EE6110 Aug.-Nov., 2021

Class #5 (Aug. 27, 2021)

Note Title 27-03-2020

~~Linear~~ (Linear est.)

Minimum Mean Square Error Criterion → Conditional Mean

Given $\underbrace{u(1), u(2) \ldots u(M)}_{} \rightarrow \bar{u}$, estimate d as $\hat{d}(M) \leftarrow \hat{d}$

MMSE

$$J(M) = \min_{\hat{d}} E\left[\left(\underbrace{d - \hat{d}(M)}_{e(M)}\right)^2\right] \qquad |e(M)|^2, \; E[(e e^*)]$$

$$\hat{d}(M) = g(\bar{u})$$

$$\min_{\hat{d}} \int_d \int_{\bar{u}} \underbrace{f_{d,\bar{u}}(d,\bar{u})} [d - \hat{d}]^2 \, dd \, d\bar{u}$$

$$f_{d|\bar{u}}(d|\bar{u}) \, f_{\bar{u}}(\bar{u}) \leftarrow \text{Bayes' rule}$$

$$= \min_{\hat d} \int_{\bar u} f_{\bar u}(\bar u)\, d\bar u \left\{ \underbrace{\int_d f_{d|\bar u}(d|\bar u)\,(d-\hat d)^2\, dd}_{\text{diff. wrt } \hat d} \right\}$$

unconstrained MMSE → $\hat d = \underbrace{\int d\, f_{d|\bar u}(d|\bar u)\, dd}_{\rightarrow E[d|\bar u]}$

(*) orthogonality principle.

$$E[\underbrace{(d-\hat d)}_{e}\, \boxed{\bar u}] = 0 \qquad \bar u \rightarrow g(\bar u) = 0$$

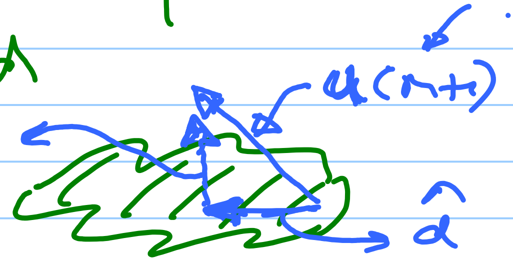
e estimation error

$\hat d \rightarrow E(d|\bar u)$

$\hat d$ ✓

$\bar u \leftrightarrow d$

e



Comparisim of some Estimators

Vector model

$$\bar{y}_{N\times1}(k) = \tilde{h}\, x(k) + \bar{n}(k) \rightarrow H_{N\times M}\, \bar{x}_{N\times1}(k)$$

measurement. observation vector, $N\times1$ data to be estimated, measurement noise.

orfanides

Scalar model

known, $\mathcal{N}(0, \sigma_n^2)$

$$y(k) = \boxed{h}\, \underbrace{x(k)}_{} + n(k)$$

$x(M) \in \{1, -1\}$, p_1, p_2 (γ_2, γ_{21})

discrete r.v. $\rightarrow P_x \rightarrow$ ~~$f_x(x)$~~

$\underline{\underline{\mathrm{var}(.)}} \le$ CRLB

→ MAP, ML, MMSE, L-MMSE, ZF ←