12. An uniform i.i.d sequence $\{d(k)\}$ drawn from 8 -ary PAM alphabet (with $E_a = E[d^2(k)]$=1.0) is pulse shaped by a modified duo-binay filter $g(t)$. Recall that $g(\mathrm{kT}) = 1$ for k = 0 and 1, and is zero for other values of k, where T is the symbol duration. The received signal at the input to the ADC is given by $r(t) = \sum d(k)g(t - \mathrm{kT}) + n(t)$, where $n(t)$ is AWGN.

a) Design a precoder for the channel. Specify the precoder operations (Hint : Use base-8 arithmetic)

b) Make a neat sketch of the decoder decision regions for the noisy channel, and also indicate the Gray coding on the 8-ary PAM symbols taking the decision regions into account.

A. Precoder :

Let $I(k)$ be the information sequence at the k^{th} instant. $I(k) \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

$p(k)$ is the precoder output. Then

$$p(k) = [I(k) - p(k-1)] \bmod 8. \qquad (1)$$

$d(k)$ is the modulated output.

$d(k) = 2p(k) - 7$. $d(k) \in \{-7, -5, -3, -1, 1, 3, 5, 7\}$

$r(k)$ is the received signal. For the no noise case, $r(k)$ can be written as.

$r(k) = d(k) + d(k-1)$ $r(k) \in \{-14, -12, -10, ...0, ...12, 14\}$

$= 2\{p(k) + p(k-1) - 7\}$

$\tilde{I}(k) = [\frac{r(k)}{2} + 7\ ] \bmod 8.$

Assuming that the decision boundary is chosen as the perpendicular bisector of the linesegment joining two adjacent received symbols.