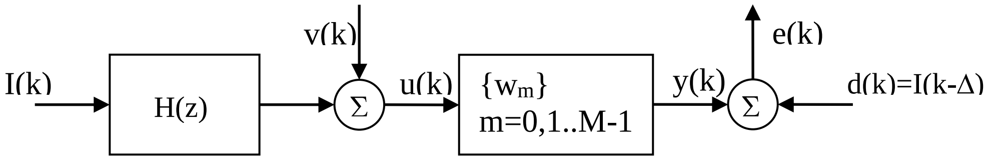
Department of Electrical Engineering
Indian Institute of Technology, Madras

Due before
Friday 5pm
Dec. 03, 2021

EE 6110: Adaptive Signal Processing

November 16, 2021 **Assignment #2** Marks: 20

Consider the adaptive channel equalisation model as shown in the figure below (see also pg. 224 in the E-copy of Prof.Ali Sayed's "Adaptive Filter Theory" for a similar problem). Here, the independent, uniformly distributed data symbols I(k) entering the channel F(z) are drawn from a 16-QAM alphabet, and the transmit signal power $E[|I(k)|^2] = \sigma_1^2 = 1$. The 3-tap channel is specified by $H(z) = 1 - 0.8z^{-1} + 0.5z^{-2}$ and the additive Gaussian noise component v(k) is zero mean with variance $\sigma_v^2 = 0.02$. The linear equaliser has order M=14, and the desired response d(k)=I(k-Δ), where the decoding delay Δ=5.



(a) Determine the LMSE (Wiener) solution $\mathbf{w}_{opt}$.for this choice of M, Δ, and σ_v^2. What is the corresponding J_{min}?

(b) Simulate the Least-Mean Squares (LMS) algorithm based adaptive equaliser with μ=0.10 being the gain constant. Plot the error convergence curve by averaging $e^2(k)$ over 25 Monte-Carlo runs, where the average squared error is defined by $\xi(k) = (1/25)\sum_{i=1}^{25} e_i^2(k)$. Plot 10log10(ξ(k)) versus k, for k=1,2….2000. What is the simulated and theoretical excess MSE (EMSE) that you get?

(c) For the same channel conditions and M and Δ, find the "best possible" gain constant μ for the LMS that will converge within 1500 samples. Plot its convergence curve. What are the EMSE values here?

(d) Repeat part (b) for the ε-normalised LMS algorithm discussed in class. Choose an appropriate value for ε and for μ in this case, and justify the reason for your choice(s). What are the EMSE values here?

(e) Repeat part (b), using now a Recursive Least Squares (RLS) algorithm to define the adaptive equaliser. Use forgetting factor λ=0.995 and initial choice of inverse of the data covariance **P**(0)=100. What is the simulated and theoretical excess MSE (EMSE) in this case?

(f) For the same λ=0.995, if you are allowed to change **P**(0), specify a new **P**(0) that will make RLS converge at least <u>five</u> times faster than the selection in (d). What are the EMSE values here?

(g) Make a single consolidated plot, with both the LMS curves in "solid blue", ε-normalised LMS in "solid green", and both of the RLS curves in "solid black". Mark also the J_{min} line using "dashed red" on this same plot.