# MIMO for 3GPP LTE-Advanced and Beyond

Gerhard Bauch and Guido Dietl

DOCOMO Euro-Labs

Universität der Bundeswehr München

# About the Presenters

**Gerhard Bauch** received the Dipl.-Ing. and Dr.-Ing. degree in Electrical Engineering from Munich University of Technology (TUM) in 1995 and 2001, respectively, and the Diplom-Volkswirt degree from FernUniversitaet Hagen in 2001.
In 1996, he was with the German Aerospace Center (DLR), Oberpfaffenhofen, Germany. From 1996-2001 he was member of scientific staff at Munich University of Technology (TUM). In 1998 and 1999 he was visiting researcher at AT&T Labs Research, Florham Park, NJ, USA. In 2002 he joined DoCoMo Euro-Labs, Munich, Germany, as Senior Researcher and later manager of the Advanced Radio Transmission Group. In 2007 he was appointed Research Fellow at DoCoMo Euro-Labs. Since October 2003 he has also been an adjunct professor at Munich University of Technology. In 2007 he was a visiting professor teaching courses at the University of Udine in Italy and at the Alpen-Adria-University Klagenfurt in Austria. Since 2009, he has been a full professor at the Universität der Bundeswehr München.
He received best paper awards of the European Personal Mobile Communications Conference (EPMCC) 1997, IEEE Globecom 2008 and 2009, and IEEE International Conference on Communications (ICC) 2009, the Texas Instruments Award of TUM 2001, the award of the German Information Technology Society (ITG in VDE) (ITG Foerderpreis) 2002 and the Literature award of ITG in 2007.
He is a Senior Member of the IEEE and a member of the German Information Technology Society (ITG in VDE (Association for Electrical, Electronic & Information Technologies)) where he serves as a member of the committee “Information and System Theory.”
Dr. Bauch has been member in the technical program committee and organizing committee of several conferences and served as vice chair of the working group 4 “New Air Interfaces, Relay Based Systems and Smart Antennas“ of the Wireless World Research Forum (WWRF). He has (co-)authored a textbook on “Contemporary Communications Systems” as well as more than 100 scientific papers in major journals and international conferences.
His research interests include channel coding and modulation, turbo processing, multihop transmission, multiple access and various aspects of signal processing in multi-antenna systems (MIMO).

**Guido Dietl** received the Dipl.-Ing. and Dr.-Ing. degree (both summa cum laude) in Electrical Engineering from Munich University of Technology (TUM), Munich, Germany, in 2001 and 2006, respectively.
He has been with the TUM from 2001 to 2006 where he was working as a Research Engineer on reduced-rank signal processing in Krylov subspaces and on its application to wireless multiuser communications. In Winter 2000/2001 and Summer 2004, he was a Guest Researcher at Purdue University, West Lafayette, IN, USA. In Fall 2005, he visited the Australian National University (ANU) in Canberra, ACT, Australia. He joined DoCoMo Communications Laboratories Europe GmbH (DoCoMo Euro-Labs), Munich, Germany, in 2006, where he is currently manager of the Advanced Radio Transmission Group.
Dr. Dietl received the VDE Award for his diploma thesis in 2001, the Kurt Fischer Award of TUM for his doctoral thesis in 2007 and the award of the German Information Technology Society (ITG in VDE) 2007 (ITG Foerderpreis).
He is member of the IEEE since 2001 and member of the VDE (Association for Electrical, Electronic & Information Technologies) since 2007.
He has authored a monograph on “Linear Estimation and Detection in Krylov Subspaces” published by Springer in 2007 and written more than 30 scientific papers in books, journals, and conferences.
His main research interests are numerical linear algebra, reduced-rank signal processing, iterative (Turbo) detection, and transmit signal processing in multiuser multiple-input multiple-output (MIMO) systems.

# Outline (1)

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- Introduction
  - MIMO for bandwidth-efficient wireless communications
  - Multiuser diversity
  - Single-user (SU) MIMO versus multi-user (MU) MIMO
  - Uplink MU-MIMO versus downlink MU-MIMO
  - Linear versus non-linear MU-MIMO
- Single-user MIMO
  - Single-user MIMO in 3GPP Long-Term Evolution (LTE)
  - Spatial multiplexing with Rx and Tx processing
- Theoretical fundamentals
  - Introduction to Dirty Paper Coding (DPC)
  - Tomlinson-Harashima precoding (THP)
  - Precoding for the MIMO broadcast channel

# Outline (2)

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- Non-linear MU-MIMO algorithms based on Dirty Paper Coding (DPC) and zero-forcing (ZF)
  - Sequential encoding with DPC and ZF for single receive antennas
  - Sequential encoding with DPC and block zero-forcing (block ZF)
  - SESAM: A capacity approaching algorithm
  - Comparison of achievable rates

- Theoretical limits
  - Capacity of the SU-MIMO channel
  - Capacity region of the MIMO multiple-access channel (MAC)
  - Sum capacity of the MIMO broadcast channel (Sato bound)
  - DPC and dual MAC region of the MIMO broadcast channel
  - Capacity region of the MIMO broadcast channel

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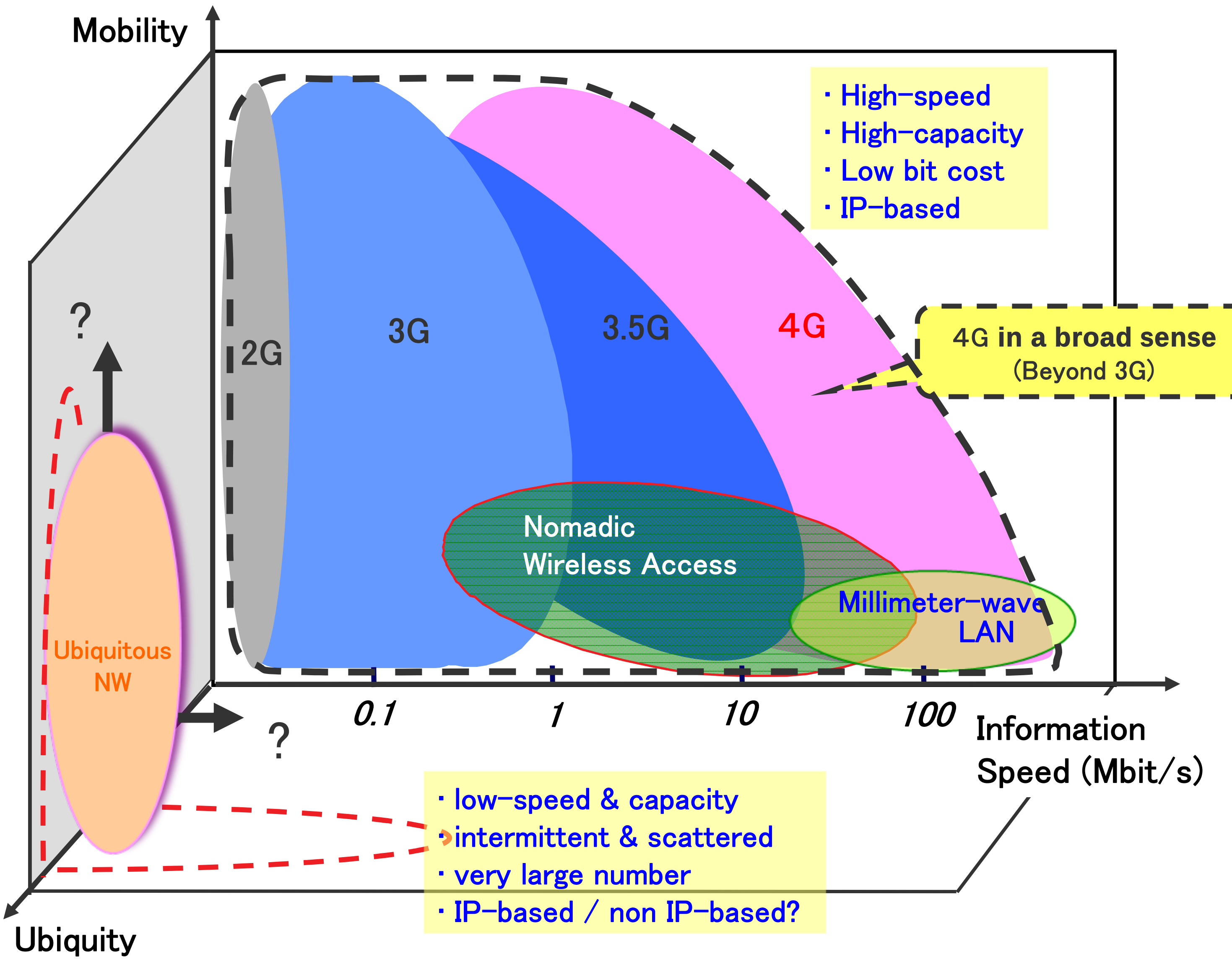
# Outline (3)

