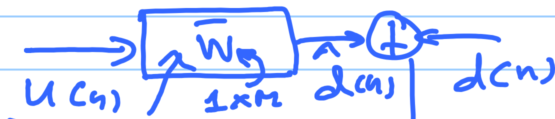
EE6110 Aug.-Nov., 2021

Class #09 (Sep. 15, 2021)

Canonical form for (the Error Surface):



MSE → $J(\bar{w}) = J_{min} + (\bar{w}-\bar{w}_o)^H R (\bar{w}-\bar{w}_o)$;

$\bar{w}_o = R^{-1}\bar{p}$;

$\begin{bmatrix} u(n) \\ \vdots \\ u(n-M+1) \end{bmatrix}$ ← WSS

$\min_{\bar{w}} E[|e(n)|^2]$

since R → ac matrix of a WSS → { positive semi-definite $\lambda_i \geq 0$, $i = 1 \ldots M$; almost always positive definite $\lambda_i > 0$ }

Aside: $X(n) = A\cos(\omega_c n + \theta)$, $\theta \to [0, 2\pi]$

↑ discrete r.p. WSS

$u(n) = X(n) + V(n)$

$V(n) \sim \mathcal{N}(0, \sigma_v^2)$

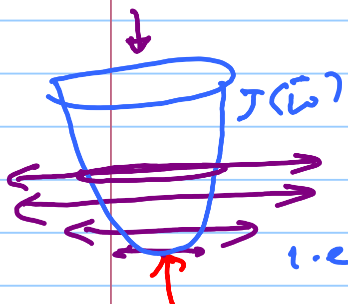
$\bar{X} = \begin{bmatrix} X(n) \\ X(n-1) \\ \vdots \\ X(n-M) \end{bmatrix} \to R_{XX} = E[\bar{X}\bar{X}^H] = \begin{bmatrix} [\,] + \sigma_v^2 & & \\ & \ddots + \sigma_v^2 & \\ & & \ddots \end{bmatrix}$ M×M (M−1)

$R_{uu} = E[\bar{u}\bar{u}^H]$ full rank

Since R is (+)ve definite $\longrightarrow$ diagonalisable using a Unitary Similarity Transformation

$$R = Q \Lambda Q^H \qquad Q \rightarrow \text{unitary} \quad Q^H Q = I = Q Q^H$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_2 \end{bmatrix} \qquad Q = \begin{bmatrix} | & | & & | \\ q_1 & q_2 & \cdots & q_M \\ | & | & & | \end{bmatrix};$$

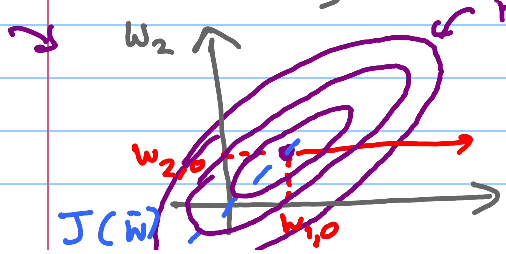


i.e. $\rightarrow J(\bar{w}) = J_{min} + \underbrace{(\bar{w} - \bar{w}_o)^H}_{\bar{v}^H} \overbrace{Q \Lambda Q^H}^{R} \underbrace{(\bar{w} - \bar{w}_o)}_{\bar{v}}$ ✓

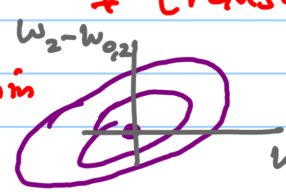
$$\bar{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_M \end{bmatrix};$$

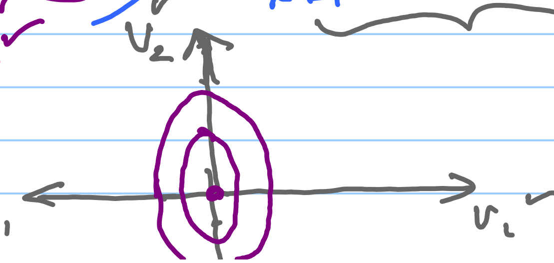
$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix};$$

