Introduction to OFDM - Notation

1. OFDM Transmission

$k \rightarrow$ Block

t or $m \rightarrow$ time index

$n \rightarrow$ sub-carrier (frequency) index (tone)

Cont. time		Sample index
T	$\longleftrightarrow$	N
T_{cp}	$\longleftrightarrow$	N_{cp}
T_{OFDM}	$\longleftrightarrow$	$N+N_{cp}$

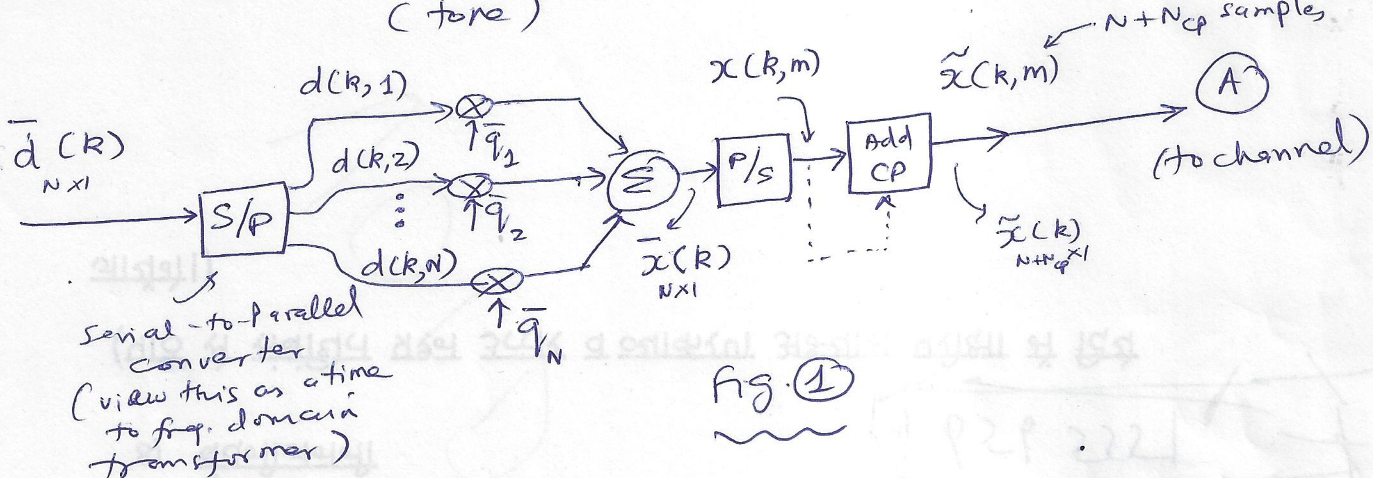

fig. 1

- In matrix notation, for $N=8$, this can be written as

$$\underset{N\times1}{\bar{x}(k)} = \underbrace{\begin{bmatrix} \bar{q}_1 & \bar{q}_2 & \cdots & \bar{q}_N \end{bmatrix}}_{\triangleq F} \begin{bmatrix} d(k,1) \\ d(k,2) \\ \vdots \\ d(k,N) \end{bmatrix};$$

i.e. $\bar{x}(k) = F\,\bar{d}(k)$;

- $\cancel{\tilde{x}^T(k) = [x(k, N-N_{cp}+1) \cdots x_N]}$

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- $$\tilde{x}^T(k) = \Big[x(k, N-N_{cp}+1) \cdots x(k,N) \;\big|\; x(k,1)\;\; x(k,2) \cdots \overbrace{x(k, N-N_{cp}+1) \cdots x(k,N)}^{N_{cp}\text{ samples}}\Big];$$

$\rightarrow$ (1)

★ Assume DFT basis (N=8 example)

Then, nth subcarrier (basis vector)

$$\bar{q}_n = \begin{bmatrix} e^{j\frac{2\pi}{N}n0} \\ e^{j\frac{2\pi}{N}n1} \\ \vdots \\ e^{j\frac{2\pi}{N}n(N-1)} \end{bmatrix}$$