- Linear MU-MIMO schemes for 3GPP Long Term Evolution (LTE) and 3GPP LTE Advanced
  - Linear versus nonlinear precoding
  - MU- versus SU-MIMO
  - Summary of MIMO techniques in 3GPP-LTE
  - Precoder codebook based 3GPP-LTE MU-MIMO
  - Channel codebook based ZF precoding
  - Performance comparisons
  - MU-MIMO Status in 3GPP-LTE-Advanced

# What is 4G Access?



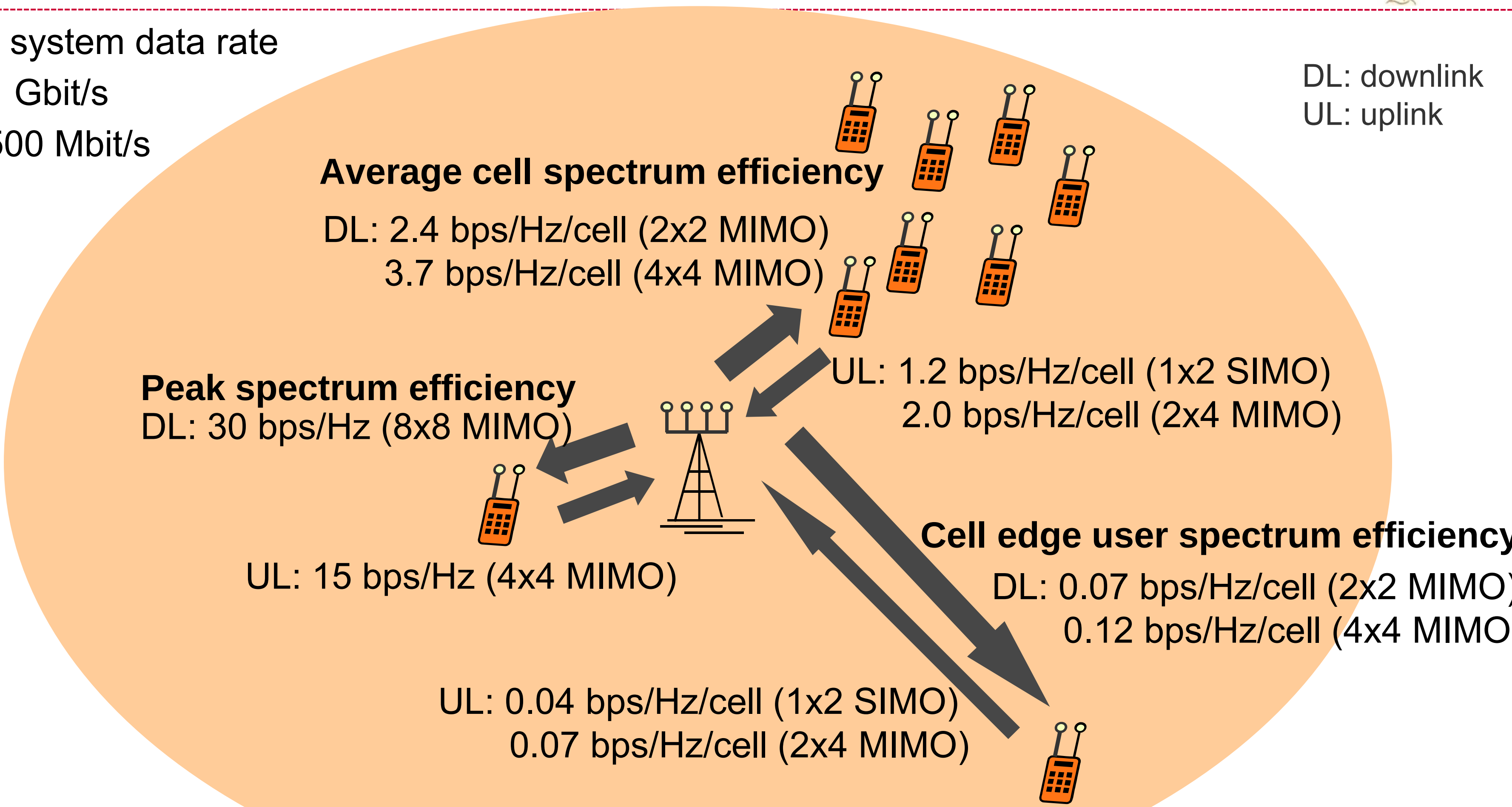


# Performance Targets for LTE-Advanced



Peak system data rate
DL: 1 Gbit/s
UL: 500 Mbit/s

DL: downlink
UL: uplink




**Development of spectrum efficiency:**

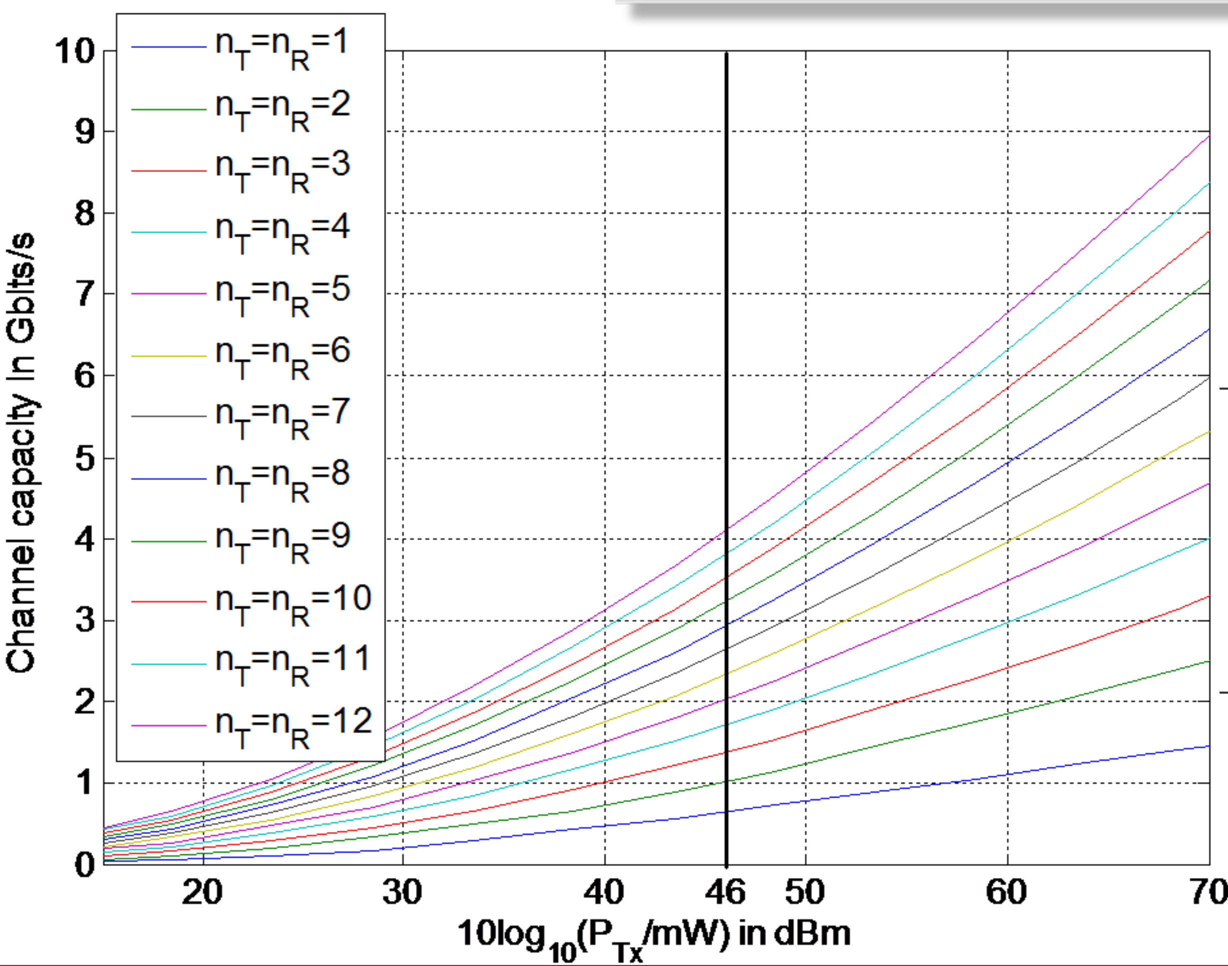
HSPA 2005 → x 2-4 → LTE 2010 /2011 → x 2-4 → LTE Advanced 2015

