Digital Modulation and Coding

Tutorial-1

1. Consider the signal set shown below in Fig .1

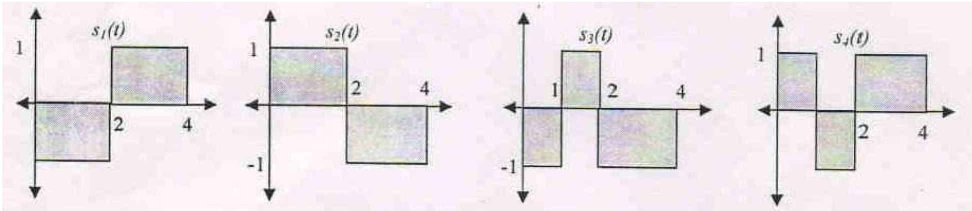
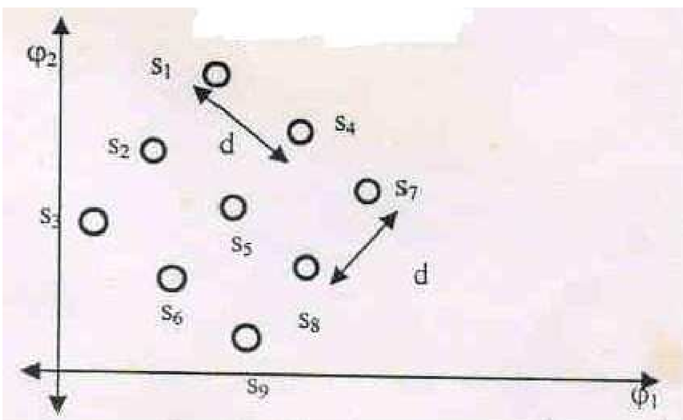


Figure-1

a) Find the compact (i.e. smallest) basis set required to ensure sufficient statistics.

b) What is the minimum Euclidean distance $d_{\min}$ of this signal set?

2. Consider the signal constellation shown in figure below ocrresponding to signal $s(t)$. Assume that the received signal is given by $r(t) = s(t) + n(t)$ where $n(t)$ is AWGN with psd(and after matched filtering, variance in each dimension) $N_0/2$. Assume $P(s_m) = 1/9$ for $m =$ 1,2 ... 9



a) Given that

$$q = \frac{1}{\sqrt{\pi N_0}} \int_{d/2}^{\infty} e^{-\frac{\alpha^2}{N_0}} d\alpha$$

obtain in terms of q the exact expression for the average probability of symbol error, P_E assuming Maximum Likelihood Decoding (MLD)

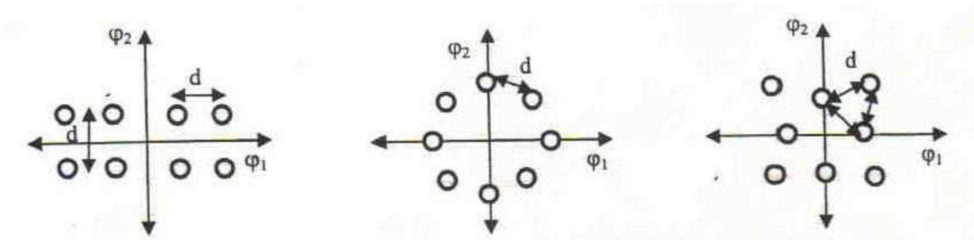
b) Now is $P(s_2) = P(s_5) = P(s_8) = 2/9$, and the remaining signals are equiprobable, then make a rough plot of the new decision regions. Indicate the exact shift from the decision regions in part (a), if any.

3. Consider an equiprobable, square, 16- QAM signal with symbol period T and with an average symbol power of E_s/T =40 microwatts. The AWGN variance $N_0/2$ (per dimension) is 2 microwatts.

a) Determine the expression for the average probability of the symbol error P_E, in terms of the $''q''$ function in Problem .2

b) Get the numerical value for P_E for the given SNR per symbol. IF u cannot compute the erfc() function, use the Chernoff upper bound discussed in the class.

