Tutorial-2

Q.1) **LE & DFE**: An uniform, real iid sequence $\{I(k)\}$ is with E[I$^2(k)$]= 1.0 is filtered by an LTI system $H(z) = 0.5 - z^{-1} + 0.2z^{-3}$, and the resultant output is corrupted by an AWGN sequence $\{v(k)\}$with variance 0.3 to finally yield the measurements $\{u(k)\}$. Assuming that $\{I(k)\}$ and $\{v(k)\}$ are mutually uncorrelated, find the concise expression for auto-correlation matrix R_u of size 2 x 2 (i.e., for M=2). Assume the desired sequence $d(k) = I(k - \Delta)$.

(a) What will be the resultant 2-tap MMSE linear equalizer (LE) weights $W_{\text{MMSE}} = [W_0\, W_1]^T$? use $\Delta = 0$.

(b) Instead, define MMSE decision feedback equalizer(DFE) with 2 feedforward coefficients $[a_0\, a_1]^T$and a single feedback coefficient b_1. Use again $\Delta = 0$.

(c) Get expressions for minimum MSE $\xi_{\min}$ (which is obtained by substituting the MMSE filter coefficients in to the expression $\xi = E[e^2(k)]$) and compare the LE and the DFE.

Q.2) Repeat Q.1 for following cases:

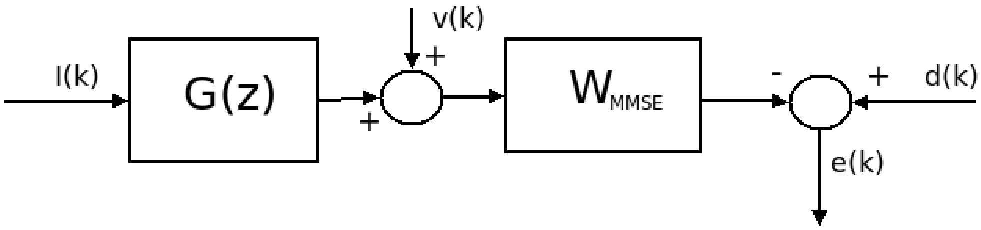
(i) $\Delta = 1$ with other dimension/quantities being the same.

(ii) $\Delta = 1$ and with 3 coefficients in the feed-forward section and 2 coefficients in the feedback section.

Q.3) **IIR Channel**: Consider the linear MMSE estimation problem below where the desired response $d(k) = I(k)$, and all are real signals. Here, G(z) is an infinite impulse response (IIR) transfer function, given by $G(z) = 1/(1 - 0.9z^{-1})$ and $\{I(k)\}$ and $\{v(k)\}$ are mutually uncorrelated with $\sigma_1^2 = 1.0$.

(a) If the wiener filter W_{MMSE} is to have an order M=2, find the wiener solution $W_{\text{MMSE}} = [W_0\; W_1]^T$ for $\sigma_v^2 = 0.4$.

(b) If now $\sigma_v^2 = 0$, what will be the new wiener solution?



Q.4) **Decision Feedback Equalizer**: An uniform, real iid sequence $\{I(k)\}$ is with $E[I^2(k)] = 2.0$ is filtered by an ISI channel with transfer function $H(z) = 0.5 - z^{-1} + 0.25z^{-2} + 0.10z^{-3}$ and the resultant output is corrupted by white noise sequence $\{v(k)\}$ with variance 0.20, to finally yield the measurements $\{y(k)\}$. A MMSE decision feedback equalizer (DFE) with 2 feedforward coefficients and 2 feedback taps is to be defined. Set up the Wiener-Hopf equation for this DFE assuming error-free feedback and fully specify the entries of the auto-correlation matrix and the cross-correlation vector.(Note: you need not solve the system of equations).

Q.5) In a co-channel interference model, the received signal $r(k)$ is given below where the "desired signal" $I_1(k)$ is filtered by $\{h_l\} = \{1, -0.5, -0.2\}$ and the "interfering signal" $I_2(k)$ is filtered by channel $\{g_l\} = \{0.1, 0.5, -2.5, 0.3\}$. The variance of noise $w(k)$ is $\sigma_w^2 = 0.1$, while the two self and mutually uncorrelated signals $I_1(k)$ and $I_2(k)$ have power $\sigma_1^2 = 1.0$ and $\sigma_2^2 = 0.7$, respectively. It is proposed to define a 3-tap equalizer $W = [W_0\, W_1\, W_2]^T$ using MMSE theory.

(a) If $d(k) = I_1(k)$, what is the auto-correlation matrix R?

(b) What is the corresponding P?