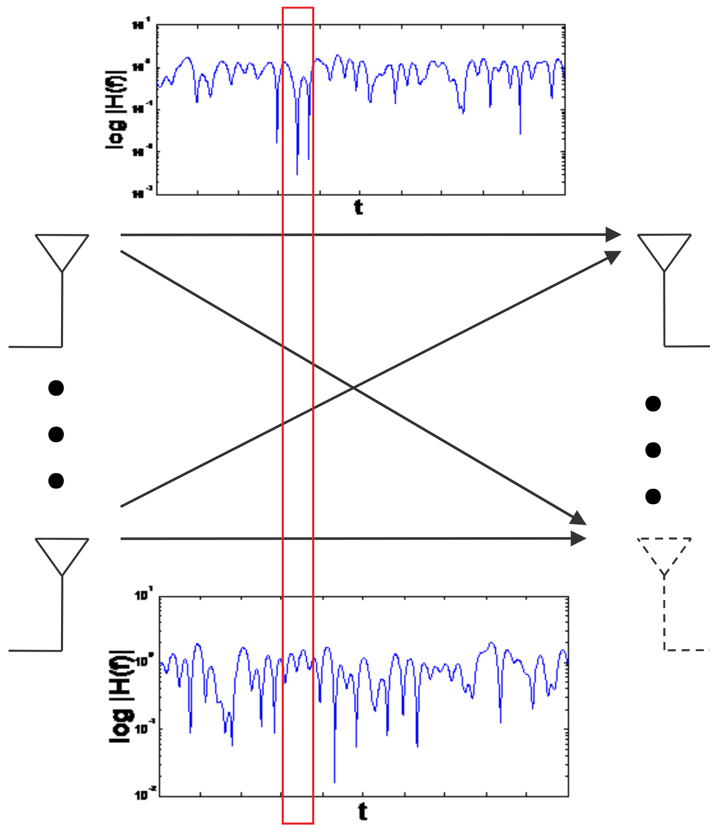
# Capacity of Single-User (SU) MIMO

Carrier frequency: $f_c$=3.8 GHz
Bandwidth: $B$ = 100 MHz
Transmit power: $P_{Tx}$
$n_T$ transmit antennas
$n_R$ receive antennas

WINNER channel model:
Urban macro cell, non-line-of-sight (NLOS)

Use multiple antennas at transmitter (and receiver)
⇒ capacity increases significantly in fading environment



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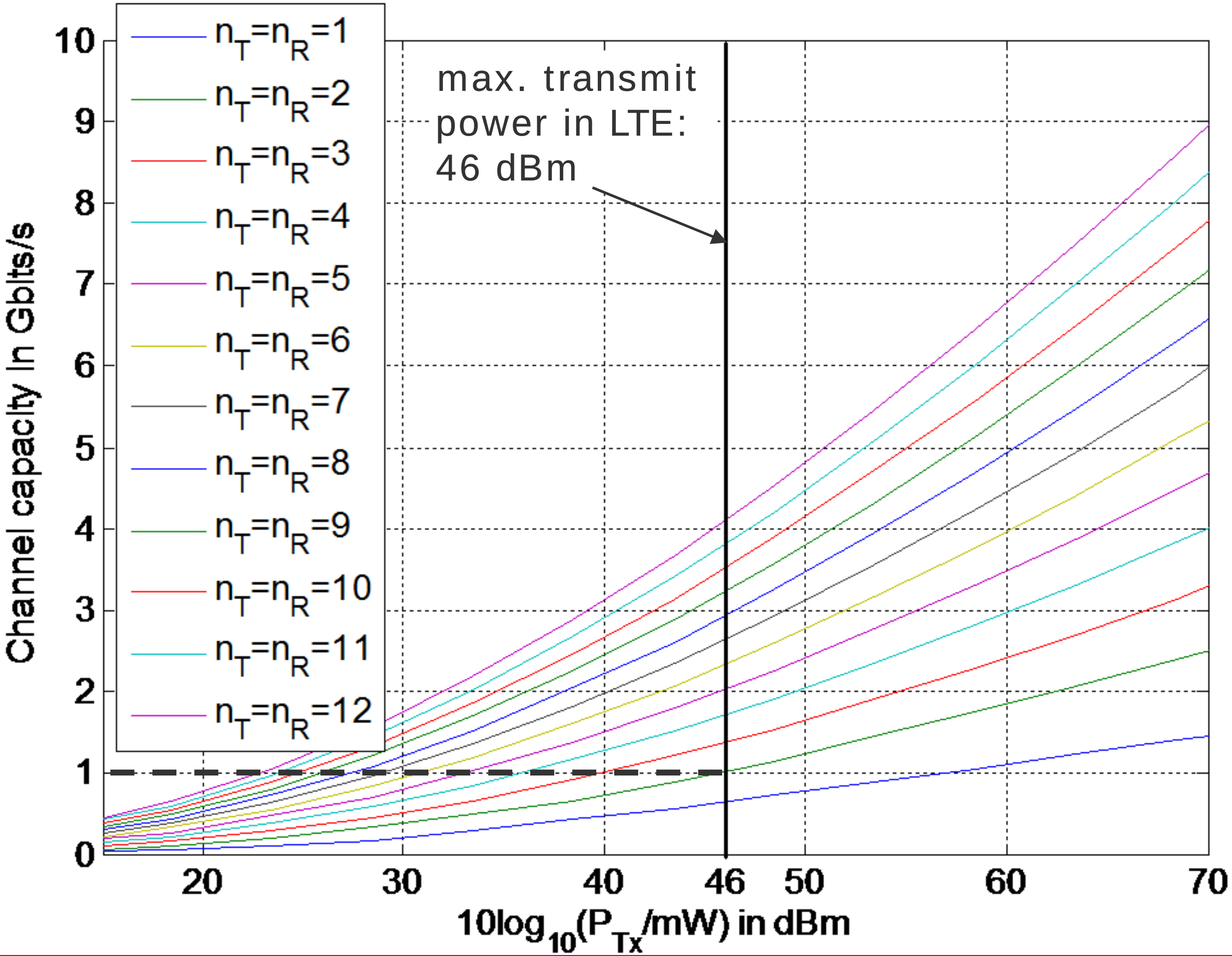
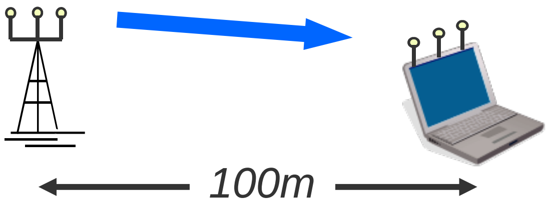
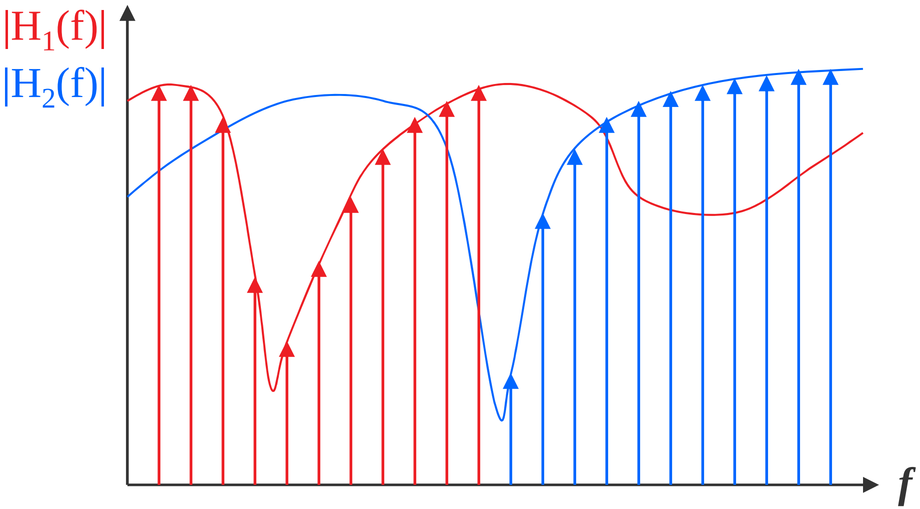
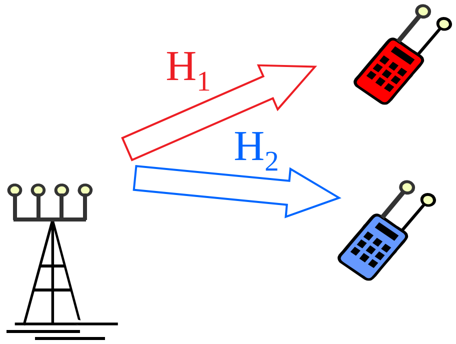


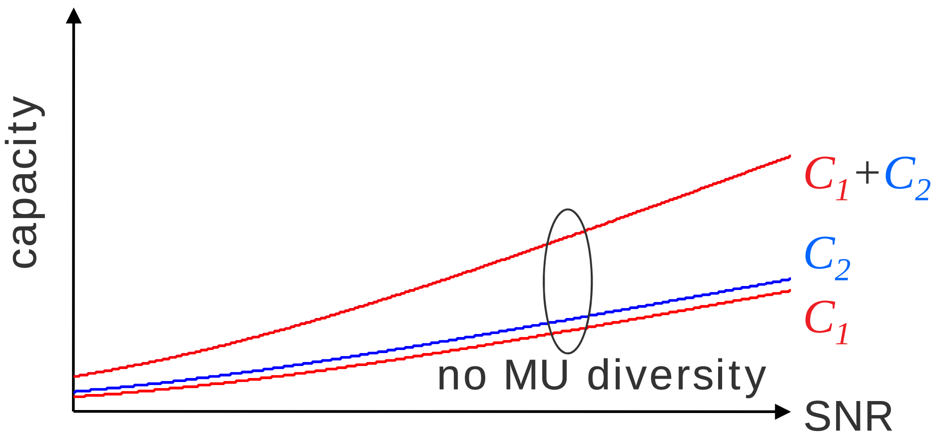

IMT-Advanced performance targets cannot be met without MIMO.

