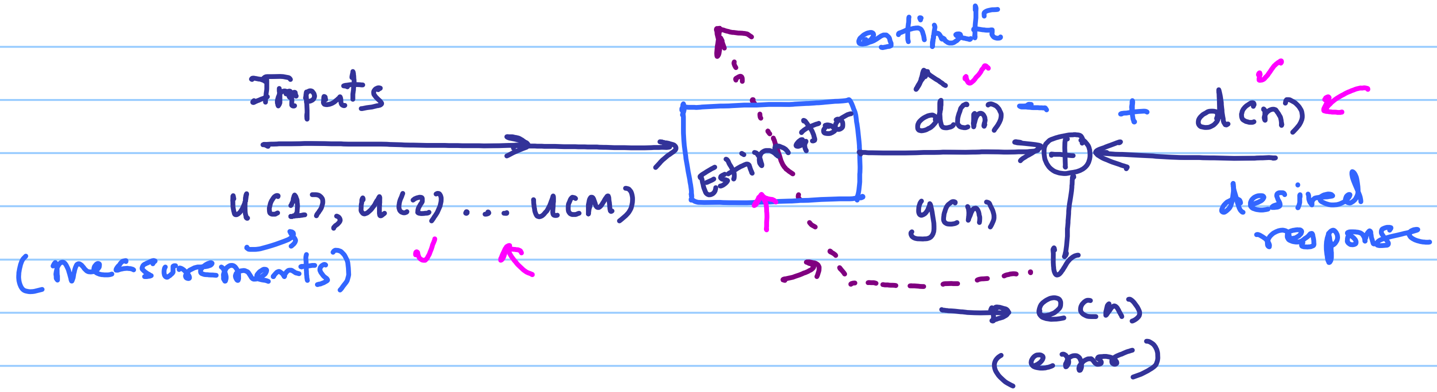
EE6110 Aug.-Nov., 2021

Class #3 (Aug. 19, 2021)

& Class #4 Aug. 26, 2021

Note Title 27-03-2020

Review of Estimation Theory :



(*) Properties of Estimators

Small sample
- unbiasedness
- efficiency

Consistency

$M > 50$?

Large sample
- Asymptotic unbiasedness
- Asymptotic efficiency.

(*) unbiasedness

* If $d(k)$ is deterministic, $E[\hat{d}(k)] = d \quad \forall k^{1 \ldots M}$
* If $d(k)$ is random, $E[\hat{d}(k)] = E[d] \quad \forall k$

(*) ✓ Efficiency. (Cramer-Rao Inequality)

$u \leftrightarrow d$?

If $\hat{d}(M)$ is an unbiased estimator of deterministic d, then given $u(1), u(2) \ldots u(M)$,

$\hat{d}(M)$ M

$$E\left[(d - \hat{d}(k))^2\right] \geq \frac{1}{E\left\{\left[\frac{\partial}{\partial d} \ln f_u(\bar{u})\right]^2\right\}}$$

$f_{u|d}(\bar{u}|d)$

$$\geq \frac{1}{-E\left\{\frac{\partial^2}{\partial d^2} \ln f_u(\bar{u})\right\}}$$

CRLB

Example :

$$\hat{d} = \frac{1}{M}\sum_{i=1}^{M} u(i)$$ where $\boxed{u(i) = d + w(i)}$ white

d: deterministic, $w(i) \sim \mathcal{N}(0, \sigma_w^2)$

$\hat{d}$

d

$$E\left[(d-\hat{d})^2\right] \quad \hat{d} = d + \frac{1}{M}\sum_{i=1}^{M} w(i)$$

$$E\left[\left(\frac{1}{M}\sum_{i=1}^{M} w(i)\right)^2\right] = \frac{1}{M^2} E\left[\underbrace{\left(\sum w(i)\right)^2}_{M\sigma_w^2}\right] = \boxed{\frac{\sigma_w^2}{M}}$$

(*) RHS

$$f(\bar{u} \mid d) = f(\underset{M\times 1}{\bar{u}}) = \frac{1}{(2\pi\sigma_w)^{M/2}} \cdot e^{-\frac{1}{2}\sum_{i=1}^{M}\frac{(u(i)-d)^2}{\sigma_w^2}}$$

$$\ln f(\bar{u}) = c + -\frac{1}{2}\sum (\quad)^2$$

$$\frac{\partial}{\partial d}\ln f(\bar{u}) = \sum_{i=1}^{M}\frac{(u(i)-d)}{\sigma_w^2}$$

$\forall i$ $\frac{\sigma^2}{M}$

$u(i) = d + w(i)$

$d(i)$?