4. Consider signal sets converying 3 bits/symbol where the transmitted signal $s(t) = \mathrm{Re}\{\sum\ I(k)g(t - \mathrm{KT})\exp(j2\pi f_c t)\}$ uses band limited pulse shaping function $g(t)$ with symbol period T and $E_g = \int\ g^2(t)\,\mathrm{dt} = 1$. All the sets are considered to have energy per bit $E_b = 2$ Joules, while the white noise PSD in each dimension is $N_0/2 = 0.1$ W/Hz. Since we consider 2-dimensional signal constellations (N=2), assuming a bandwidth of $1/t$ for the matched filter, the noise power will be $N(N_0/2)(1/T) = N_0/T$. after the matched filter, the signal power to the noise power ratio (per symbol) is given by SNR = $(E_s/T)/(N_0/T)$ where $E_s = E[|I^2(k)|]$, $E_g = \log_2 \mathrm{M}E_b$. Finally for the problem at hand with $M = 8$, SNR $= 3E_b/N_0$.



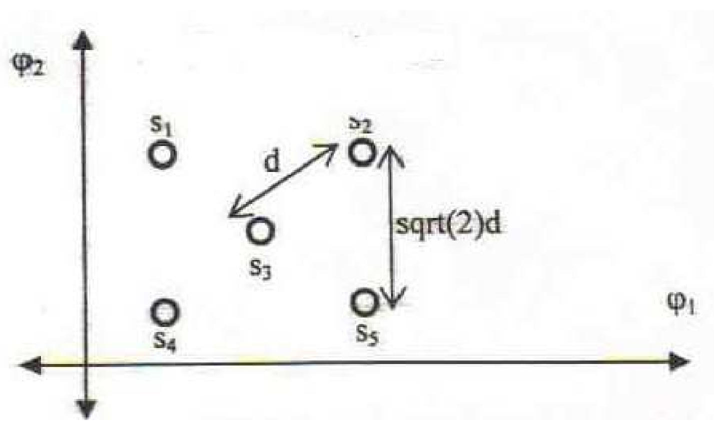
a) For each of the below signal constellations, find the approximated value of $''d''$ based on the above value of E_b

b) Find the minimum distance of all the 3 signal sets. Which of them has the smallest minimum distance.

c) Find the approximate expression for the average symbol error probability P_E using the union bound only on the " nearest neighbours " , in each case. Use the tables for the erfc() function to compute the numerical values.

d) If instead of the average value of the signal power being fixed, if the peak energy of the constellations is fixed to 6 joules (for all the 3 constelaltions), redo (a) to (c) above.

5. Consider the signal constellation shown in Fig.5 below ocrresponding to signal $s(t)$. Assume that the received signal is given by $r(t) = s(t) + n(t)$ where $n(t)$ is AWGN with psd(and after matched filtering, variance in each dimension) $N_0/2$.



a) If $P(s_m) = 1/5$ for all m then plot the decision regions for the given signal set.

b) Given that

$$q = \frac{1}{\sqrt{\pi N_0}} \int_{d/2}^{\infty} e^{-\frac{\alpha^2}{N_0}}\, d\alpha$$

obtain in terms of q the exact expression for the average probability of symbol error, P_E assuming Maximum Likelihood Decoding (MLD)

c) What will be the approximate value of P_E by using the Union bound argument, but restricting the union to only the nearest neighbours.

d) Now is $P(s_1) = P(s_2)$=0.35, and $P(s_3) = P(s_4)$=$P(s_5) = 0.1$, then make a neat plot of the new decision regions. Indicate the exact shift from the decision regions in part (a), if any.

