RLS Algorithm Summary

Step 0 : Initialization : $\boxed{\underset{M\times M}{\Phi}(0) = \delta I_{M\times M}}$; $\Rightarrow \boxed{\underset{M\times M}{P}(0) = \delta^{-1} I_{M\times M}}$

Pre-windowed formulation

Also, Choose $\delta \leq 0.01\,\sigma_u^2$ & $0 < \lambda < 1$ → forgetting factor $\lambda \approx 0.99$

↳ regularisation term

$\boxed{\bar{w}(0) = \bar{0}_{M\times 1}}$

$n = 1, 2, \ldots\ldots$

$-M+ \ \ -2 \ \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3 \ldots \quad n$

Step 1 : $(*)$ $\underset{M\times 1}{\bar{a}(n)} = \lambda^{-1} \underbrace{P(n-1)\,\bar{u}(n)}$; $\bar{u}(1) = \underset{M\times 1}{\begin{bmatrix} u(1) \\ u(0) \; 0 \\ u(-1) \; 0 \\ \vdots \\ u(-M+1) \end{bmatrix}}$ $\Big\}\, M-1$

intermediate vector

Kalman Gain → $(*)$ $\underset{M\times 1}{\bar{k}(n)} = \bar{a}(n) \Big/ \left(1 + \bar{u}^H(n)\,\bar{a}(n)\right)$;

Step 2 : Find a-priori error

$$\xi^*(n) = d^*(n) - \bar{u}^H(n)\,\hat{w}(n-1);$$

Step 3 : Weight update Eqn.

$$\hat{w}(n) = \bar{w}(n-1) + \boxed{\bar{k}(n)\,\xi^*(n)};$$

Riccati Eqn. → Step 4 : (Inv.) Covariance update Eqn.

$$\hat{P}(n) = \lambda^{-1}\left[P(n-1) - \lambda\,\bar{k}(n)\,\bar{a}^H(n)\right];$$

Step 5 → Go back to Step 1

Recall:

$$\bar{w}(n) = \bar{w}(n-1) + \overbrace{P(n)\,\bar{u}(n)}^{\text{Eqn } 6}\,\xi^*(n)$$

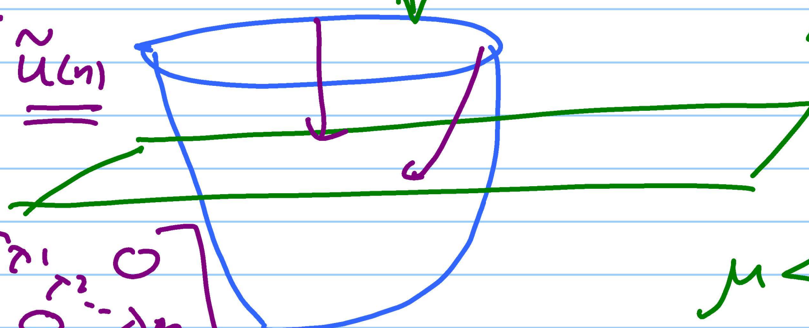
$$= \quad \mu \quad I$$

$$R_{\bar{u}\bar{u}} = \begin{bmatrix} \lambda & & & 0 \\ & \lambda & & \\ & & \lambda & \\ 0 & & & \lambda \end{bmatrix} = \lambda I_{m\times m} \qquad \mu \qquad \mu < \frac{1}{\lambda}$$

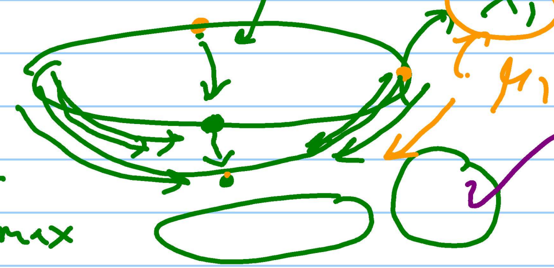
$$\rightarrow P(n)\,\bar{u}(n) \rightarrow \tilde{u}(n)$$

"Whitening" Transform

$$R_{uu} = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots\ \lambda_m \end{bmatrix}$$



$\lambda_2 > \lambda_1$ $\quad$ λ_2 μ_2 $\quad$ λ_1 μ_1



$$\mu < \frac{1}{\lambda_{max}}$$

Initialization of the RLS

Stability of the Recursion cannot be vouchsafed

$$\underline{\Phi}(0) = \delta \, I_{M\times M}, \quad \delta \le 0.01\, \sigma_u^2$$

$$\sigma_u^2 = E\left[u(n)\, u^*(n)\right]$$

$$\text{Trace } R_{uu} = M \sigma_u^2$$

$$w(n) = 1 \cdot w(n-1) + (\cdot)$$

0.9999

$$P(n) = P(n-1) + (\cdot)$$

$$\underline{\Phi}^{-1}(n)$$

$$\underline{\Phi}(n)$$

$n=0$

$$\underline{\Phi}(n) = \sum_{i=1}^{n} \lambda^{n-i}\, \bar{u}(i)\, \bar{u}^H(i) + \delta \lambda^n I_{M\times M}$$

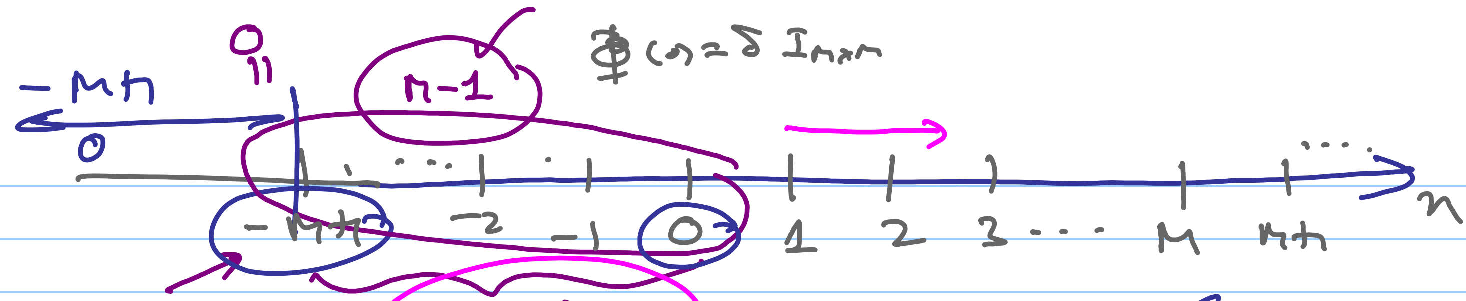
forgetting factor. regularisation term.

$$\bar{u}(1) = \begin{bmatrix} u(1) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{M\times 1}$$

$P(1)$

$$\underline{\Phi}(1) = \lambda \cdot \bar{u}(1)\, \bar{u}^H(1) + \delta \lambda I$$

$$\lambda = 0.999 \qquad \delta = 0.005$$



$$\Rightarrow \lambda^{\frac{(-M+1)}{2}} \cdot \delta^{1/2} \Rightarrow \Phi(0) = \delta I_{M\times M}$$

Hayhin