# Multiuser Diversity




**Static FDMA:** A fixed fraction of the subcarriers is allocated to each user.

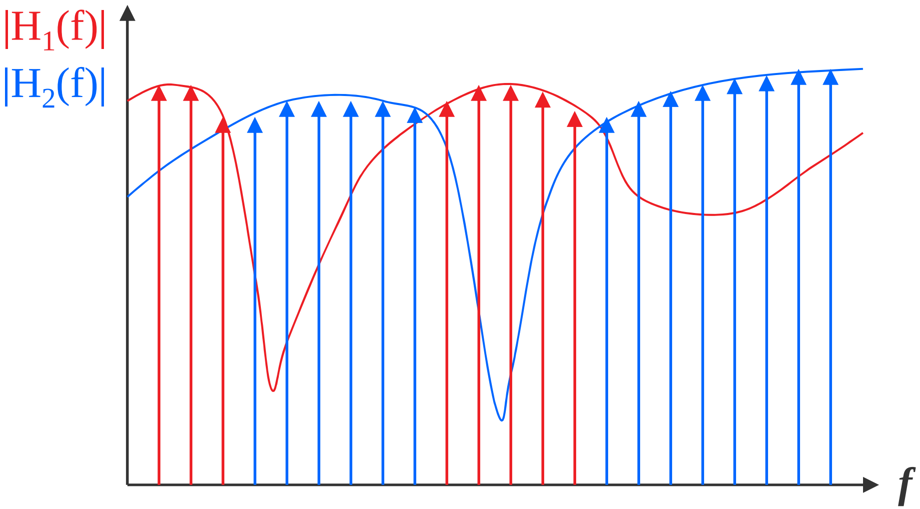


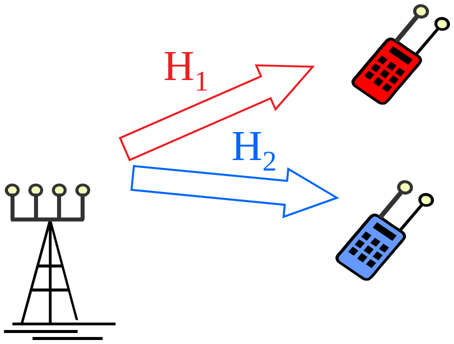

**Sum capacity:**

$$C = \sum_{k=1}^{K} C_k$$

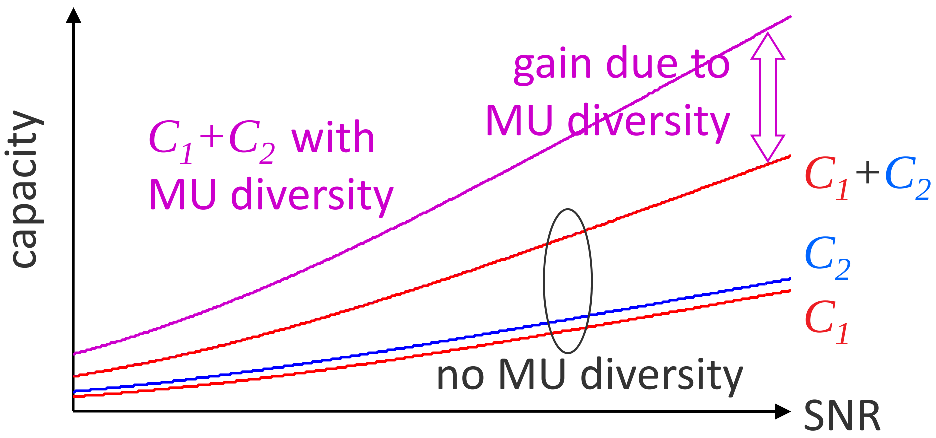
$C_k$: capacity of user $k$

# Multiuser Diversity




**Dynamic FDMA:** A subcarrier is allocated to the user with the highest capacity on that subcarrier (exploitation of MU diversity).

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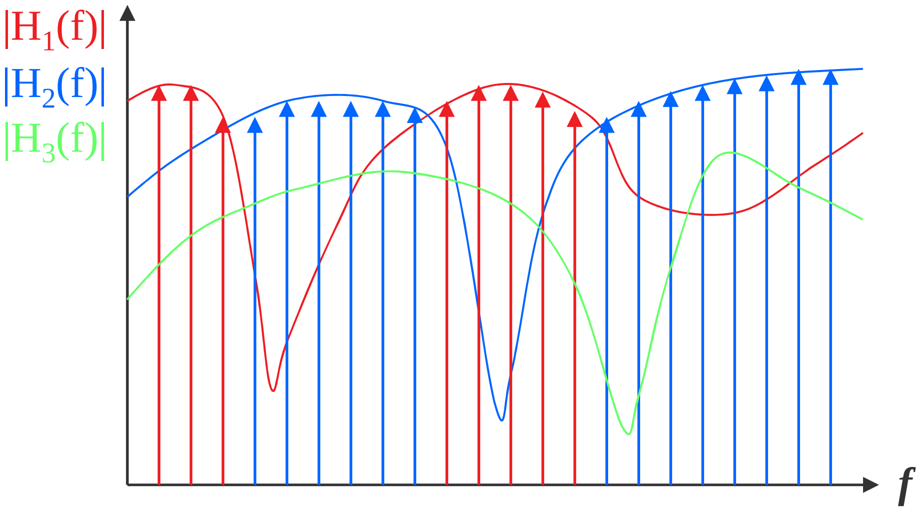
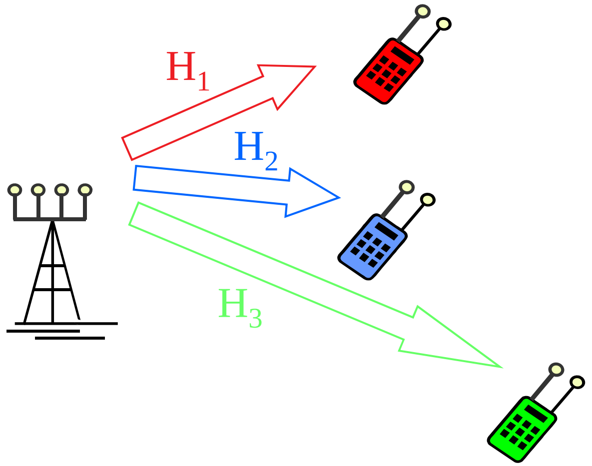
# Multiuser Diversity



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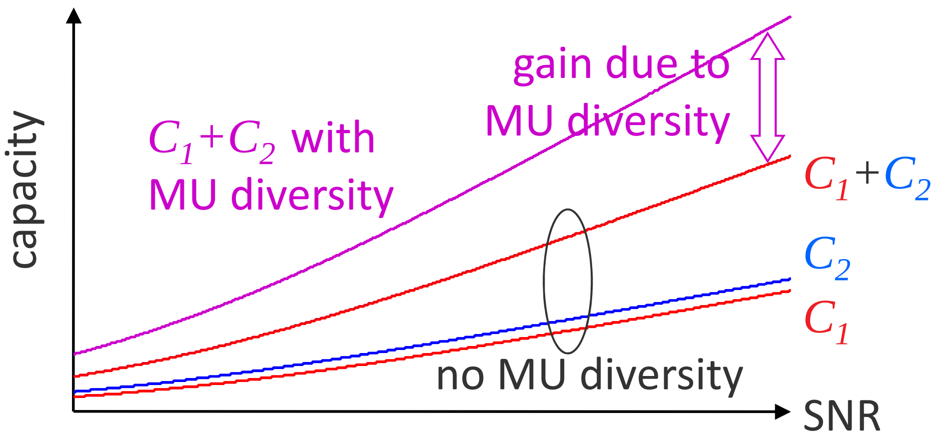