6. In a " quad-orthogonal " scheme, 4-ary PAM signals are sent on orthogonal carriers (dimension), to convey M signals in the $N = M/4$ dimensions. given that the PAM signals are located at $\{3d, d, -d, -3d\}$:

   a) Provide the exact clsoed form expression for the average probability of symbol error, P_E, assuming coherent MLD for the AWGN channel with psd (after matched filtering, the variance) $N_0/2$.

   b) What is the Union bound on P_E ? also provide the approximate exxpression of P_E if only the nearest neighbour symbols are used

7. An uniform i.i.d sequence $\{d(k)\}$ drawn from a 4-ary PAM alphabet (with $E_a = E[d^2(k)]$=1.0) is pulse shaped by a modified duo-binay filter $g(t)$. Recall that g(kT) = 1 for k = -1 and 1, and is zero for other values of k, where T is the symbol duration. The received signal at the input to the ADC is given by $r(t) = \sum d(k)g(t-\text{kT}) + n(t)$, where $n(t)$ is AWGN.

   a) Specify the precoder operations (Hint : Use symbols 0,1,2, & 3 and operations in base-4 arithmetic)

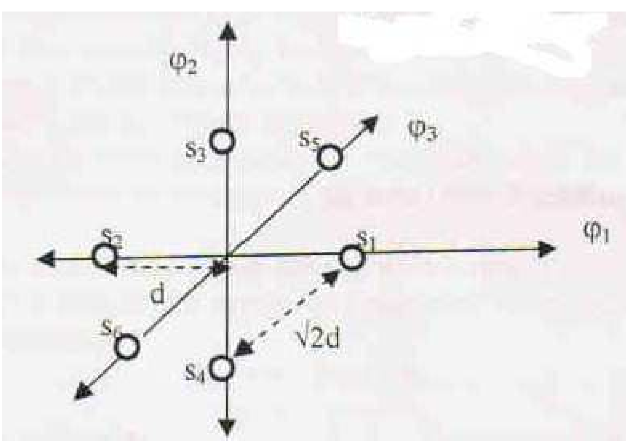
   b) Make a neat sketch of the decoder decision regions for the noisy channel, and also indicate the Gray coding on the 4-ary PAM symbols.

   c) What is the $d_{\min}$ for this scheme ? (in terms of E_a)

8. Consider the signal constellation shown in Fig.6 below ocrresponding to signal $s(t)$. Assume that the received signal is given by $r(t) = s(t) + n(t)$ where $n(t)$ is AWGN with psd(and after matched filtering, variance in each dimension) $N_0/2$. Assume $P(s_m)$ = 1/96 for m = 1,2 ... 6 . Given that

$$q = \frac{1}{\sqrt{\pi N_0}} \int_{d/2}^{\infty} e^{-\frac{\alpha^2}{N_0}} d\alpha$$

obtain in terms of q the exact expression for the average probability of symbol error, P_E assuming Maximum Likelihood Decoding (MLD)



9. A digital communication system operates in a band-limited channel with pass-band bandwidth of 2Mhz. Assuming SRRC pulse shaping with 50% excess abndwidth (i.e, $\beta = 0.5$), what will be the bit rate if 16 QAM modulation is used ?

10. The raised cosine spectal characteristic is given by

$$\begin{aligned} X_{\rm rc}(f) &= T &&, 0 \le |f| \le (1-\beta)/2T \\ &= T/2\ [1 + \cos(\pi T/\beta(|f| - (1\beta)/2T))], && (1-\beta)/2T \le |f| \le (1+\beta)/2T \\ &= 0 &&, \quad |f| \ge (1+\beta)/2T \end{aligned}$$

a) Show that the corresponding impulse response is
$x(t) = [\sin(\pi t/T)\cos(\beta\pi t/T)]/[(\pi t/T)(1 - 4\beta^2 t^2/T^2$

b) Determine the Hilbert transform of $x(t)$ when $\beta = 1$

c) Does $x'(t)$ (which is the Hilbert trnasform of x(t) possess the desirable properties of $x(t)$ that make it appropriate for data transmission. Explain

d) Determine the envelope of the SSB suppressed carrier signal generated from $x(t)$.

