EE6110 Aug.-Nov., 2021

Class #17 (Oct. 01, 2021)

Note Title 27-03-2020

- System ID
- Inv. Filtering
- L. Prediction
- Noise Canc.

$\{\bar{u}(n)\} \longleftrightarrow \{d(n)\}$

LRM

$$\boxed{d(n) = \bar{w}_0^H \bar{u}(n) + v(n)}$$

R, R^{-1}

$E[\ ]$

$\bar{w}_0 = R^{-1}\bar{p}$

Sample-by-sample adaptation

$\bar{w}_{M\times 1}$, $\bar{w}(n)$

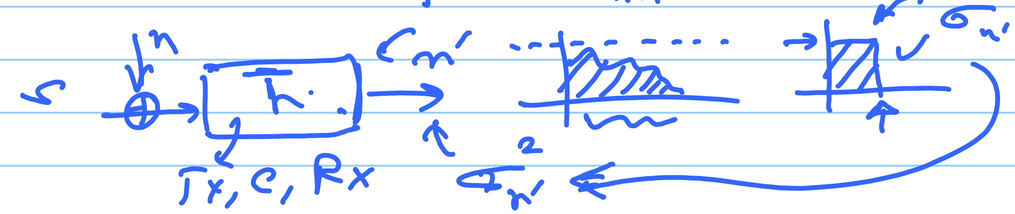
$\sigma_v^2 = J_{min}$

$R^{-1}(n)\,\bar{p}(n) \to \bar{w}_0(n)$

⟹ "Block Adaptive" Rules.

$J(\bar{w})$

$\hookrightarrow J(\bar{w}_0) = \sigma_d^2 - \bar{w}_0^H \bar{p} = J_{min}$



"Seperation principle" ↳ $\|\bar{u}(n)\|^2$ and $e_a(n)$ are independent ← $\tilde{w}(n-1)^H \bar{u}(n)$

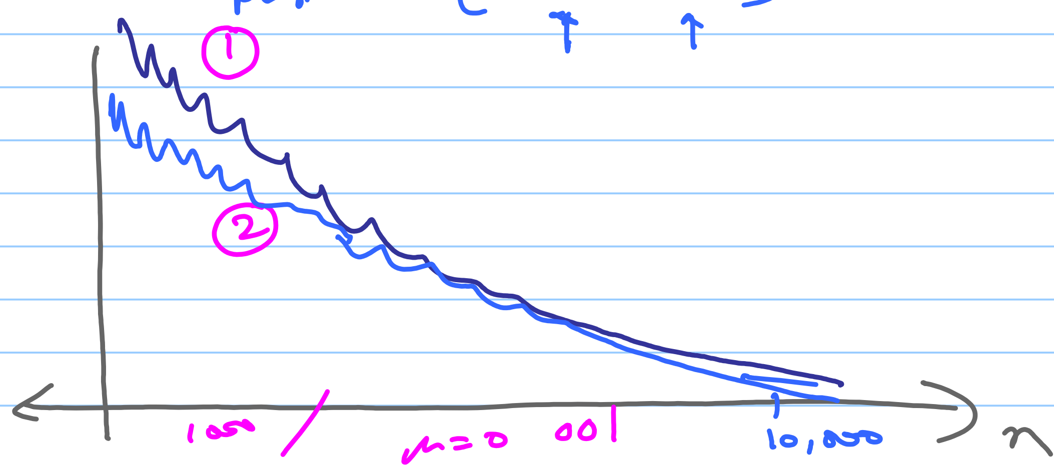
$$\underbrace{E\left[(\cdot)(\cdot)\right]}_{1} = \underbrace{\underbrace{E\left[(\cdot)\right]}_{Tr[R_u]} \cdot E\left[(\cdot)\right]}_{2}$$

by ~~pps~~ urs

$(*)$ $\quad e(n) = e_a(n) + v(n)$

$\Rightarrow \quad e_a(n) = e(n) - v(n)$ ← $v(n)$ independent of $\bar{u}(n)$ $\forall n$