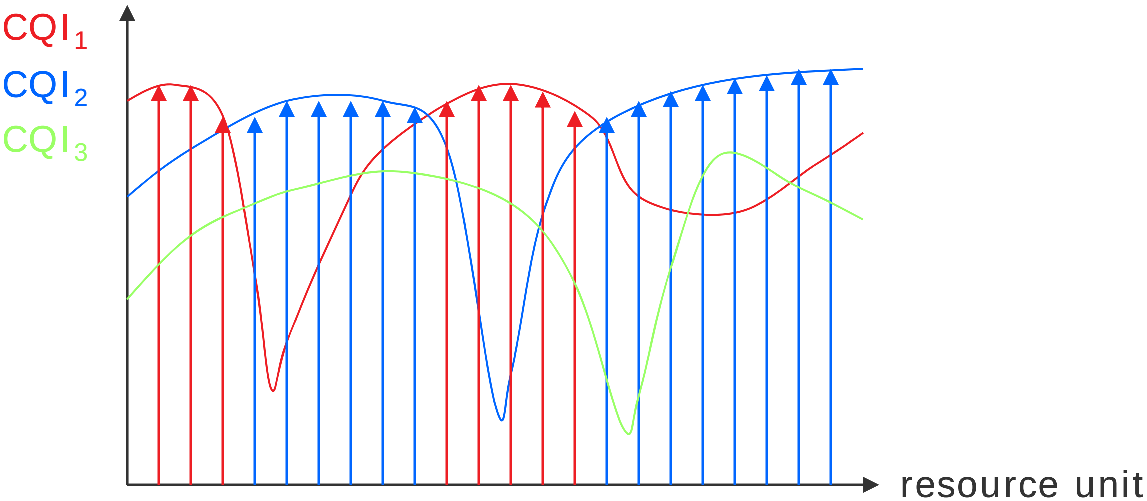

**Advantage:**
Sum capacity is increased.

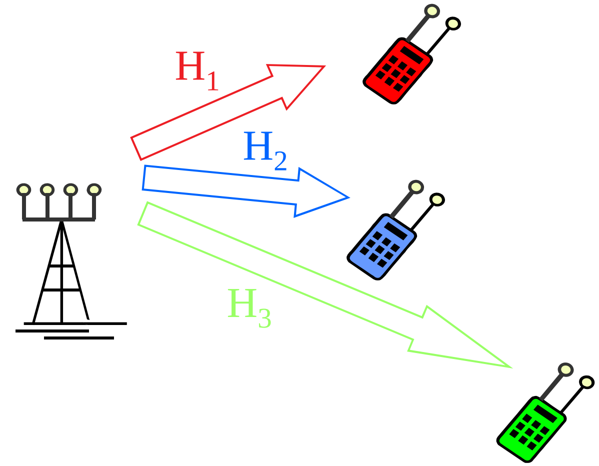
**Problem:**
A user in a bad channel state may not be served for a long time (fairness problem, QoS is not guaranteed).

→ Exploitation of multiuser diversity is difficult for delay sensitive applications.

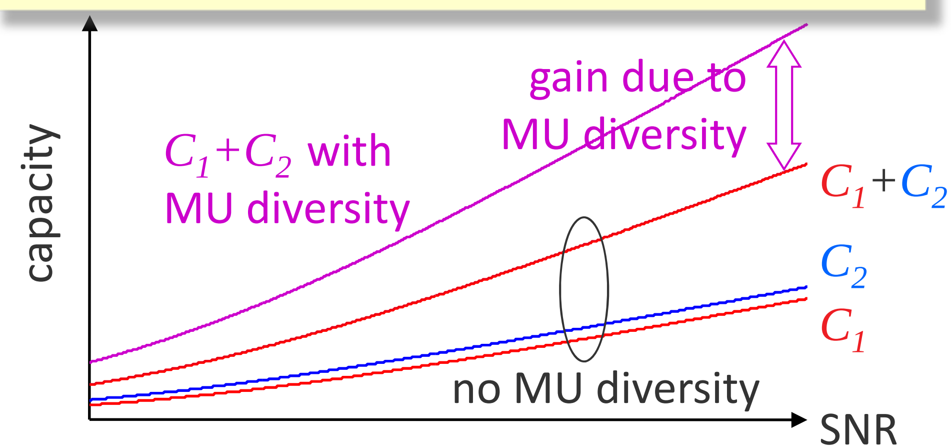
# Multiuser Diversity






**A resource unit in time, frequency and space is allocated to the user with the highest capacity on that resource unit.**

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# Multi-User MIMO versus Single-User MIMO

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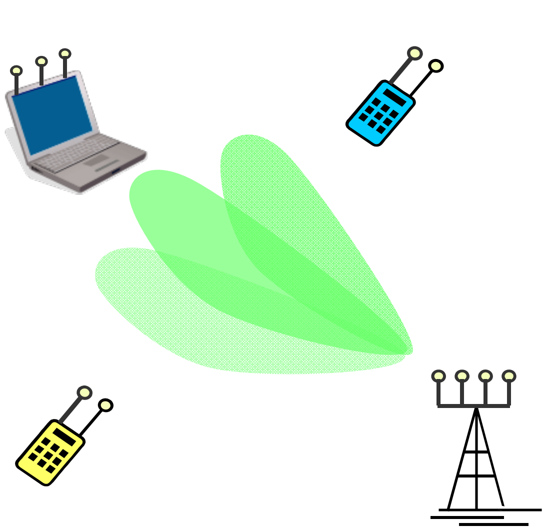
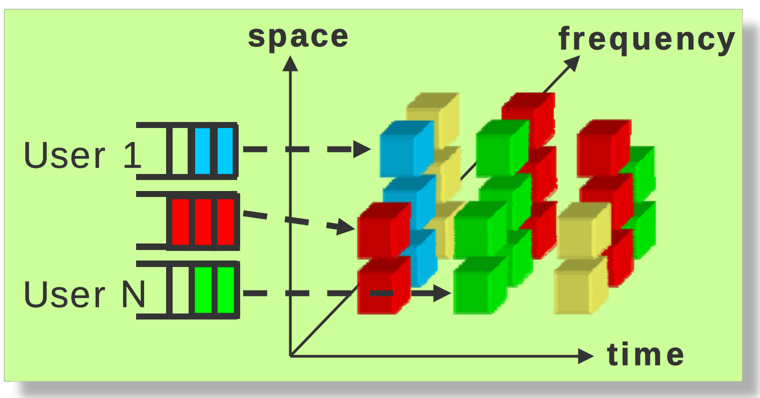
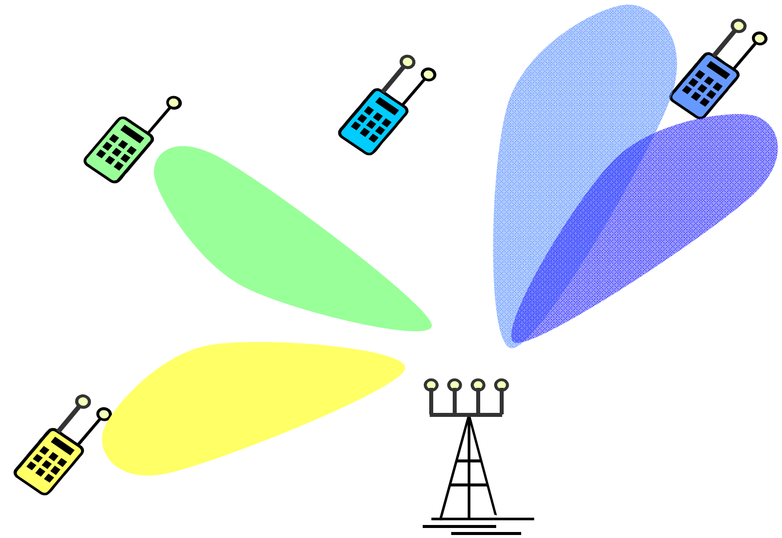
## Single-User (SU) MIMO



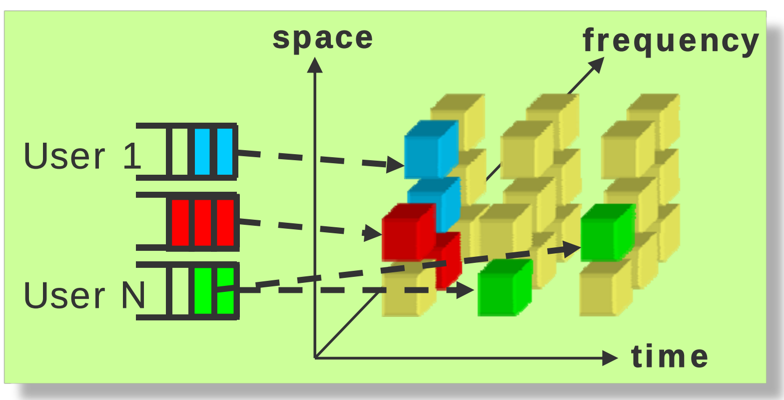
- All spatial dimensions assigned to **one** user.
- Separation of users by TDMA, FDMA.




## Multi-User (MU) MIMO



- Spatial dimensions assigned to **several** users.
- Separation of users by TDMA, FDMA **and** SDMA.