'time' index m

- Also, instead of ordering (labelling) $\bar{q}_1$ to $\bar{q}_8$, we will use $\bar{q}_{N/2}$ to $\bar{q}_{-\frac{N}{2}+1}$

i.e., for $N = 8$

$$Q = \begin{bmatrix} | & | & | & | & | & | & | & | \\ \bar{q}_4 & \bar{q}_3 & \bar{q}_2 & \bar{q}_1 & \bar{q}_0 & \bar{q}_{-1} & \bar{q}_{-2} & \bar{q}_{-3} \\ | & | & | & | & | & | & | & | \end{bmatrix};$$

- Recall, in DFT & IDFT notation

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}nk}$$

equivalent to

$$\cong \sum_{n=-\frac{N}{2}+1}^{N/2} x[n]\, e^{-j\frac{2\pi}{N}nk}$$

- observe that

Zero DC

non-zero DC

$$\bar{q}_4 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}; \quad \bar{q}_1 = \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \\ j \\ -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \\ -1 \\ -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \\ -j \\ \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \end{bmatrix}; \quad \bar{q}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix};$$

etc.

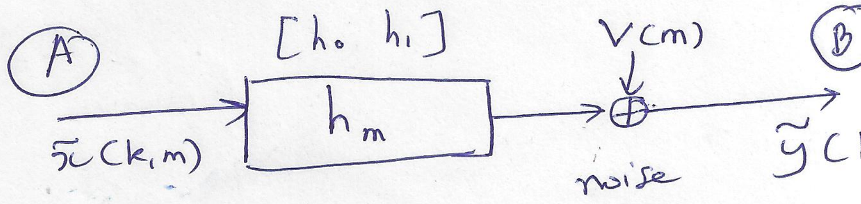
2. OFDM Reception — Measurement Model

Assumptions for Example (with $N=8$):

(i) $\boxed{N_{cp} = 3}$; (ii) sample-spaced Multipath channel $\bar{h}$ with $\boxed{L=2}$ taps

$\Rightarrow \bar{h} = [h_0 \ h_1]^T$;

continuing from Fig (1)



linear convolution

$$\tilde{y}(k,m) = h_m * \tilde{x}[k,m]$$

Fig (2)

Affected by IBI

block k-1

block k

block k+1

$$\begin{bmatrix} \tilde{y}(k-1,11) \\ \tilde{y}(k,1) \\ \tilde{y}(k,2) \\ \tilde{y}(k,3) \\ \tilde{y}(k,4) \\ \tilde{y}(k,5) \\ \vdots \\ \tilde{y}(k,11) \\ \tilde{y}(k+1,1) \end{bmatrix} = \begin{bmatrix} x(k-1,8) & x(k-1,7) \\ x(k,6) & x(k-1,8) \\ x(k,7) & x(k,6) \\ x(k,8) & x(k,7) \\ x(k,1) & x(k,8) \\ x(k,2) & x(k,1) \\ \vdots & \\ x(k,8) & x(k,7) \\ x(k+1,6) & x(k,8) \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \end{bmatrix} + \begin{bmatrix} * \\ V(1) \\ V(2) \\ \vdots \\ V(11) \\ * \end{bmatrix}$$

Affected by IBI

- out of 11 samples of $\tilde{y}(k,:)$, 4 choices of 8 contiguous samples possible
  - $\{\tilde{y}(k,1)$ to $\tilde{y}(k,8)\}$ → has IBI ; discard
  - $\{\tilde{y}(k,2)$ to $\tilde{y}(k,9)\}$ → 1st IBI-free choice
  - $\{\tilde{y}(k,3)$ to $\tilde{y}(k,10)\}$ → Practical, slip-insensitive choice
  - $\{\tilde{y}(k,4)$ to $\tilde{y}(k,11)\}$ → critical choice (last IBI-free choice)

3. Timing Recovery in OFDM

- Timing recovery $\Rightarrow$ block-boundary (FFT-window) determination
  $\Rightarrow$ picking a "suitable" N contiguous samples of $\tilde{y}[k,m]$ from the $N+N_{cp}$ received samples.

