EE6110 Aug.-Nov., 2021

Class #20 (Oct. 14, 2021)

Note Title 27-03-2020

Recap.

$e(k) = d(k) - y(k)$ $\qquad \bar{y} = A\bar{w}$ $\qquad A \to (N-M+1) \times M$

$\bar{w} \to M\times 1$

$\underset{\bar{w}}{\min} \sum_{i=M}^{N} |e(i)|^2$ $\qquad \to$ $N-M+1$

LS

Covariance formulation

$M \times M$ $\qquad N \gg M$

$\Phi \bar{w} = \bar{z} \Rightarrow \boxed{\bar{w}_{LS} = \Phi^{-1}\bar{z}} = \boxed{\underbrace{(A^H A)^{-1}}_{\Phi^{-1}} \underbrace{A^H \bar{d}}_{z}}$

$\{u(1) \ldots u(N)\}$

Given the lgm $\to \bar{d} = A\bar{w}_0 + \bar{e}_0(n)$ Block Adaptive

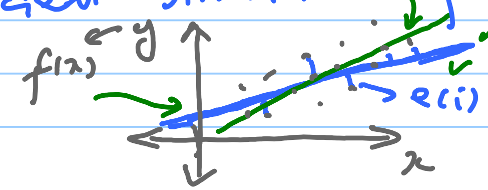
$\to$ BLUE

L, G, M

LS, ML,

✓ (*) LS $\to$ achieves CRLB $\Rightarrow$ efficient estimation

✓ (*) Uniqueness of LS $\longleftrightarrow$ SVD



$N-M+1$

Recursive LS Algorithm

time-varying w_o

Low Complexity

$\mathcal{J} \rightarrow \mathcal{E} \rightarrow \mathcal{E}$ $\mathcal{E}_g$

$$\mathcal{E} \triangleq \sum_{i=1}^{n\,(N)} \lambda^{n-i} |e(i)|^2 + \delta \lambda^n \|\bar{w}(n)\|^2$$

$\delta > 0$, regularisation term

Pre-windowed formulation (M)

Forgetting factor $0 < \lambda < 1$; $\lambda = 1 \rightarrow$ LS   0.9–0.999

$\bar{u}(M)$ $\bar{u}(1)$ 1 2 3 .. M ...... n ; M

equivalent to

$$\boxed{\Phi(n) = \sum_{i=1}^{n} \lambda^{n-i} \bar{u}(i)\bar{u}^H(i) + \delta \lambda^n I_{M\times M}}$$

i

$\lambda = 0.99$

$$\Phi(1) = \lambda^0 \bar{u}(1)\bar{u}^H(1) + \delta.\lambda.I_{M\times M} = 0.99\left[\bar{u}(n)\bar{u}^H(n)\right]_{M\times M} + (0.99)^2 \bar{u}(n-1)\bar{u}^H(n-1)$$

$(0.99)^{n-1}\, u(1)\, u^H(1)$

rewrite (1) as

$$\Phi(n) = \lambda \left[\sum_{i=1}^{n-1} \lambda^{n-1-i} \bar{u}(i)\bar{u}^H(i) + \delta \lambda^{n-1} \cdot I_{M\times M} \right] + 1 \cdot \bar{u}(n)\bar{u}^H(n)$$

$$\therefore \boxed{\Phi(n) = \lambda \underbrace{\Phi(n-1)}_{\Phi^{-1}} + \bar{u}(n) \cdot \bar{u}^H(n)} \rightarrow (2)$$

A

and $$\bar{z}(n) = \sum_{i=1}^{n} \lambda^{n-i} \bar{u}(i) d^*(i) \qquad \Phi^{-1}z \quad \times$$

$$\Rightarrow \boxed{\bar{z}(n) = \lambda \bar{z}(n-1) + u(n) d^*(n)} \rightarrow (3)$$

use the Matrix Inversion Lemma for $\Phi(n)$ in (2) to find $\Phi^{-1}(n)$

$$\boxed{\Phi^{-1}(n) = \lambda^{-1}\Phi^{-1}(n-1) - \frac{\lambda^{-1}\Phi^{-1}(n-1)\bar{u}(n)\,\bar{u}^H(n)\lambda^{-1}\Phi^{-1}(n-1)}{\left(1 + \lambda^{-1}\bar{u}^H(n)\Phi^{-1}(n-1)\bar{u}(n)\right)}} \quad (4a)$$

Aside: Matrix Inversion Lemma:

$A > 0$ (positive def.) $\leftarrow \underset{M\times M}{A} = \underset{M\times M}{B^{-1}}$ ($B>0$) $+ \underset{M\times N}{C}\,\underset{N\times N}{D^{-1}}\,C^H_{N\times M}$ ($D>0$)

Ex: $AA^{-1} = I_{M\times M}$

$$\Rightarrow \quad A^{-1} = B - BC\left(D + C^H BC\right)^{-1} C^H B \;;$$

Define $\underset{M\times M}{P(n)} \triangleq \Phi^{-1}(n)$ ✓

$\dashrightarrow$ "Kalman Gain"

and also define

$$\boxed{\underset{M\times 1}{\bar{k}(n)} = \frac{\lambda^{-1} P(n-1)\, \bar{u}(n)}{\left(1 + \lambda^{-1} \bar{u}^H(n)\, P(n-1)\, \bar{u}(n)\right)}} \quad (4b)$$

Rewriting (4a)

$$\boxed{P(n) = \lambda^{-1} P(n-1) - \bar{k}(n)\, \bar{u}^H(n)\, \lambda^{-1} P(n-1)} \quad (5)$$

"Riccati Eqn"

Exercise: from (4b) show that $\boxed{k(n) = P(n)\, \bar{u}(n)}$ (6)

RLS Time-Update Equation

$$\hat{w}(n) = \Phi^{-1}(n)\,\bar{z}(n)$$

$$= P(n)\,\underbrace{\bar{z}(n)}_{\text{expand using } 3}$$

$$= \lambda\,\underbrace{P(n)}_{\text{expand using } 5}\,\bar{z}(n-1) + P(n)\,\bar{u}(n)\,d^{*}(n)$$

$$= \underbrace{P(n-1)\,\bar{z}(n-1)}_{\hat{w}(n-1)} - \bar{k}(n)\,\bar{u}^{H}(n)\,\overbrace{P(n-1)\,\bar{z}(n-1)}^{\hat{w}(n-1)} + \underbrace{P(n)\,\bar{u}(n)}_{\bar{k}(n)}\,d^{*}(n)$$

$$\hat{w}(n) = \hat{w}(n-1) + \bar{k}(n)\left[d^{*}(n) - \bar{u}^{H}(n)\,\hat{w}(n-1)\right]$$

"a-priori" error. $\xi^*(n)$

$$\boxed{\hat{w}(n) = \hat{w}(n-1) + \bar{k}(n)\,\xi^*(n)} \rightarrow (7)$$

$P(n)\,\bar{u}(n)\,\xi^*(n)$

LMS

$\bar{w}(n) = w(n-1) + \mu\,\bar{u}(n)\,e^*(n)$

$E[e^*\bar{u}]$