

Quiz-2 Solution

Solution 1:

$$R = \begin{bmatrix} 3.7 & -2 \\ -2 & 3.7 \end{bmatrix} \quad \text{and} \quad \bar{p} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$$

(a) LE-MMSE $W_{\text{LE-MMSE}} = \begin{bmatrix} 0.5727 \\ 0.3096 \end{bmatrix}$

(b) $J_{\min}^{(\text{LE-MMSE})} = \sigma_I^2 - \bar{p}^T W_{\text{LE-MMSE}} = 0.1409$

(c) $\{h_l\} \otimes \{W_{\text{LE-MMSE}}\} = \{0.85905, -0.1083, -0.0232, 0.1548\}$ last three coefficients are ISI terms. Here we can normalised the coefficient by dividing the all term by first term.

Without Normalisation ISI power = $0.0362 \times \sigma_I^2$

With Normalisation ISI power = $0.04904 \times \sigma_I^2$

(d) For ZFE, $R = \begin{bmatrix} 3.5 & -2 \\ -2 & 3.5 \end{bmatrix} \quad W_{\text{ZFE-ZF}} = \begin{bmatrix} 0.6363 \\ 0.3636 \end{bmatrix} \quad \bar{p} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$

(e) $\{h_l\} \otimes \{W_{\text{LE-ZF}}\} = \{0.95445, -0.0909, -0.04545, 0.1818\}$ last three coefficients are ISI terms.

Without Normalisation ISI Power = $0.04338 \times \sigma_I^2$

With Normalisation ISI Power = $0.04762 \times \sigma_I^2$

Solution 2:

$$Y = \begin{bmatrix} y_n \\ y_{n-1} \\ y_{n-2} \\ I_{n-3} \\ I_{n-4} \end{bmatrix} \quad Y' = \begin{bmatrix} y_n \\ y_{n-1} \\ y_{n-2} \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} I_{n-3} \\ I_{n-4} \end{bmatrix}$$

$$R = E[YY^T] = \begin{bmatrix} E[Y'Y'^T] & E[Y'I^T] \\ E[IY'^T] & E[II^T] \end{bmatrix}$$

$$E[u_n u_{n-k}] = \begin{cases} 0.2082 & \text{for } k=0 \\ 0.1666 & \text{for } k=1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{Noise Covariance Matrix } C = \begin{bmatrix} 0.2083 & 0.1666 & 0.1333 \\ 0.1666 & 0.2083 & 0.1666 \\ 0.1333 & 0.1666 & 0.2083 \end{bmatrix}$$

$$E[Y'Y'^T] = 5 * \begin{bmatrix} 1.89 & 0.5 & -0.8 \\ 0.5 & 1.89 & 0.5 \\ -0.8 & 0.5 & 1.89 \end{bmatrix} + C$$

$$E[Y' I^T] = \begin{bmatrix} -4 & 0 \\ 0 & -4 \\ 5 & 0 \end{bmatrix} = \{E[I Y'^T]\}^T$$

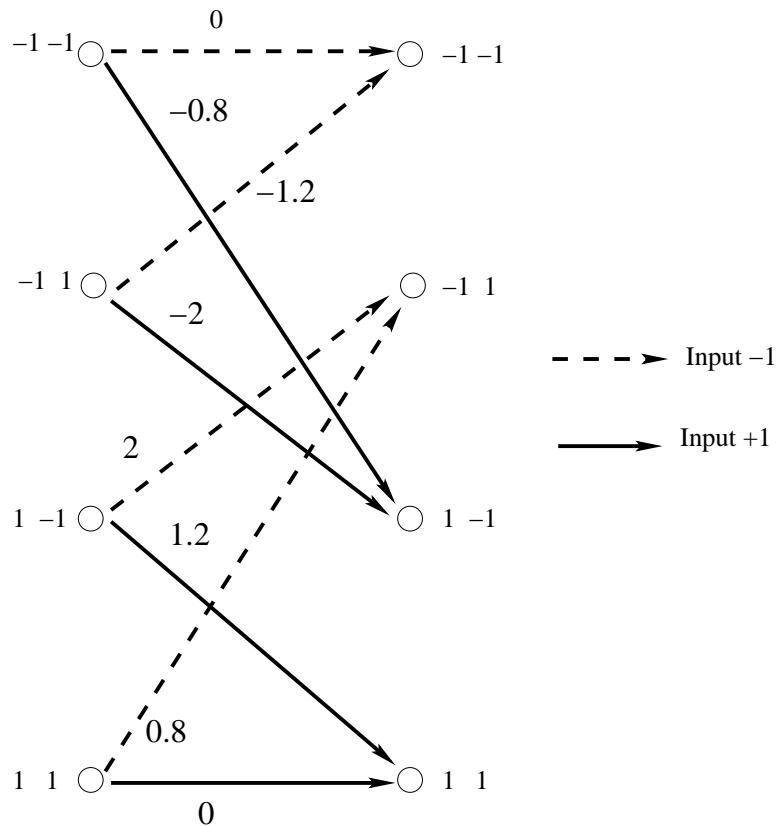
$$E[II^T] = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$R = \begin{bmatrix} 9.6583 & 2.66 & -4.13 & -4 & 0 \\ 2.66 & 9.6583 & 2.66 & 0 & -4 \\ -4.13 & 2.66 & 9.6583 & 5 & 0 \\ -4 & 0 & 5 & 5 & 0 \\ 0 & -4 & 0 & 0 & 5 \end{bmatrix}$$

$$\bar{p} = E[I_{n-2} Y] = \begin{bmatrix} 0 \\ 5 \\ 2.5 \\ 0 \\ 0 \end{bmatrix}$$

3)

a)



b & c)

