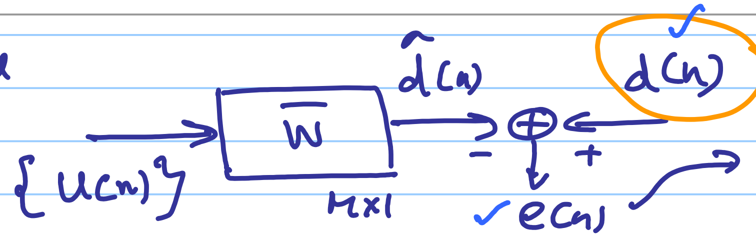
EE6110 Aug.-Nov., 2021

Class #10 (Sep. 16, 2021)

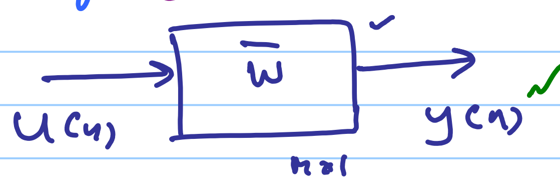
Note Title 27-03-2020

Recall



$J(\bar{w}) = E\left[|e(n)|^2\right]$ wrt $\bar{w}$

Example: Linearly Constrained Minimum-Variance Filter (Beam-former)



$J(\bar{w}) = \min_{\text{w.r.t. } \bar{w}} E\left[|y(n)|^2\right]$

with a linear constraint

$\bar{w}^H \bar{d} = g$ is real or complex const.

"desired" response → "look" direction

$y(n) = \bar{w}^H \bar{u}(n)$

$J(\bar{w}) = \bar{w}^H R_{uu} \bar{w}$

$R_{uu} = E\left[\bar{u}\,\bar{u}^H\right]$

Lagrange Multiplier Technique

New cost fn: $J(\bar{w}, \lambda)$

$$J = \underbrace{J(\bar{w})}_{\bar{w}^H R \bar{w}} + \mathrm{Re}\{\lambda^* (\bar{w}^H \bar{d} - g)\}$$

$\lambda \to$ Lagrangian

$$\rightarrow \nabla_w J = 2R\bar{w} + \lambda^* \bar{d} = \bar{0} \Rightarrow \boxed{\bar{w}_o = -\frac{\lambda^*}{2} R^{-1} \bar{d}} \rightarrow (1)$$

$\frac{d}{d\bar{w}}$ $\quad$ $\bar{w}_{opt}$

To eliminate λ, we use $\underbrace{\bar{w}_o^H \bar{d} = g}$ use this in (1)

$$\bar{d}^H \bar{w}_o = g^*$$

$$\underbrace{\bar{d}^H R^{-1} \bar{d} \left(\frac{-\lambda^*}{2}\right) = g^*}$$

$$(R^{-1})^H = R^{-1}$$

$$\lambda = \frac{-2g}{\bar{d}^H R^{-1} \bar{d}}$$

$$\boxed{\bar{w}_o = \frac{g^* R^{-1} \bar{d}}{\bar{d}^H R^{-1} \bar{d}}}$$

$\theta_d \rightarrow$ desired "look" dirn.

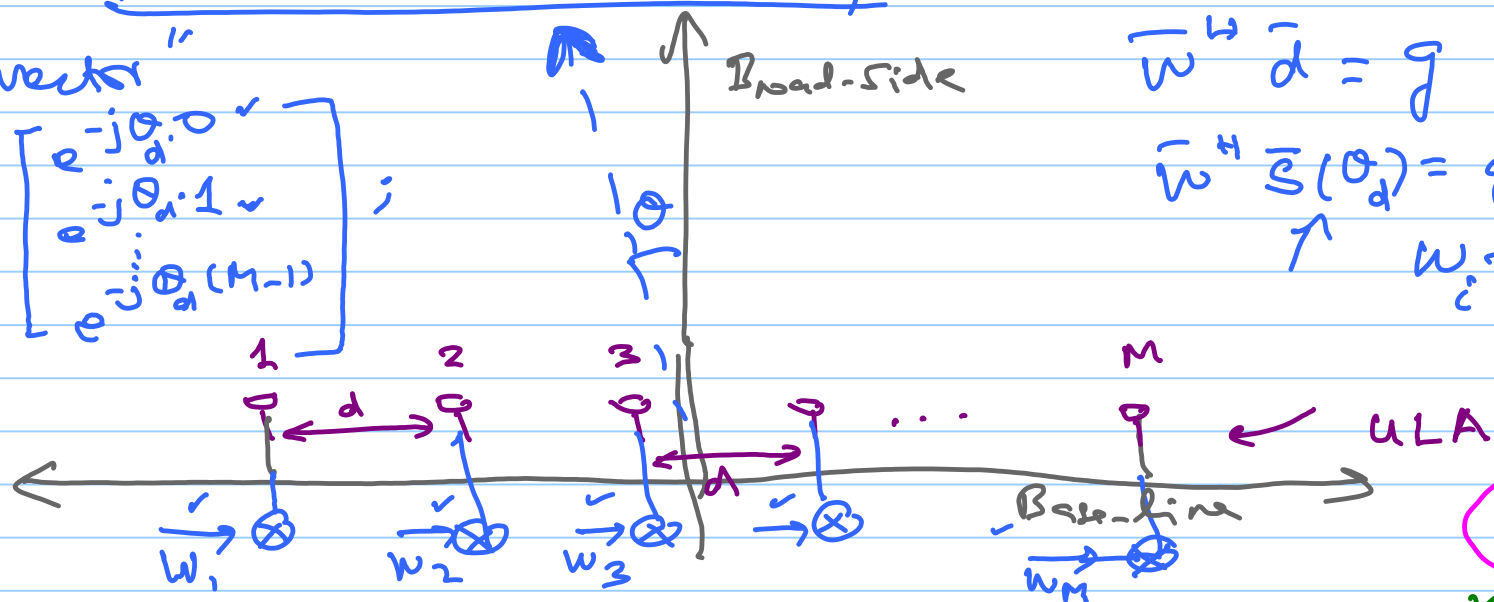
"Steering vector"