(c) What is the minimum mean square error $\xi_{\min}$?

$$r(k) = \sum_{l=0}^{2} h_l I_1(k-l) + \sum_{l=0}^{3} g_l I_2(k-l) + w(k)$$

Q.6) **MLSE using Viterbi Algorithms**: A 2-ary PAM sequence $\{I(k)\}$, with alphabet {-1, 1} passes through an ISI channel with a 4^{th} order transfer function $H(z) = 0.8 - 0.5z^{-1} - 0.6z^{-2} + 0.4z^{-4}$, and the resultant output is corrupted by an AWGN sequence $\{v(k)\}$ to yield the measurements $\{r(k)\}$. A viterbi equalizer is to be used to implement MLSE.

(a) How many survivor sequences will be maintained by the viterbi algorithm?

(b) Assuming that all the symbols transmitted up to symbol time k where -1 (i.e., $I(n) = -1$, $n = 1, 2...., k-1$), trace the true transmitted sequence from time k to $k+3$ if the corresponding symbols where $I(k) = 1, I(k+1) = 1$ and $I(k+2) = -1$; in other words, indicate the states visited by the true sequence thru the viterbi trellis from k to k+3.

Q.7) Binary PAM is used to transmit information over an unequalized linear filter channel. When a=1 is transmitted, the noise-free output of demodulator is

$$x_m = \begin{cases} 0.3 & (m=1) \\ 0.9 & (m=0) \\ 0.3 & (m=-1) \\ 0 & \text{otherwise} \end{cases}$$

(a) Design a three-tap zero forcing linear equalizer so that the output is

$$q_m = \begin{cases} 1 & (m=0) \\ 0 & (m=\pm 1) \end{cases}$$

(b) Determine q_m for $m = \pm 2, \pm 3,$ by convolving the impulse response of the equalizer with the channel response.

Q.8) The transmission of a signal pulse with raised cosine spectrum through a channel results in the following (noise free) sampled output from the demodulator:

$$x_k = \begin{cases} -0.5 & (k=-2) \\ 0.1 & (k=-1) \\ 1 & (k=0) \\ -0.2 & (k=1) \\ 0.05 & (k=2) \\ 0 & (\text{otherwise}) \end{cases}$$

(a) Determine the tap coefficients of three-tap linear equalizer based on the zero forcing criterion.

(b) For the coefficient determined in (a), determine the output of the equalizer for the case of the isolated pulse. Thus, determine the residual ISI and its span in time.

Q.9) Determine the tap weight coefficients of a three tap zero forcing equalizer if ISI span three symbols and characterized by the value $x(0)=1, x(-1)=0.3$ and $x(1)=0.2$. Also determine the residual ISI at the output of the equalizer for the optimum tap coefficients.

Q.10) In line-of-site microwave radio transmission, the signal arrives at the receiver via two propagation paths: the direct path and delayed path that occurs due to signal reflection from surrounding terrain. Suppose that the received signal has the form

$$r(t)=s(t)+\alpha s(t-T)+n(t)$$

where $s(t)$ is the transmitted signal, α is the attenuation ($\alpha<1$) of the secondary path and $n(t)$ is AWGN.

(a) Determine the output of the demodulator at $t=T$ and $t=2T$ that employs a filter matched to $s(t)$.

(b) Determine the probability of error for a symbol-by-symbol detector if the transmitted signal is binary antipodal and detector ignores the ISI.

(c) What is the error-rate performance of simple (one-tap) DFE that estimates α and removes ISI? Sketch the detector structure that employs a DFE.

Q.11) In magnetic recording channel, where the readback pulse resulting from a positive transition in the write current has the form

$$p(t)=\left[1+\left(\frac{2t}{T_{50}}\right)^2\right]^{-1}$$

a linear equalizer is used to equalize the pulse to a partial response. The parameter T_{50} defined as the width of pulse at the 50% amplitude level. The bit rate is $1/T_b$ and the ratio of $T_{50}/T_b=\Delta$ is the normalized density of the recording. Suppose the pulse is equalized to the partial response values

$$x(nT)=\begin{cases}1 & (n=\pm 1)\\ 2 & (n=0)\\ 0 & (\text{otherwise})\end{cases}$$

where $x(t)$ represents the equalized pulse shape.

(a) Determine the spectrum $X(f)$ of the band-limited equalised pulse.

(b) Determine the possible output level at the detector, assuming that successive transition can occur at the rate $1/T_b$.

(c) Determine the error rate performance of the symbol-by symbol detector for this signal, assuming that the additive noise is zero-mean gaussian with variance σ^2.

Q.12) Sketch the trellis for the viterbi detector of the equalized signal in Q.11 and label all the states. Also, determine the minimum euclidean distance between the merging paths.