Error performance surface $= J_{min} + \underbrace{\bar{v}^H \Lambda \bar{v}}$ ✓ $\rightarrow \underbrace{\sum_{k=1}^{M} \lambda_k |v_k|^2}$



+ translation ✓

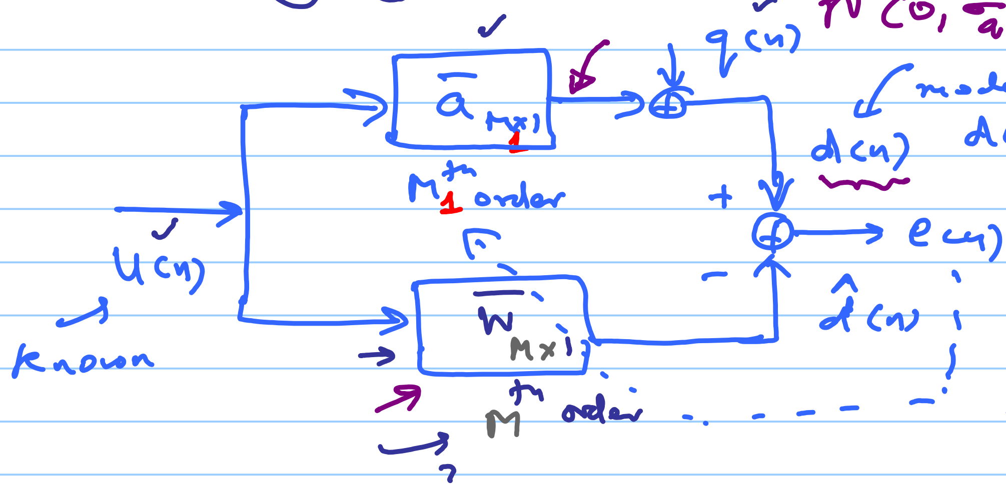




LMMSE → $J(\bar{w})$ → Quadratic fn : in $\bar{w}$ ⇒ global minimum

Sec 2.6

System ID Model :



Exercise : Show that. ①

(a) $\sigma_d^2 = \sigma_q^2 + \bar{a}^H R_{M_1} \bar{a}$

$R_{M_1} = E\left[u_{M_1}(n)\, u_{M_1}^H(n) \right]$;

(b) If the adaptive estimator is of order M → ②

$J_{min}(M) = \sigma_d^2 - \bar{w}_o^H R_M \bar{w}_o$ (M^{th} order)

Substituting ① into ②

$J_{min}(M) = \sigma_q^2 + \bar{a}^H R_{M_1} \bar{a} - \bar{w}_o^H R_M \bar{w}_o$

$M < M_1$ under modelling

$J_{min}(M) > \sigma_q^2$

$M = M_1$

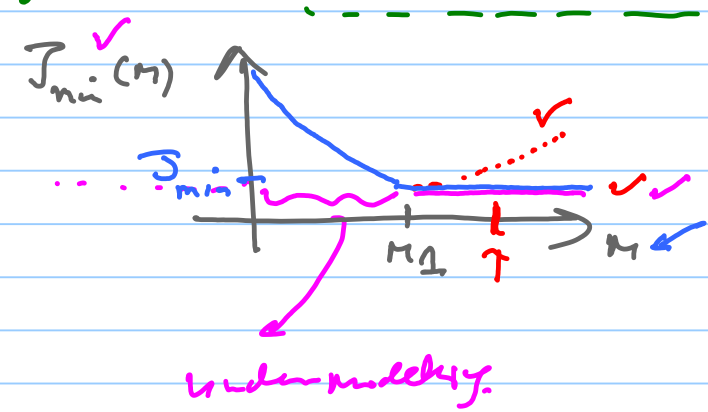
$R_{M1} = R_M$; $\bar{w}_0 = \bar{a}$

$J_{min}(M) = \sigma_q^2$

$M > M_1$ over modelling

$\bar{w}_0 = \begin{bmatrix} \bar{a} \\ \bar{0} \end{bmatrix}$ $\}M_1$, $\}M-M_1$, $M \times 1$

$J_{min}(M) = \sigma_q^2$



AIC , MDL