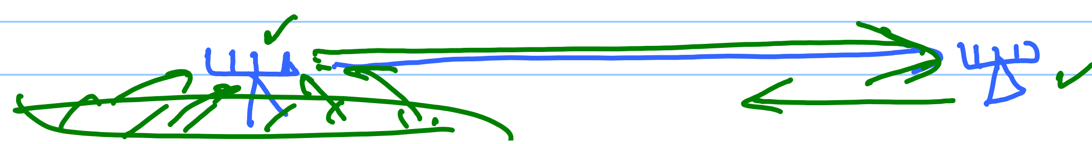
$$\underline{s}(\theta_d) = \begin{bmatrix} e^{-j\theta_d \cdot 0} \\ e^{-j\theta_d \cdot 1} \\ \vdots \\ e^{-j\theta_d (M-1)} \end{bmatrix};$$

$$\bar{w}^H \bar{d} = g$$

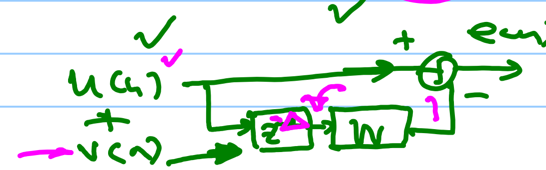
$$\bar{w}^H \bar{s}(\theta_d) = g$$

$$w_i \rightarrow r e^{j\theta}$$





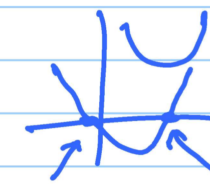
ALE

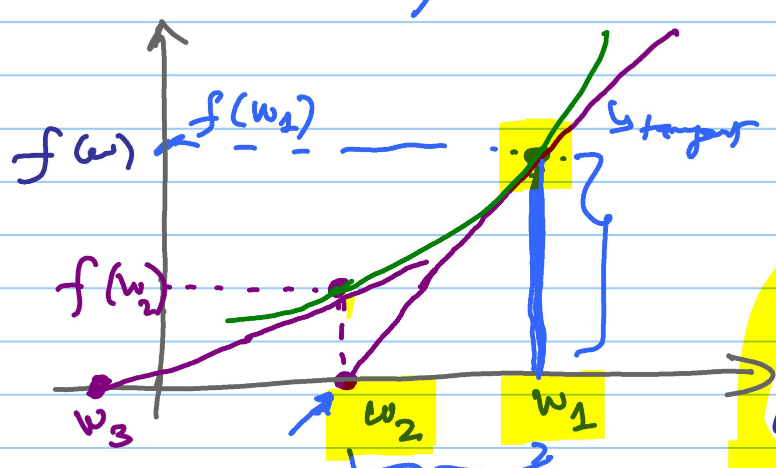


$d(n) \to \underline{\underline{FA}}$

Newton's Method & Steepest Descent

Let $f(w)$ be a "smooth" function of $w \leftarrow \bar{w}$





To find roots of $f(w) = 0$ $\to f'(w)$

slope $f'(w_1) = \dfrac{f(w_1) - 0}{w_1 - w_2}$ ✓

$$\boxed{w_2' = w_1' - \left(\frac{1}{f'(w_1)}\right) f(w_1)}$$

$$\boxed{w_{k+1} = w_k - \left(\frac{1}{f'(w_k)}\right)\cdot f(w_k), \quad k = 1, 2, 3 \ldots}$$

LMMSE ✓ 1-step convergence !!