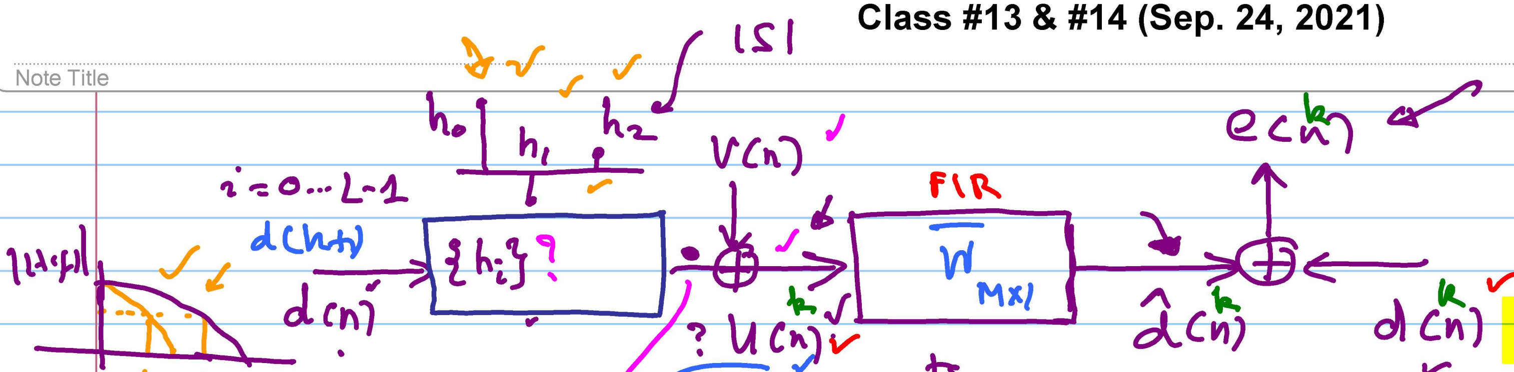
EE6110 Aug.-Nov., 2021

Class #13 & #14 (Sep. 24, 2021)

Note Title 27-03-2020



$R = E[\bar{u}(n)\bar{u}^H(n)]_{M\times M}$

$\bar{p} = E[d(n)\bar{u}(n)]_{M\times 1}$

$(-2\bar{p} - 2R\bar{w})$

measurements

$u(k) = h_0 d(k) + h_1 d(k-1) + h_2 d(k-2) + v(k)$

1 km

4 km & 2 Mbps

1 km

DSL

T

$k \to kT$, $k \to kT_s$

$T_s = T/2$

(*) Newton's Algo. $\bar{w}(k+1) = \bar{w}(k) + R^{-1}(-\nabla_k)$ ← one-step conv.

(*) Steepest Descent Algo. $\bar{w}(k+1) = \bar{w}(k) + \mu(-\nabla_k)$

$= \bar{w}(k) - 2\mu(R\bar{w}(k) - \bar{p})$.

$0 < \mu < \dfrac{1}{\lambda_{max}}$; $\bar{w}(k+1) \xrightarrow[k\to\infty]{} \bar{w}_o$; $\bar{w}_o = R^{-1}\bar{p}$

* Least Mean Squares Algo. ✓

$\nabla_k = -2E\left[\underbrace{e^*(k)\,\bar{u}(k)}_{\text{instantaneous gradient}}\right]$ ✓

insight : $\nabla_k = -2p - 2R\,\bar{w}(k)$

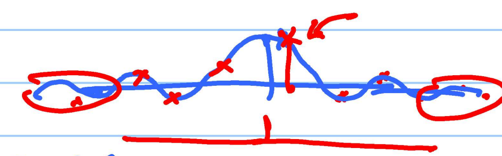
$$\boxed{\bar{w}(k+1) = \bar{w}(k) + \mu\left(2e^*(k)\,\bar{u}(k)\right)}$$

$\times R^{-1}$
$\times R$
$\times \bar{p}$

$\rightarrow e(k) = d(k) - \hat{d}(k)$
$\hat{d}(k) = \bar{w}(k)\,\bar{u}(k)$

$\bar{w}_0$

$\rightarrow \lambda_{max}$? ; $d(k)$?
↳ training sequence / pilot sequence ~~$R, \bar{p}, R^{-1}$~~



$\{h_i\}$ $\bar{h}_{L\times 1}$ $\bar{w}$

Example : $L = 3$; $M = 2$; bipolar signals $d \in \{+1, -1\} \Rightarrow \sigma_d^2 = 1$
$E[d] = 0$;

$$u(n) = \left(h_0\, d(n) + h_1\, d(n-1) + h_2\, d(n-2) + v(n)\right) \quad \mathcal{N}(0, \sigma_v^2)$$

$$u(n-1) = \left(h_0\, d(n-1) + h_1\, d(n-2) + h_2\, d(n-3) + v(n-1)\right)$$

find : $R = E[\bar{u}\bar{u}^*]_{2\times 2}$; $\bar{p}_{2\times 1}$; * Assume $d(n)$ & $v(n)$ are uncorrelated

* $d(n)$ is ~~white~~ i.i.d $p(1) = p(-1) = 1/2$;