- If the timing recovery algorithm is used to extract the "critical" choise ; i.e.,
  $\{\tilde{y}(k, N_{cp}+1)$ to $\tilde{y}(k, N+N_{cp})\}$, then

$$\left[\tilde{y}\right]_{N\times 1} = \underset{\text{CIR}}{h[m]} \overset{\text{circular conv.}}{\circledast} \underbrace{F\bar{d}}_{\tilde{x}=x} + \overset{\text{noise}}{V[m]}$$

. CIR $\rightarrow$ Channel Impulse Response

$$\text{i.e.}\quad F^{H}\bar{y} = F\bar{h} \overset{\text{product}}{\cdot} \underbrace{F^{H}F}_{=I}\bar{d} + F^{H}\bar{V}$$

$$\overleftarrow{Y} = \left[H\right]_{N\times N} \cdot \bar{d} + \underbrace{F^{H}\bar{V}}_{\bar{V}},$$

CFR $\rightarrow$ Channel Frequency Response

for $N=8 \longrightarrow \begin{bmatrix} H[4] & & \\ & \ddots & \\ & & H[-3] \end{bmatrix}$

$\Rightarrow Y[k,n] = \underset{\text{CFR}}{H[k,n]} \cdot \underset{\text{QAM}}{d[k,n]} + V[k,n]$ on n^{th} subcarrier

- Now, if a "practical, slip-insenstive" choice (in the case where $N_{cp} > L-1$) of the block is made (with $\Delta > 0$) then the measurement $(N\times 1)$ can be written as

$$\left[\tilde{y}\right]_{N\times 1} = h[m] \circledast F_{\Delta}\bar{d} + V[m]$$

where $F_{\Delta} = \begin{bmatrix} | & | & & | \\ \bar{q}_{\Delta}(4) & \bar{q}_{\Delta}(3) & \cdots & \bar{q}_{\Delta}(-3) \\ | & | & & | \end{bmatrix}$; for $N=8$

→ Timing Recovery contd:

where $\begin{bmatrix} \vdots \\ \bar{q}(n) \\ \vdots_{\Delta} \\ \vdots \end{bmatrix}$ → is a circularly rotated version of $q(n)$ (either rotated "up" or "down" by Δ samples)

$\Delta > 0 \Rightarrow$ • Rotated 'down' is a practical choice

$\Delta < 0 \Rightarrow$ • Rotated 'up' corresponds to IBI choice

★ Now, $F^H F = I_{N \times N}$;

But, $$F^H F_{\Delta} = \begin{bmatrix} e^{j\frac{2\pi}{8}\Delta \cdot 4} & & & & \bigcirc \\ & e^{j\frac{2\pi}{8}\Delta \cdot 3} & & & \\ & & \ddots & & \\ \bigcirc & & & & e^{j\frac{2\pi}{8}\Delta(-3)} \end{bmatrix}_{8\times 8}$$ ← For $N=8$ example

This is a "phase-ramp" in the frequency domain.

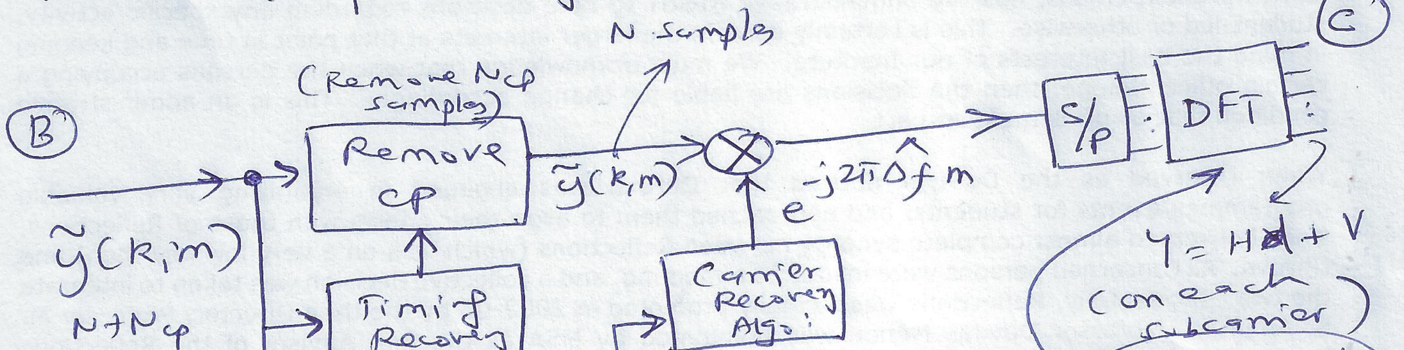
★ Subsequently, the scalar model on the n^{th} sub-carrier would now be

$$Y[k,n] = \underbrace{H[k,n] \cdot e^{j\frac{2\pi}{N}\Delta \cdot n}}_{\triangleq H'[k,n]} \cdot d[k,n] + V[k,n]$$

Using pilot subcarriers, this equivalent channel can be estimated easily.