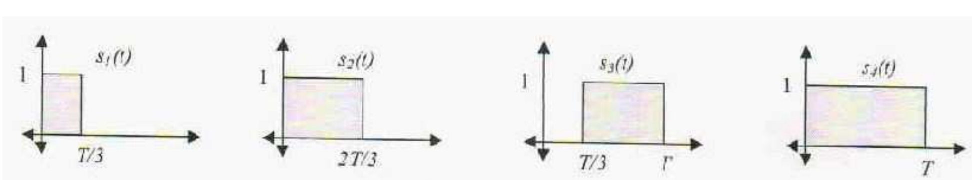
11. Suppose a digital communication system employs Gaussian shaped of the form $x(t)$ = exp(-$\pi a^2 t^2$. To reduce the level of inter symbol interference to a relatively small amount we impose a condition that $x(t = |T|) = 0.05$, where T is the symbol interval and $|.|$ is the mod ulus operation. The bandwidth W of the pulse $x(t)$ is defiend as that value of W for which $X(W)/X(0) = 0.01$, where $X(f)$ is the fourier transformof $x(t)$. Determine the value of W and compare this to that of the value of rased cosine with 100 percent rolloff (i.e. $\beta = 1$). Hint : first use the ISI constraint to find $''a''$ of $x(t)$.

12. An uniform i.i.d sequence $\{d(k)\}$ drawn from 8 -ary PAM alphabet (with $E_a = E[d^2(k)]$=1.0) is pulse shaped by a modified duo-binay filter $g(t)$. Recall that g(kT) = 1 for k = -1 and 1, and is zero for other values of k, where T is the symbol duration. The received signal at the input to the ADC is given by $r(t) = \sum d(k) g(t - \mathrm{kT}) + n(t)$, where $n(t)$ is AWGN.

a) Specify the precoder operations (Hint : Use symbols 0,1,2, & 3 and operations in base-4 arithmetic)

b) Make a neat sketch of the decoder decision regions for the noisy channel, and also indicate the Gray coding on the 4-ary PAM symbols.

13. Find the ortho-normal basis set that will span the below signal set.

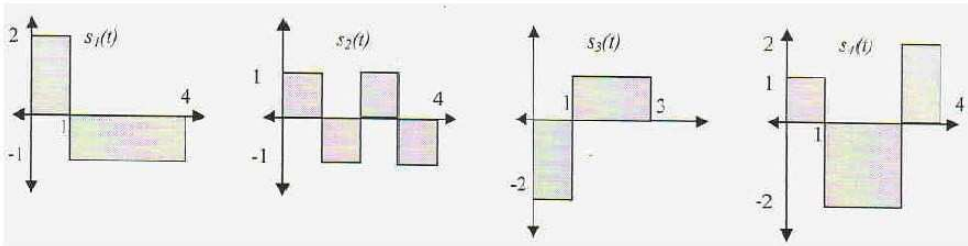


14. Consider the four waveform below

a) Determine the basis function.

b) Use the bass functions to represent the four corresponding signal vectors s_1, s_2, s_3 and s_4

c) Determine the minimum distance between any pair of vector.



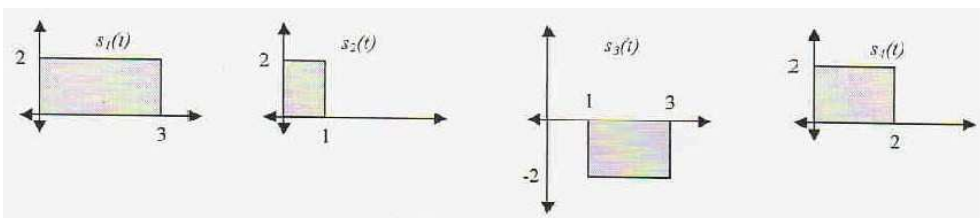
15. Consider the four wave form below (Figure-9).

a) Determine the basis functions.

b) Make rough sketch (3D?) of the signal constellation and mark the minimum distance

c) Using only the minimum distance(s), what will be the lower bound on the average symbol error probability P_E?

d) Using the union bound argument, get an expression for an upper-bound on P_E



16. FROM THE TEXTBOOK THE FOLLOWING PROBLEMS :

Chapter 8:

Problems - 8.1 - 8.6, 8.8 - 8.26 , 8.28 -8.34

Chapter 9 :

Problems - 9.2 - 9.4, 9.6 -9.9 , 9.11-9.22