* $v(n)$ is white

$E[d(n)\, \bar{u}(n)]$

$$R = E[\bar{u}\bar{u}^H] = \begin{bmatrix} ⊙_1 & ⊙_2 \\ ⊙_3 & ⊙_1 \end{bmatrix}_{2\times 2} ; \quad E[u(n)\, u^*(n)] \quad E[u(n-1)\, u^*(n-1)]$$

$$\bar{u}(n) = \begin{bmatrix} u(n) \\ u(n-1) \end{bmatrix}.$$

$$(h_0^2 + h_1^2 + h_2^2)\, \sigma_d^2 + \sigma_v^2$$

linear $\underline{\underline{E}}$

(h_0) σ_d^2

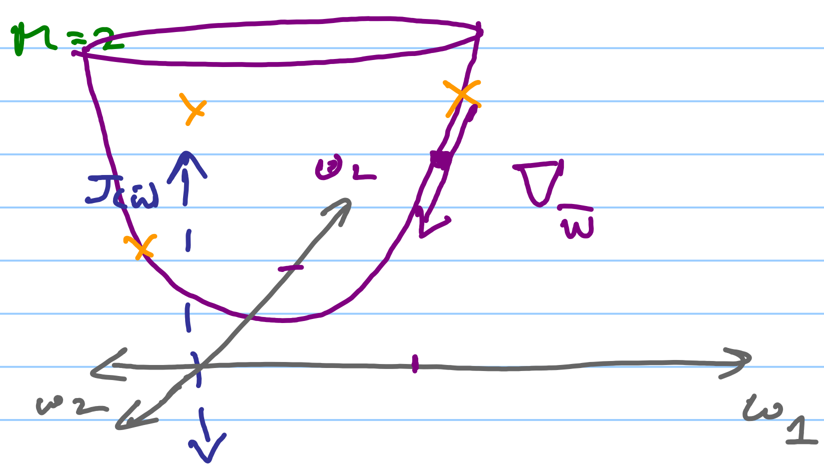
$$(h_0 h_1 + h_1 h_2)\, \sigma_d^2$$

Exercise : $\bar{p}_{2\times 1} = \begin{bmatrix} \\ \end{bmatrix}$?

$= 0$

$$\underbrace{E[u^*(n)\,u(n-1)]}_{R} = (h_0 h_1 + h_1 h_2)\sigma_d^2 \;; \quad \bar{p} = \begin{bmatrix} h_0 \sigma_d^2 \\ 0 \end{bmatrix};$$

Least Mean Squares Algorithm



Exercise: Newton method can also be rewritten as

$$\bar{w}(k) = \bar{w}_0 + (1-2\mu)^k \left(\bar{w}(0) - \bar{w}_0\right)$$

$R^{-1}\bar{p}$

initial choice

note:

$\mu = \frac{1}{2} \Rightarrow \bar{w}(1) = \bar{w}_0 \leftarrow$ 1-step conv.

$$\times\ \nabla_{\bar{w}} = -2\,E\left[e^{*}(k)\,\bar{u}(k)\right]$$

$e = d(k) - \bar{w}^{H}\bar{u}(k)$

$\bar{w}(0)$ $\quad e^{*} = d^{*}(k) - \bar{u}^{H}\bar{w}$

$$= -2\bar{p} - 2R\bar{w}$$

"stochastic"

Steepest Descent

$$\times\ \boxed{\nabla_{w}(k) = -2\hat{p} - 2\hat{R}\,\bar{w}(k)}$$

" instantaneous gradient "

$$\bar{w}(k+1) = \bar{w}(k) + \mu\left(-\nabla_{k}\right)$$

$\nabla(k)$

$$\begin{bmatrix} u(k) \\ u(k-1) \\ u(k-2) \\ u(k-M+1) \end{bmatrix}$$

$$* \text{ LMS} \quad \underset{M\times 1}{\bar{w}(k+1)} = \underset{M\times 1}{\bar{w}(k)} + 2\mu\, e^{*}(k)\, \underset{M\times 1}{\bar{u}(k)}$$

$$\boxed{\begin{aligned} e(k) &= d(k) - \hat{d}(k) \\ \hat{d}(k) &= \bar{w}^{H}(k)\cdot\bar{u}(k) \end{aligned}}$$

or

$$* \text{ LMS} \quad \bar{w}(k+1) = \bar{w}(k) + 2\mu\, e(k)\,\bar{u}^{*}(k)$$

$H = T*$

Exercise : Show that

$$e(k) = d(k) - \hat{d}(k)$$

$$\hat{d}(k) = \bar{W}^{H}(k)\,\bar{u}(k)$$

$$\hat{d}(k) \rightarrow \bar{W}^{H}(k)\,\bar{u}(k)$$

$\bar{W}_0$

* $\bar{w}(k+1) \longrightarrow \hat{W}(k)$

$\hat{d}(k)$ $\hat{W}$ → is this a linear function of $\bar{u}(n)$?

* Exercise : ST

$$\bar{W}(k) = \left(\mu \sum_{i=-\infty}^{k-1} e^{*}(i)\,\bar{u}(i)\right) \cdot \bar{u}(k)$$

$\hat{d}(k) =$

* Steepest Descent Rule $\longrightarrow$ $\overline{w}_0 = R^{-1}p$