# Advantages of Multi-User MIMO

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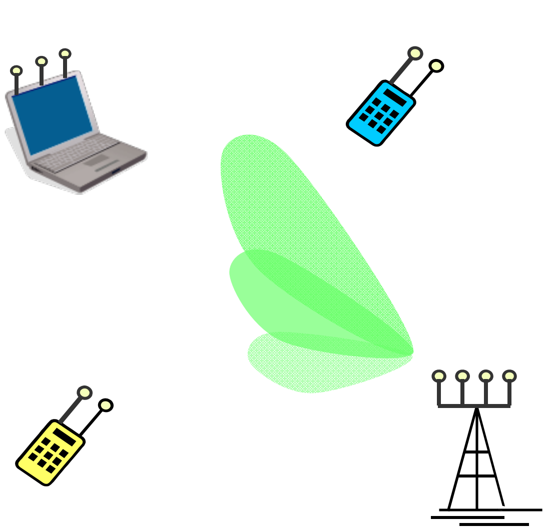
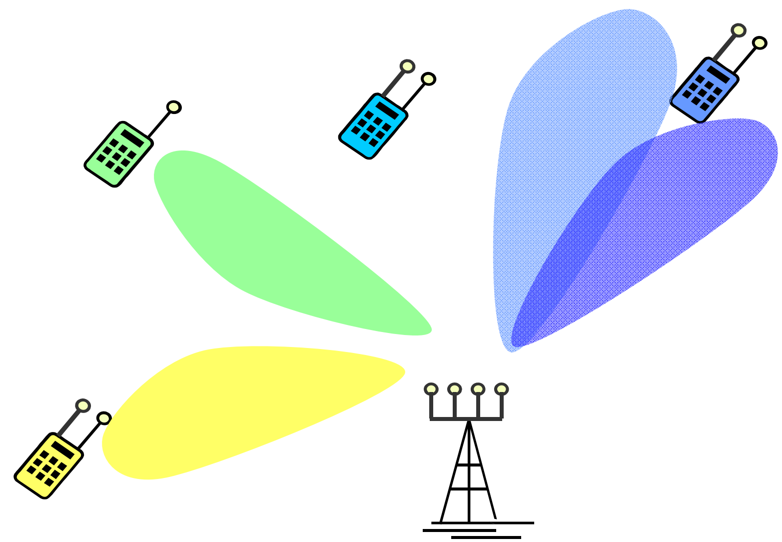
## Single-User (SU) MIMO



- Limited exploitation of multi-user diversity.
- Number of spatial dimensions is limited by number of antennas at UE.

  ⇒ Potential spatial dimensions are wasted if UEs have less antennas than node B.
- Used spatial dimensions may be weak in case of low rank channel (spatial correlation).

## Multi-User (MU) MIMO



- Better exploitation of multi-user diversity.
- Full number of spatial dimensions which is supported by node B can be exploited.

  ⇒ Capacity gain if UEs have less antennas than node B.
- Stronger spatial dimensions are exploited, particularly in case of low rank channel.

Design goals:
- Precoder for spatial user separation
- Scheduler for exploitation of multi-user diversity

# Uplink versus Downlink Multi-User (MU) MIMO

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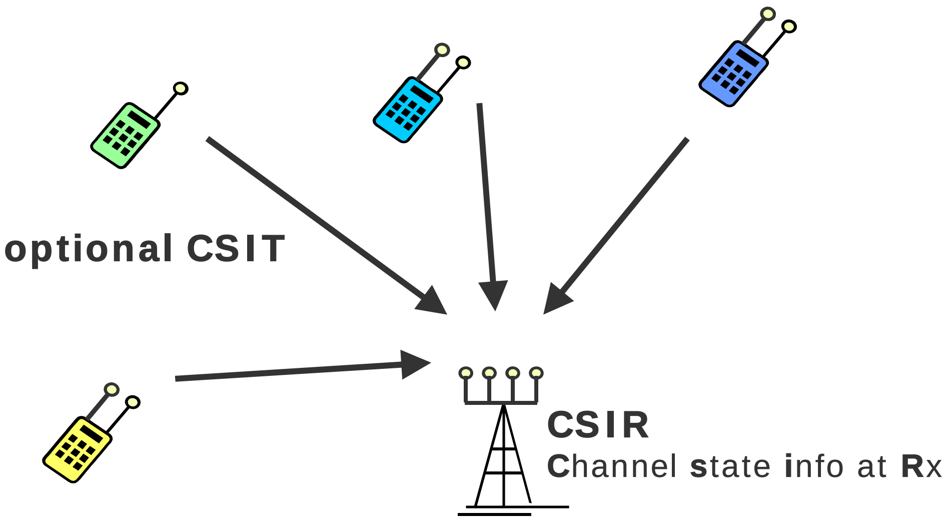
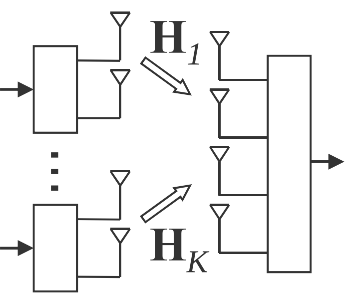
## Uplink MU-MIMO



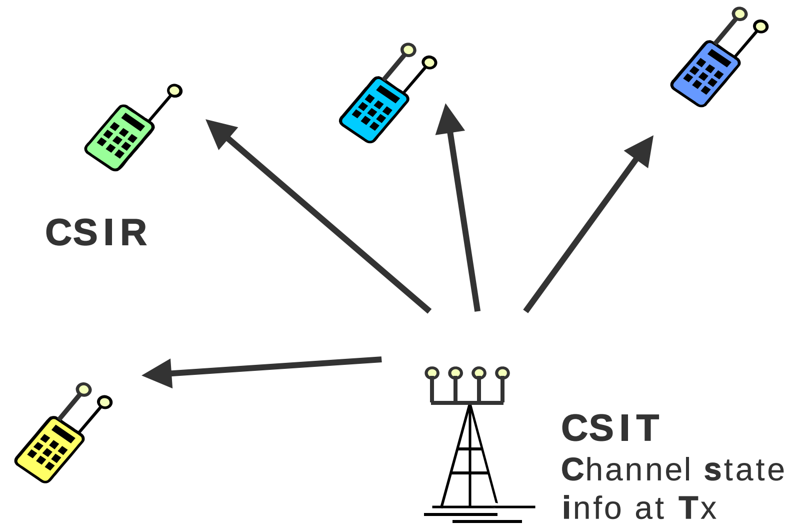

- All Rx antennas can cooperate.
- Tx antennas of different users cannot cooperate.

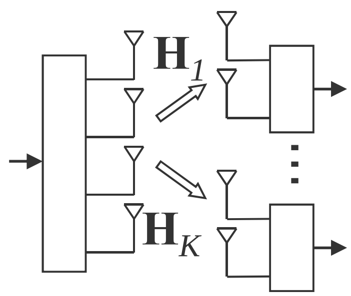
  → Inter-user interference can be resolved at Rx.

  → Same detection methods as in SU-MIMO can be applied.
- Transmit power constraint per user.

**Main challenge: Scheduling**

## Downlink MU-MIMO




- All Tx antennas can cooperate.
- Rx antennas of different users cannot cooperate.

  → Inter-user interference needs to be resolved at Tx.
- Sum transmit power constraint.