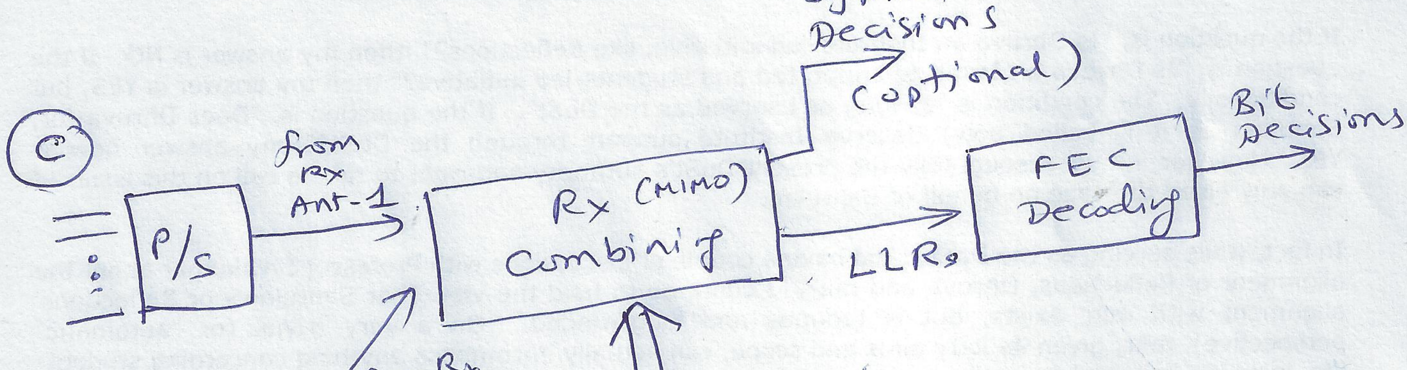
4. OFDM Receiver Block Diagram

Continuing from fig (2), we develop below the main Rx processing blocks



$\Delta f_{integer}$ part may be determined from the freq.-domain if needed

$\hat{\Delta f} \rightarrow$ either the entire frequency offset, or only the fractional part (less than Δf_{sub})



Channel Estimates $\rightarrow$ using pilot symbols and/or pilot sub-carriers starting from the $y = Hd + V$ measurements

- Note on System Bandwidth

recall $T + T_{cp} = T_{OFDM}$

Subcarrier Bandwidth $\boxed{\Delta f_{sub} = \frac{1}{T}}$;

OFDM System BW $\boxed{W \approx N \Delta f_{sub}}$;

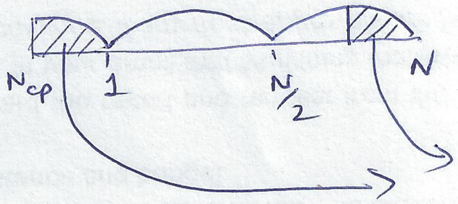
Instantaneous BW is a random-variable; However, guard-bands are used to ensure spectral mask compliance

5. Timing Recovery Algorithms

- (a) CP Correlation (Self-similarity ~~over~~ based)
- (b) Schmidl-Cox Method (Auto-Correlation based)
- (c) Cross-Correlation based
- (d) Maximum Likelihood (ML) Methods

(a) <u>CP Correlation</u>: Exploits the (possible) self-similarity of the CP portion with the last N_{cp} samples in the OFDM symbol. $\tilde{y}[k,m]$

eg: $N_{cp} = \frac{N}{4}$,



Those two portions will be similar if:

(a) $L = 1$

(b) No fading (flat)

(c) Very high SNR

In practise, one can ~~best to~~ assume $L < N_{cp}/2$, and auto-correlated $\tilde{y}[k,m]$ as follows:

note: spacing of N

$$Z(k,m) = \frac{\sum_{i=m-\frac{N_{cp}}{2}+1}^{m} \tilde{y}^*(k,i)\, \tilde{y}(k, i+N)}{\sum_{i=m-\frac{N_{cp}}{2}+1}^{m} \left|\tilde{y}(k,i)\right|^2}$$