Orthogonality principle $\quad E\left[e^*(n)\,\bar{u}(n)\right] = 0$



$$\xi^{(ms)} = \frac{\mu\, \sigma_v^2\, Tr[R_u]}{2 - \mu\, Tr[R_u]}$$

Example : MSE Performance ϵ – NLMS normalized

$$\bar{w}(n) = \underset{M\times 1}{\bar{w}(n-1)} + \frac{\overset{\text{gain const}}{\mu}}{\underset{\substack{\epsilon>0\\ \text{flooring const}}}{\epsilon} + \|\bar{u}(n)\|^2}\, e^*(n)\, \bar{u}(n)$$

$$\bar{u}(n) = \begin{bmatrix} u(n) \\ u(n-1) \\ \vdots \\ u(n-M+1) \end{bmatrix}_{M\times 1};$$

$$\triangleq g\left[e^*(n)\right] = \frac{e^*(n)}{\epsilon + \|\bar{u}(n)\|^2}$$

Recall the variance eqn for steady state performance

$$\mu\, E\left[\frac{\|u(n)\|^2 \left| e_a^*(n) + v(n) \right|^2}{(\epsilon + \|\bar{u}(n)\|^2)^2}\right] = 2\,\mathrm{Re}\left\{ E\left[e_a^*(n)\ g\left[e(n)\right] \right] \right\}$$

$g^2(\cdot)$

$$e(n) = e_a(n) + v(n)$$

$$n \to \infty$$

$e_a(n) \perp^r v(n)$; expand both sides.

$$\mu\, E\left[\underbrace{\frac{\|u(n)\|^2}{(\epsilon+\|u(n)\|^2)^2}}_{\alpha_{\bar u}} |e_a(n)|^2\right] + \mu\, \sigma_v^2\, E\left[\frac{\|\bar u(n)\|^2}{(\;\cdot\;)}\right] = 2E\left[\underbrace{\frac{|e_a(n)|^2}{\epsilon+\|\bar u(n)\|^2}}_{\eta_{\bar u}}\right]$$

Using the "separation principle"

Exercise:

$$\left(2\eta_{\bar u} - \mu\,\alpha_{\bar u}\right) \underbrace{E\left[|e_a(n)|^2\right]}_{\xi^{\epsilon\text{-NLMS}}} = \mu\, \sigma_v^2\, \alpha_{\bar u}$$

$$\xi^{\epsilon\text{-NLMS}} = \mu\, \alpha_{\bar u}\, \sigma_v^2 \Big/ \left(2\eta_{\bar u} - \mu\,\alpha_{\bar u}\right) ;$$

$\epsilon \longrightarrow$ "regularisation parameter" is small

Exercise: Show that

$$\alpha_{\bar{u}} \approx \eta_{\bar{u}} \approx \epsilon \left[\frac{1}{\|\bar{u}(n)\|^2} \right];$$

J_{ex}

EMSE

J_{min}

J_{min}

$$\therefore \boxed{\xi^{\epsilon\text{-NLMS}} = \frac{\mu\, \sigma_v^2}{2-\mu}}, \quad \epsilon \to \text{small}$$

$\hookrightarrow$ independent of $R_{\bar{u}}$

Aside

N-LMS $\longrightarrow$ solution to a constrained optimisation prblm:

Haykin ch. 6

(*) Principle of Minimum Disturbance (of weights)

$$\min \vec{\delta}(\hat{w}(n)) = \hat{w}(n) - \hat{w}(n-1)$$

subject to

$$\underbrace{\bar{w}^H}_{w(n-1)}(n)\,\bar{u}(n) = d(n) \longrightarrow (1)$$

using the Legrange Multiplier Technique.