**Main challenges:**
- **Precoding for user separation**
- **Scheduling**

# Channel State Information at the Transmitter (CSIT)

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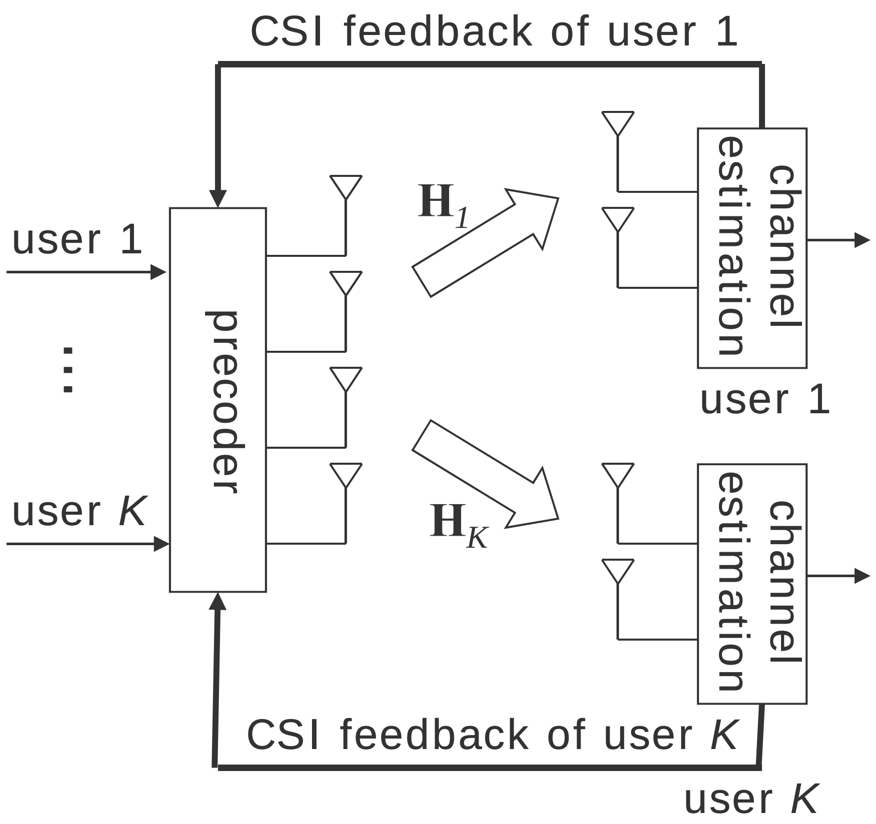
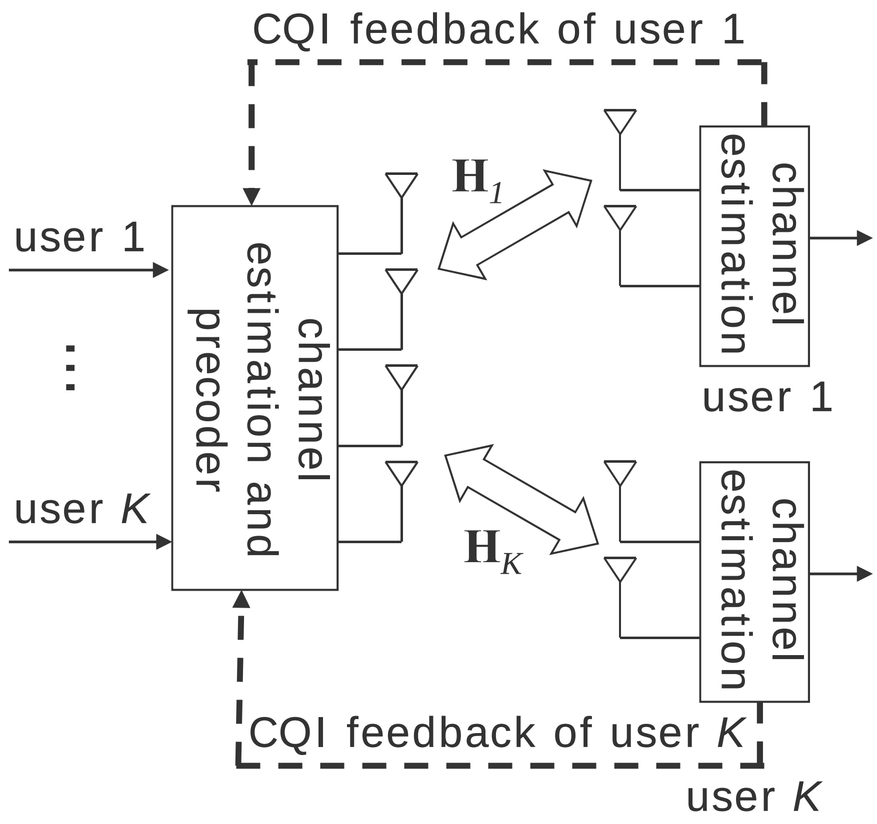
## Frequency Division Duplex (FDD)




Channel state information (CSI) feedback based on estimated

- channel matrix $\mathbf{H}_k$.
- noise power $\sigma_k^2$ .

## Time Division Duplex (TDD)




- Channel matrix $\mathbf{H}_k$ is estimated from uplink signal.
- Channel quality indicator (CQI) feedback based on estimated noise power $\sigma_k^2$.

# Broadcast

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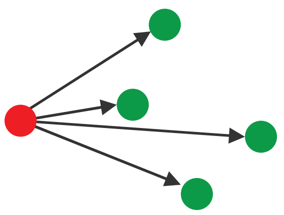
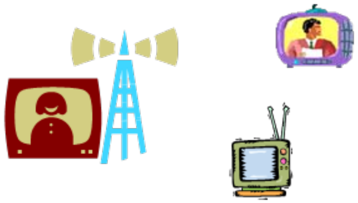
**In computer networking:**

Transmitting packets that will be received by every device on the network. The packets are not requested by the devices but are part of a continuous data stream which is sent through the network.
→ Common information is sent to multiple devices.



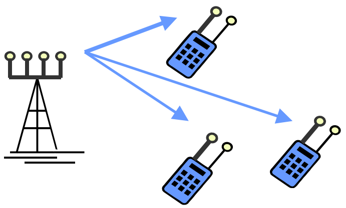
**In common knowledge:**

Distribute information e.g. via television or radio.
(Oxford Advanced Learner's Dictionary: *Send out in all directions especially by radio or TV*)
→ Common information is sent to multiple devices.



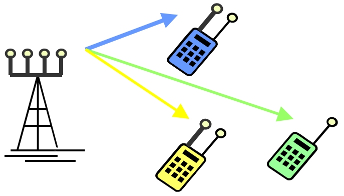
**As service in wireless communications:**

Cell Broadcast in GSM and UMTS is a one-to-many service for simultaneous delivery of messages to multiple users in a specified area.
→ Common information is sent to multiple devices.



**In information theory:**

A broadcast channel is defined by a one-to-many scenario, where independent information is transmitted to multiple users via a common signal.
→ Different information is sent to multiple devices.
Transmission of common information is a special case of a broadcast channel.

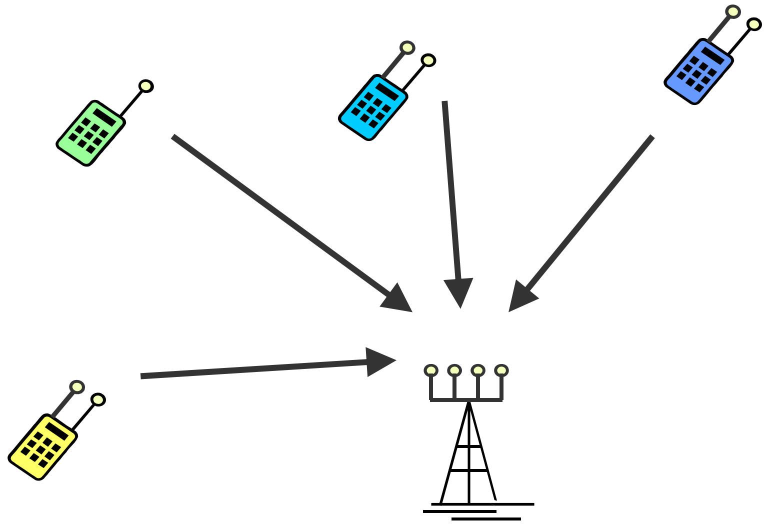


# Multiple Access Channel and Broadcast Channel



## Uplink MU-MIMO

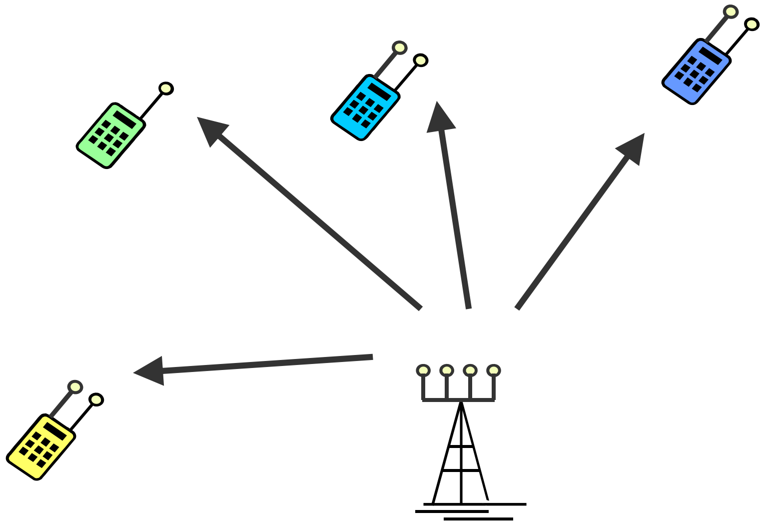
## MIMO Multiple Access Channel



- Independent information is transmitted from multiple users to a common receiver.
- Different users apply independent transmit signal alphabets.
- Transmit power constraint per user.

## Downlink MU-MIMO

## MIMO Broadcast Channel

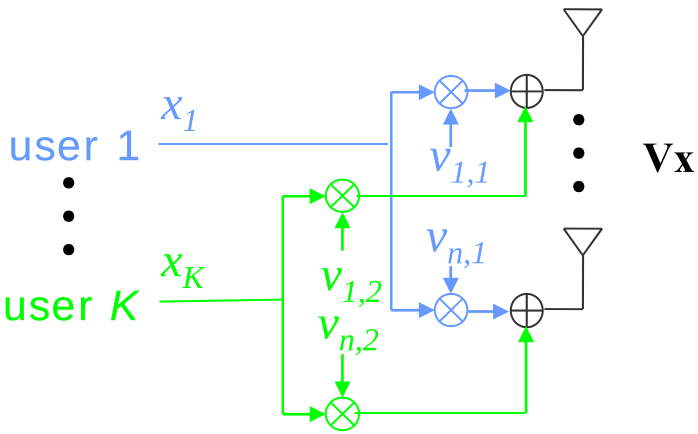


- Independent information is transmitted from a transmitter to multiple users.
- The information for all users is contained in a common signal.
- Total transmit power constraint.

