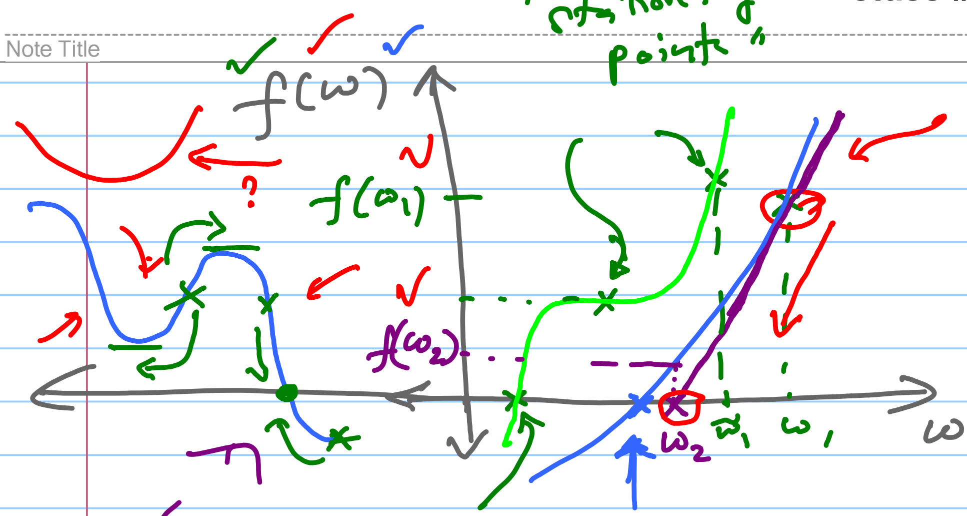
EE6110 Aug.-Nov., 2021

Class #11 (Sep. 17, 2021)

Note Title 27-03-2020



$$\boxed{\omega_{k+1} = \omega_k - \left(\frac{1}{f'(\omega_k)}\right) \cdot f(\omega_k)}$$

II   I $= 0$

$k = 1, 2, 3 \ldots$

"initialization" $\frac{"}{0}$ quadratic constraint

"basins of attraction"

$\omega \rightarrow \bar{\omega}_{M\times 1}$ $\omega \leftarrow w$

Recall : LMMSE $\min_{\bar{W}_{M\times 1}} E\left[|e(n)|^2\right] = J(\bar{w})$ ($e(n)e^*(n)$)

$f(\bar{w}_k) \triangleq \nabla_{\bar{w}} J(\bar{w})$; $f'(\bar{w}_k) \triangleq \nabla^2_{\bar{w}} J(\bar{w})$

$2 \to \frac{1}{2}$ $\bar{p} = E[d(n)^* \bar{u}]$; $R = E[\bar{u}\bar{u}^H]$; $\checkmark$

$$\underbrace{= 2R\bar{w} - 2\bar{p}}_{\text{Gradient}} \qquad \underbrace{= 2R}_{\text{"Hessian"}}$$

$R, \bar{p}$; R^{-1};

$$\bar{w}_{k+1} = \bar{w}_k - \underbrace{\left(\frac{1}{2}R^{-1}\right)}_{\mathrm{II}} \underbrace{\left(2R\bar{w}_k - 2\bar{p}\right)}_{\mathrm{I}}$$

$$= \bar{w}_k - (\bar{w}_k - \bar{w}_0) = \bar{w}_0 ;$$

$R^{-1}\bar{p} = \bar{w}_0$

"one-step" convergence

→ not practical because R & $\bar{p}$ are required

Steepest Descent Rule :

real, scalar

$$\bar{w}_{k+1} = \bar{w}_k + \mu \underbrace{\left(-\nabla_k\right)}_{\mathrm{I}} \checkmark$$

R^{-1}

↪ dimension → reciprocal of signal power ✓

$$\bar{w}_{k+1} = \bar{w}_k - 2\mu\left(R\bar{w}_k - \bar{p}\right)$$

R, p $\quad \frac{1}{2}$ $\quad R^{-1}p = \bar{w}_0$

$M \times 1$

$$= (\underset{M\times M}{I} - 2\mu R)\,\bar{w}_k + 2\mu \bar{p}$$

* use $\bar{p} = R\bar{w}_0$, and subtract w_0 from both sides.

$$\underbrace{\left(\bar{w}_{k+1} - \bar{w}_0\right)}_{\triangleq\, \bar{v}_{k+1}} = \underbrace{(\bar{w}_k - \bar{w}_0)}_{\bar{v}_k} - 2\mu R\,\underbrace{(\bar{w}_k - \bar{w}_0)}_{\bar{v}_k}$$

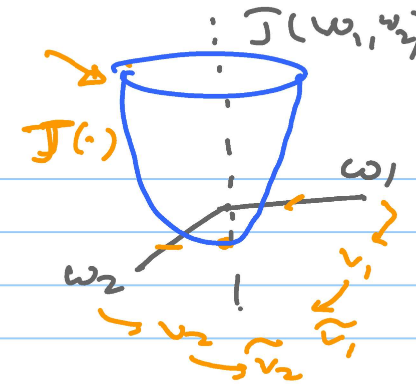
$$= \overbrace{(I - 2\mu R)}\,\bar{v}(k) \qquad \Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_M \end{bmatrix};$$

$QQ^H \to QIQ^H \qquad Q\Lambda Q^H$

$Q^H \longrightarrow$ $\boxed{\bar{v}(k+1) = Q(I - 2\mu\Lambda)Q^H\,\bar{v}(k)}$ Q^H

$Q^H \bar{v} = \tilde{v}$

$$\boxed{\tilde{v}(k+1) = (I - 2\mu\Lambda)\,\tilde{v}(k)}$$



$$\rightarrow \tilde{v}(k+1) = (I - 2\mu\Lambda)^k\,\tilde{v}(0)$$

Diagonal

$$\begin{bmatrix} (1-2\mu\lambda_1)^k & & 0 \\ & (1-2\mu\lambda_2)^k & \\ 0 & & \ddots \quad (1-2\mu\lambda_M)^k \end{bmatrix}$$

$$\tilde{v}(k) = Q^H \bar{v}_k = Q^H(\bar{w}_k - \bar{w}_0)$$

$\bar{0}$

$\bar{w}_k \rightarrow \bar{w}_0$

$\tilde{v}(k) \longrightarrow \bar{0}$

$$\tilde{V}_i(k) \longrightarrow 0 \quad \Rightarrow \lim_{k\to\infty} (1-2\mu\lambda_i)^k \longrightarrow 0$$

$i = 1 \ldots n$

$$\Rightarrow |1-2\mu\lambda_i| < 1$$

$$\Lambda = \begin{bmatrix} \lambda_{max} & & \\ & \ddots & \\ & & \lambda_{min} \end{bmatrix}$$

$$0 < \mu < \frac{1}{\lambda_i} \quad \forall i$$

$$-1 < 1-2\mu\lambda_i < 1$$

$$\mu > 0 \qquad 2\mu\lambda_i > 0 \qquad -2\mu\lambda_i < 0$$

$$-2 < -2\mu\lambda_i$$

$$\mu < \frac{1}{\lambda_i} \qquad \mu\lambda_i < 1$$

$$\boxed{0 < \mu < \frac{1}{\lambda_{max}}}$$

learning const. || gain const.