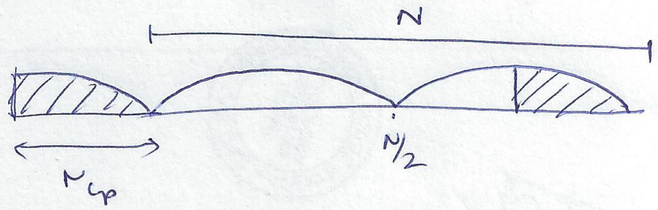
moving-averaged short-time energy term to normalise the value of $Z(k,m)$

(*) for $1 \leq L \leq N_{cp}+1$, use

$$\sum_{i=m-(N_{cp}-L+1)}^{m}$$

(b) Schmidl-Cox Method :

Eg: $N_{cp} = N/4$; $N = 2^l$;



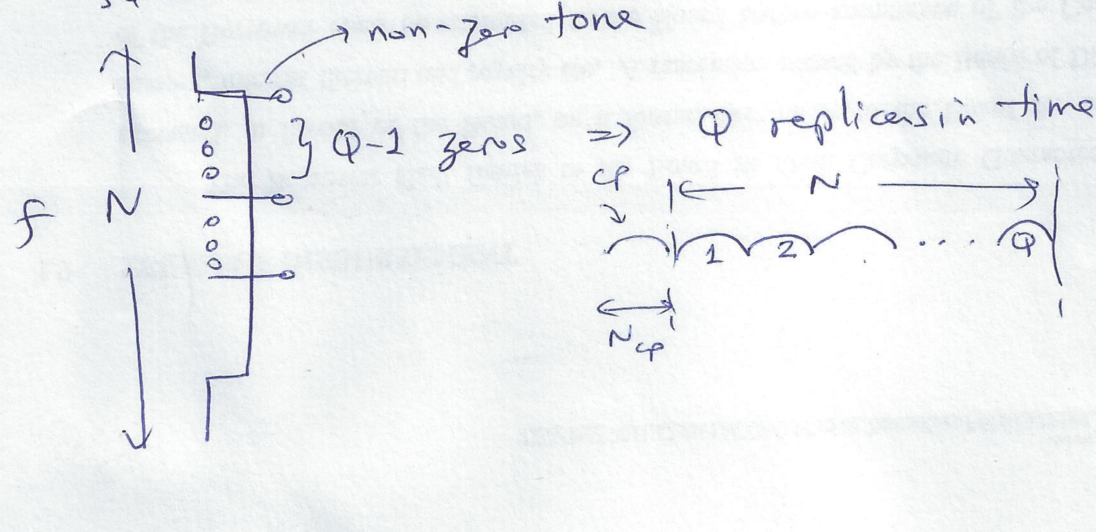
Create a time-signal with more self similarity (Here, over $N/2$ samples)

→ Even when $L = N_{cp}+1$, the self similarity over $N/2$ samples (i.e. 1 to $N/2$ is similar to $N/2+1$ to N) exists

∴ For this case,

$$Z[k,m] = \frac{\sum_{i=m-N/2+1}^{m} \tilde{y}^*[k,i]\,\tilde{y}[k,m+N/2]}{\sum_{i=m-N/2+1}^{m} |\tilde{y}(k,i)|^2}$$

- Simple method to create a preamble symbol with 2 (Q) replicas is to ~~make~~ put 1 (Q-1) zeros between every non-zero subcarrier.



(c) Cross-correlation based Timing Recovery
(known)