10,000

& $\frac{\partial^2}{\partial d^2} \ln f(\bar{u}) = -\frac{M}{\sigma_w^2}$; $\frac{1}{-E(\cdot)} = \boxed{\frac{\sigma_w^2}{M}}$; L, G

$u(i) = d + w(i)$ $\qquad \frac{\sigma_w^2}{M}$ $\qquad \underline{\underline{M=1}}$ $\qquad \hat{d} = u(i)$ $\qquad \sigma_w^2$ ↑ CRLB

correlation (i.e. u(i)) comes from d

not estimatable

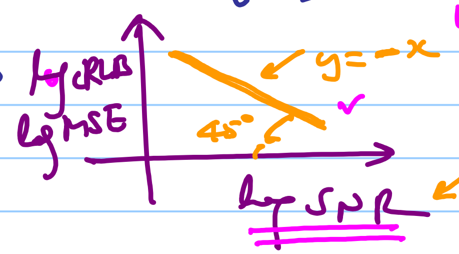
$\text{SNR} = \frac{d^2}{E[w^2] \leftarrow \sigma_w^2}$

$u = d + w$ $\qquad \frac{d^2}{\sigma_w^2}$

$\text{SNR} = \frac{d^2}{\sigma_w^2}$ $\qquad$ CRLB

$\log \text{SNR} = \log d^2 - \underbrace{\log \sigma_w^2}$

$y = c - mx$



(*) Behaviour of an "Adaptive Estimator"

$\rho \to$ pole 1st LPF

eg: $\hat{d}(k) = (1-\rho)\, u(k) + \rho\, \hat{d}(k-1)$

new estimate; new measurement; old estimate

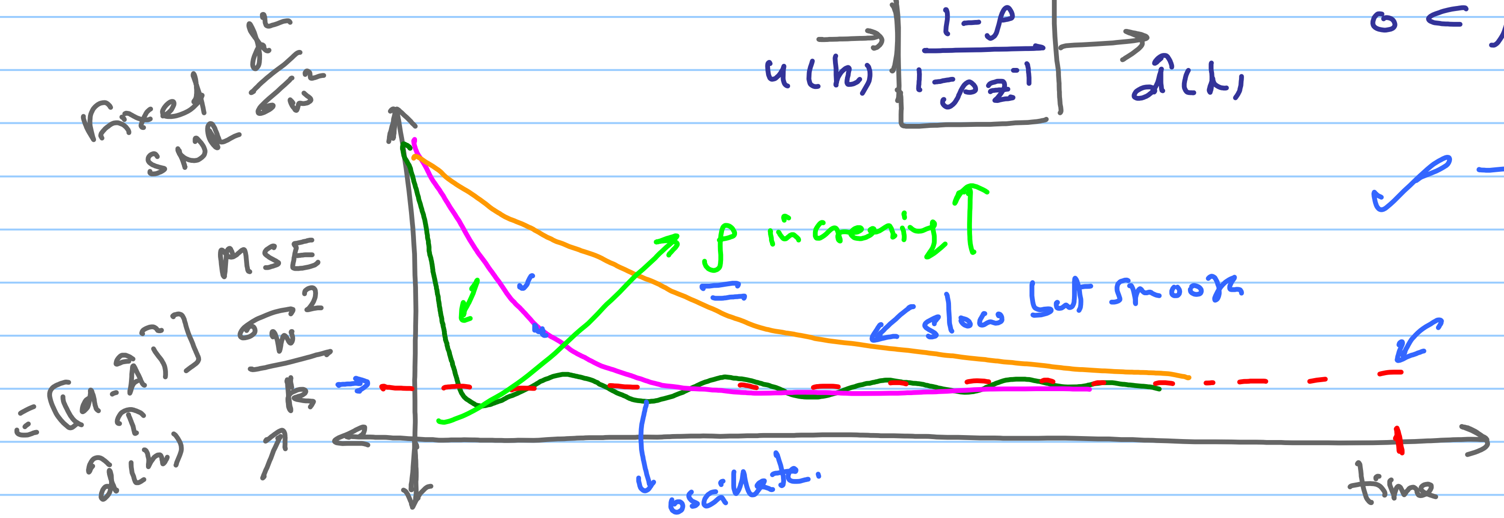
$0 < \rho \leq 1$ "forgetting factor"

$u(k) \to \boxed{\dfrac{1-\rho}{1-\rho z^{-1}}} \to \hat{d}(k)$

$\rho \to$ learning constant

gain constant

$\rho(k)$



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