$$\rightarrow J(n) = \|\vec{\delta}(\hat{w}(n))\|^2 + \mathrm{Re}\left[\lambda^* \left[d(n) - \bar{w}^H(n)\,\bar{u}(n)\right]\right]$$

$$\frac{\partial J(n)}{\partial w^*(n)} = 0 \implies \hat{w}(n) = \hat{w}(n-1) + \frac{1}{2}\lambda^* \bar{u}(n) \longrightarrow (2)$$

$M \times 1$

Eliminate λ by putting (2) into (1)

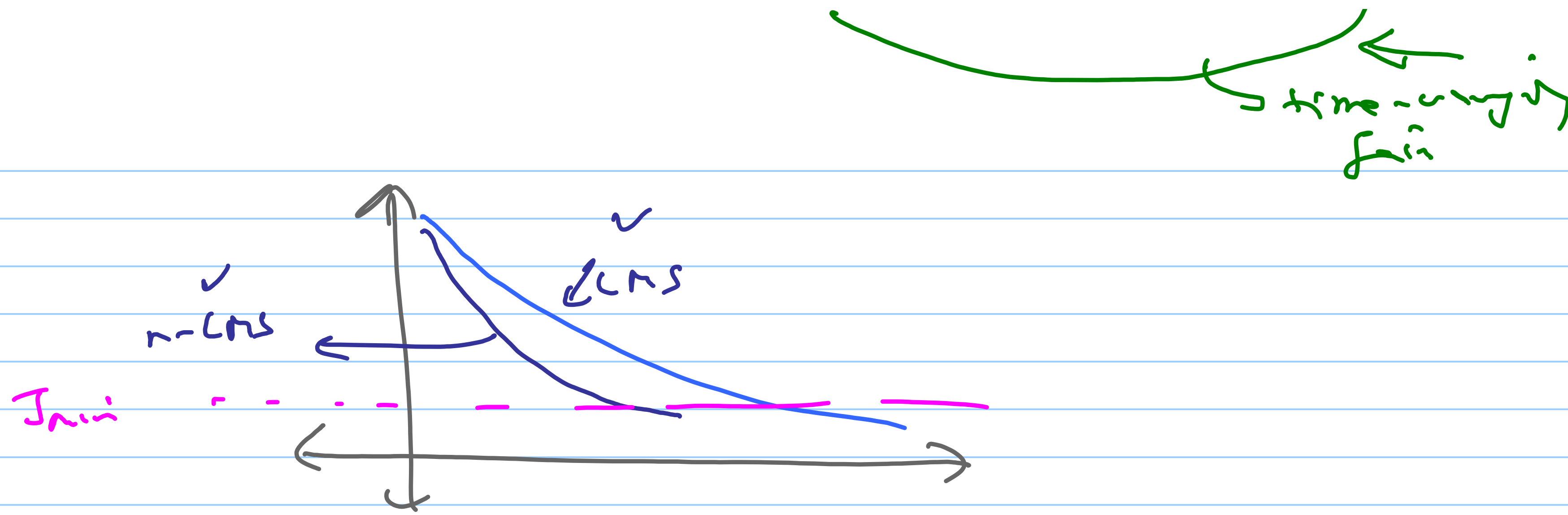
$$d(n) = \left(\hat{w}(n-1) + \frac{1}{2}\lambda^{*}\bar{u}(n)\right)^{H}\bar{u}(n)$$

$$\underbrace{d(n) - \bar{w}^{H}(n-1)\,\bar{u}(n)}_{e(n)} = \frac{1}{2}\lambda\,\bar{u}^{H}(n)\,\bar{u}(n)$$

$$\therefore \quad \lambda = \frac{2\,e(n)}{\|\bar{u}(n)\|^2} \longrightarrow (3)$$

Putting (3) into (2),

$$\hat{w}(n) = \hat{w}(n-1) + \tilde{\mu}\left(\frac{e^{*}(n)\,\bar{u}(n)}{\epsilon + \|\bar{u}(n)\|^2}\right) \rightarrow \text{N-LMS}$$



time-varying
Jmin
LMS
n-LMS
Jmin