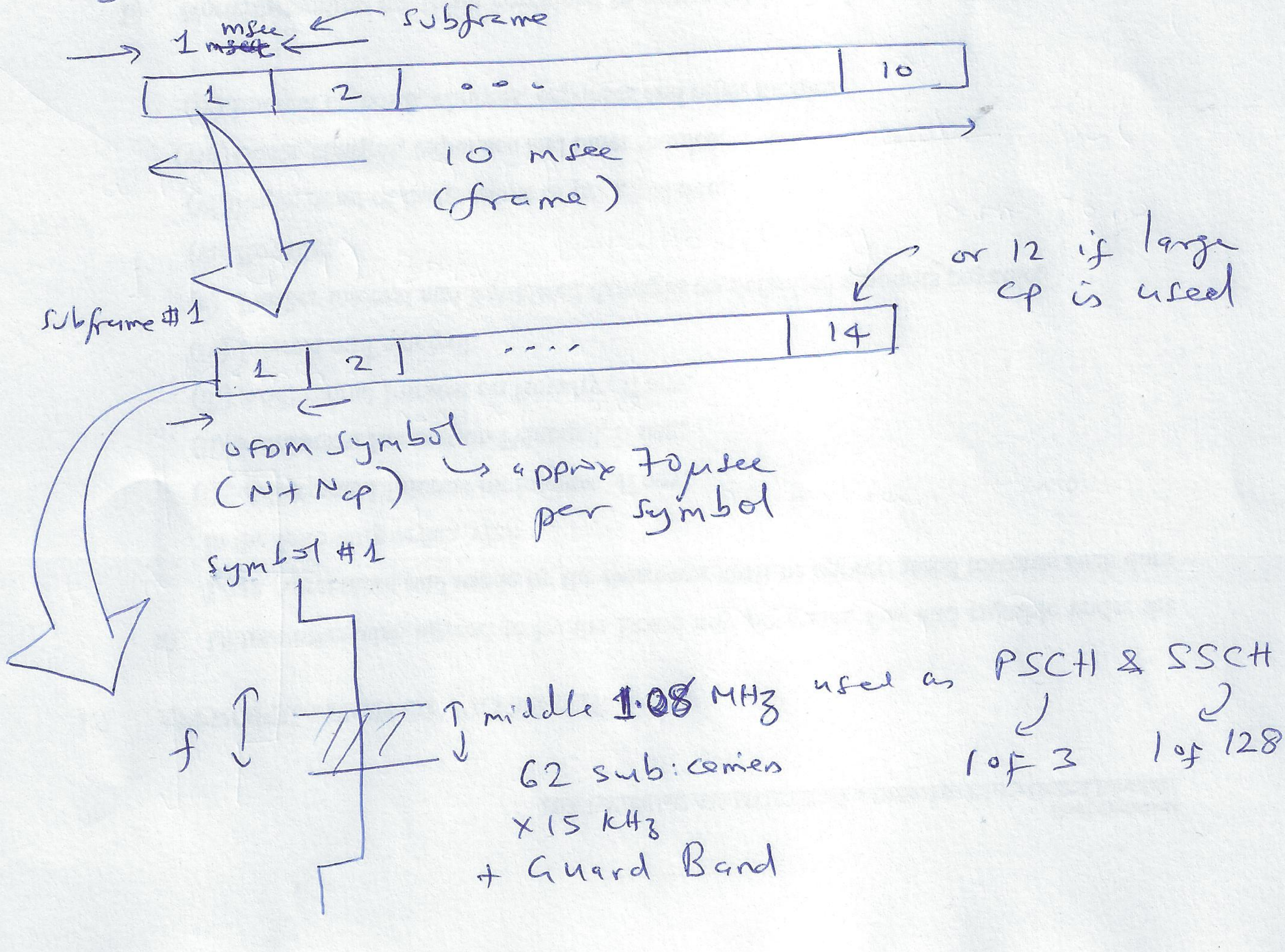
Key Idea: Send a white/PN sequence thro' the multipath channel; then, a cross-correlation at the receiver will:

(a) Reveal the Multipath CIR

(b) If the earliest arriving path is also the strongest one, the peak of the cross-correlation will provide the OFDM frame (symbol) boundary.

- In addition to timing, the PN sequence sent in the preamble can also provide base-station ID ~~dat~~ information.

- Eg: LTE (Rel-8)



EE 6141 Reading Material

1. Tsai's Book

→ Ch: 2 - Digital Modulation — entirely except Sec. 2.1.3
(pp. 9-28) ⟶ pdf 22-42

→ Ch: 5 - Synchronisation
(pp. 85-113) ⟶ pdf 69-98

→ Ch: 6 - Channel Estimation & Equalisation
(pp: 115-140) ⟶ pdf 99-125

2. Cho's Book

⇒ Ch: 4 - Intro to OFDM (pp. 111-150)

⇒ Ch: 5 - Sync for OFDM (pp: 153-183)

⇒ Ch: 6 - Channel Estimation (pp. 187-206)
↳ All Except → Blind Estimation & EM based CE } for mini-project

⇒ Ch: 7 - PAPR Reduction
↳ only Section 7.1 (pp 209-212)

⇒ Ch: 8 - Intercell-Interference Mitigation ⟶ for mini-project