Department of Electrical Engineering
Indian Institute of Technology, Madras

Due before Monday 5pm Oct. 18, 2021

EE 6110: Adaptive Signal Processing

October 08, 2021 **Assignment #1** Marks: 20

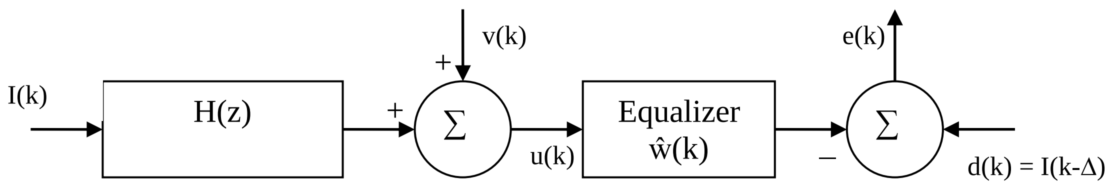
1. *Newton-Raphson Algorithm:* Use the Newton algorithm to iteratively find the roots rule to the following two polynomials. Use initial conditions as specified and tabulate the result of each iteration of the algorithm, for 5 iterations. Compare the result to the valid root(s) and comment.
(1.1) $f(x) = x^2 - 6x + 9$. Initial condition $x(0) = 10$.
(1.2) $g(x) = x^2 - 7x + 10$. Initial conditions -- Case (i): $x(0) = 30$; Case (ii) $x(0) = -12$.
(1.3) For the above two polynomials, is there a choice of initial condition (or range of initial conditions) for which the Newton algorithm will not converge to a valid root? Explain in each case.

2. *Expectation and Ergodicity:* For the 3 difference equations (models) given below, it is required to estimate the MxM autocorrelation matrix $\mathbf{R}_{uu}$ in the following manner.
First calculate the statistical $\mathbf{R}_{uu}$. Next, assuming ergodicity, numerically estimate $\mathbf{R}_{uu}$. Let's call this as $\mathbf{R}_{\mathbf{avg}}(k)$, where k data samples are used in making the estimate. Consider time averages obtained using k = 10, 100 and 1000 sample-functions (Monte-Carlo trials) of zero-mean Gaussian noise v(n), with variance as specified below.

(a) $u(n) = \cos(2\pi n/50) + v(n)$; $\sigma_v^2=0.1$
(b) $u(n) = 0.8\, v(n) - 0.2\, v(n-1)$; $\sigma_v^2=1$
(c) $u(n) = 0.9\, u(n-1) + v(n)$; $\sigma_v^2=0.5$ → since this is an AR process, *Hint*: for this AR process, to get WSS, drop the first, say, 1000 data samples generated. Then, the generated sample #1001 becomes the considered sample #1.

(2.1) For M=2, calculate $\mathbf{R}_{uu}$ and $\mathbf{R}_{\mathbf{avg}}(10)$, $\mathbf{R}_{\mathbf{avg}}(100)$, and $\mathbf{R}_{\mathbf{avg}}(1000)$, for each of the 3 models.
(2.2) Now for M=5, tabulate the Frobenius norm of the error matrix given $\mathbf{R}_{\mathbf{err}} = \mathbf{R}_{uu} - \mathbf{R}_{\mathbf{avg}}(100)$, for each of the above models. Explain your result.
(2.3) Bonus question (+2 marks): Set up the Yule-Walker equations (read up from Haykin) for the AR model in (c), and for M=2, verify that $\mathbf{w}=\mathbf{R}_{uu}^{-1}\mathbf{r}$ makes sense. Here, both $\mathbf{R}_{\mathbf{uu}}$ and $\mathbf{r}$ can be statistically estimated. Finally, what will be $\mathbf{w}$ for M=5 for the model in (c)?

3. *Wiener & Least Squares filtering:* In the figure below, the input $\{I_k\}$ is i.i.d. with $E[I_k^2]=1$, following a uniform pdf (i.e., $P(I_k=+1)=1/2=P(I_k=-1)$), and the noise v(k) is AWGN with variance σ_v^2 .



The bipolar symbols {I(k)} suffer inter-symbol interference (ISI) from the FIR channel $H(z) = 1 - 0.8z^{-1} + 0.5\, z^{-2}$. This ISI is compensated by employing a M^{th} order linear equalizer, $\hat{\mathbf{w}}$ (or $\hat{\mathbf{w}}(k)$), defined using the LMMSE criterion (Wiener filter), or, using the Least Squares (LS) criterion. The desired signal is given by d(k), where $d(k)=I(k-\Delta)$, with the decoding delay $\Delta \geq 0$ being an integer.

(3.1) Write a program to calculate the Wiener-Hopf solution $\hat{\mathbf{w}}=\mathbf{R}^{-1}\mathbf{p}$ which should work for *any* M and Δ. (Assume H(z) and σ_v^2 are known). For the given H(z), the program should be able to calculate $\hat{\mathbf{w}}$ and J_{min} for any σ_v^2, M and Δ. Note that **R** and **p** are to be computed statistically.

(3.2) Use the *auto-covariance formulation* for determining $\mathbf{R}_{avg}$ in the LS problem. To study the effect of window length on the "quality" of $\mathbf{R}_{avg}$, simulate random data {I(k)} and {v(k)}, k=1…N, and find the LS estimate $\hat{\mathbf{w}}_{LS}(N)$. When Δ=0, σ_v^2=0.05, M=7, calculate the least sum of error squares J_{min}^{LS} for the following values of N: (i) N=10, (ii)N=20, and finally, (iv) N=200. *Hint*: For J_{min}^{LS} get $\hat{\mathbf{w}}_{LS}(N)$ and substitute this in $\sigma_d^2-\mathbf{p}^T\hat{\mathbf{w}}_{LS}(N)$. Here, use the statistical **p** obtained from the LMMSE formulation in this equation.

(3.3) With N=1000, for each of the below situations, find J_{min} and J_{min}^{LS} and tabulate them. Comment on your results.

M	Δ	σ_v^2	J_{min}	J_{min}^{LS}
3	1	0.01		
10	1	0.01		
15	1	0.01		
10	2	0.01		
10	4	0.01		
10	8	0.01		
10	3	0.01		
10	3	0.05		
10	3	0.10		