTI, det = 1, Causal

$$\hat{x}(n \mid Y^n) = \hat{x}(n \mid \alpha^n)$$

(*) Therefore, if a linear estimate $\hat{x}(n \mid y^n) = \sum_{k=1}^{n} b_k \alpha(k)$

Then the MMSE problem is

$$\min_{\{b_k\}} E\left[\left| x(n) - \hat{x}(n \mid y^n) \right|^2\right]$$

will give. $w = R^{-1} p$ $R_{yy} \to R_{\alpha\alpha}$

we will have

$$\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = R_{\alpha\alpha}^{-1} p \qquad R_{\alpha\alpha}^{-1} = \begin{bmatrix} \cancel{E[\alpha(1)^2]}^{-1} & & 0 \\ & \cancel{E[\alpha(2)^2]}^{-1} & \\ 0 & & \ddots \; \cancel{E[\alpha(n)^2]}^{-1} \end{bmatrix}$$

diagonal matrix

$$\boxed{b_k = \frac{E\left[x(n)\,\alpha^*(k)\right]}{E\left[|\alpha(k)|^2\right]}, \quad 1 \le k \le n}$$

$n \to$ Scalar

Finally

$$\hat{x}(n|Y^n) = \underbrace{\sum_{k=1}^{n-1} b_k\,\alpha(k)}_{\hat{x}(n-1|Y^{n-1})} + b_n\,\alpha(n)$$