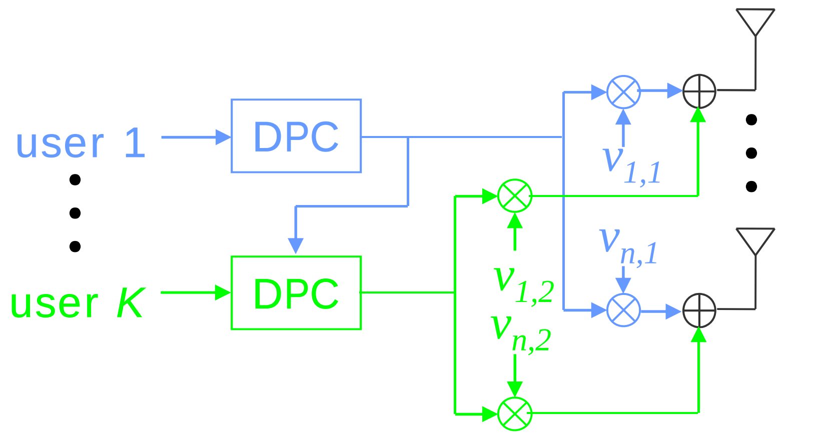
# Linear versus Non-Linear Precoding



## Linear Precoding




## Non-Linear Precoding




$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_K \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_K \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1K} \\ v_{21} & v_{22} & \cdots & v_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_T 1} & v_{n_T 2} & \cdots & v_{n_T K} \end{bmatrix}$$

# Multiuser-MIMO – Linear vs. Non-Linear precoding

## 3GPP-LTE

**Linear Precoding**

- performance degradation
- low complexity
- partial CSI via feedback

## Next Phase of Standardization (IMT-Advanced)

**Nonlinear Precoding**

- reduced complexity
- partial CSI

**Linear Precoding**

- improved precoders
- codebook design

## More Fundamental / Theoretical Work

**Nonlinear Precoding**

- close to capacity
- high complexity
- perfect CSI

e.g. SESAM algorithm

sum capacity, cell throughput, data rate

complexity

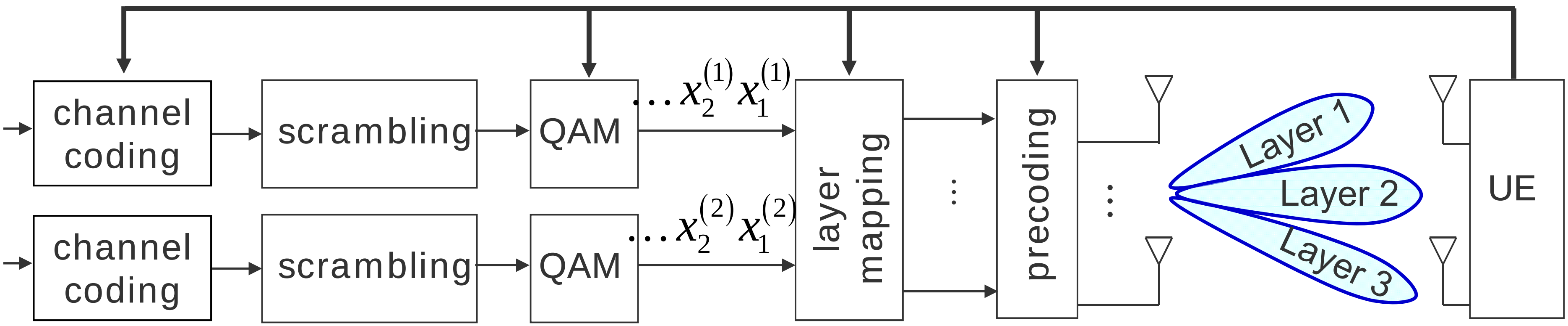
theoretical limit

# Single-User MIMO in 3GPP LTE

- Closed-Loop Downlink MIMO
- Open-Loop Downlink MIMO

# 3GPP-LTE: General Structure for Downlink Physical Channels

| feedback | 2 Tx | 4 Tx |
|---|---|---|
| rank indicator (RI): | 1 bit | 2 bit |
| precoder matrix indicator (PMI) | 2 bit | 4 bit |
| channel quality indicator (CQI) (modulation and coding scheme (MCS)) | 1st codeword: 4 bit<br>2nd codeword: 3 bit (differential) | |




1-2 codewords

QPSK
16 QAM
64 QAM

1-4 layers

2 or 4
Tx antennas

- Rank adaptation
- Spatial multiplexing
- Precoding

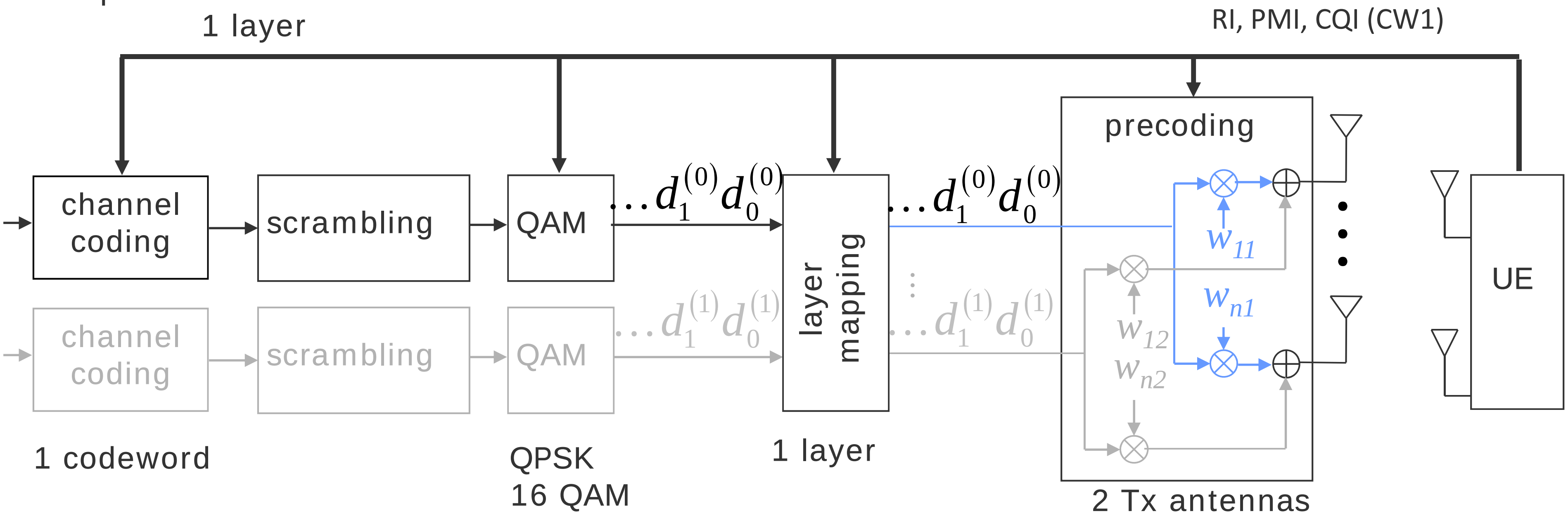
layer: spatial dimension
rank: number of layers

# Closed-Loop-MIMO

## Precoded Spatial Multiplexing: Rank 1

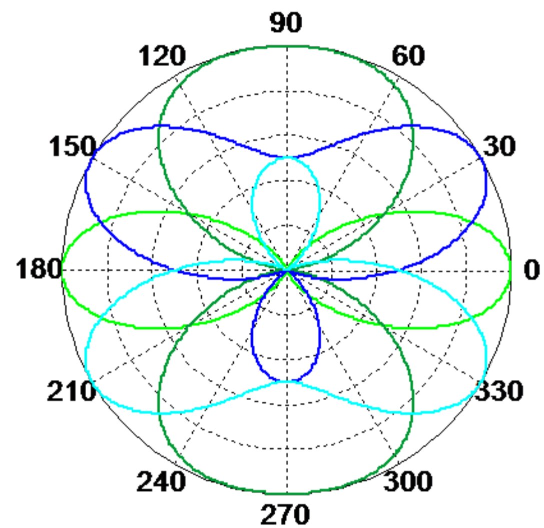


Example: 2 Tx antennas
1 layer




2 Tx antennas:
DFT-based precoder codebook

| index | 1 layer | 2 layers |
|---|---|---|
| 0 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ | $\frac{1}{2}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ |
| 1 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ | $\frac{1}{2}\begin{bmatrix}1 & 1\\j & -j\end{bmatrix}$ |
| 2 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix}$ | |
| 3 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix}$ | |

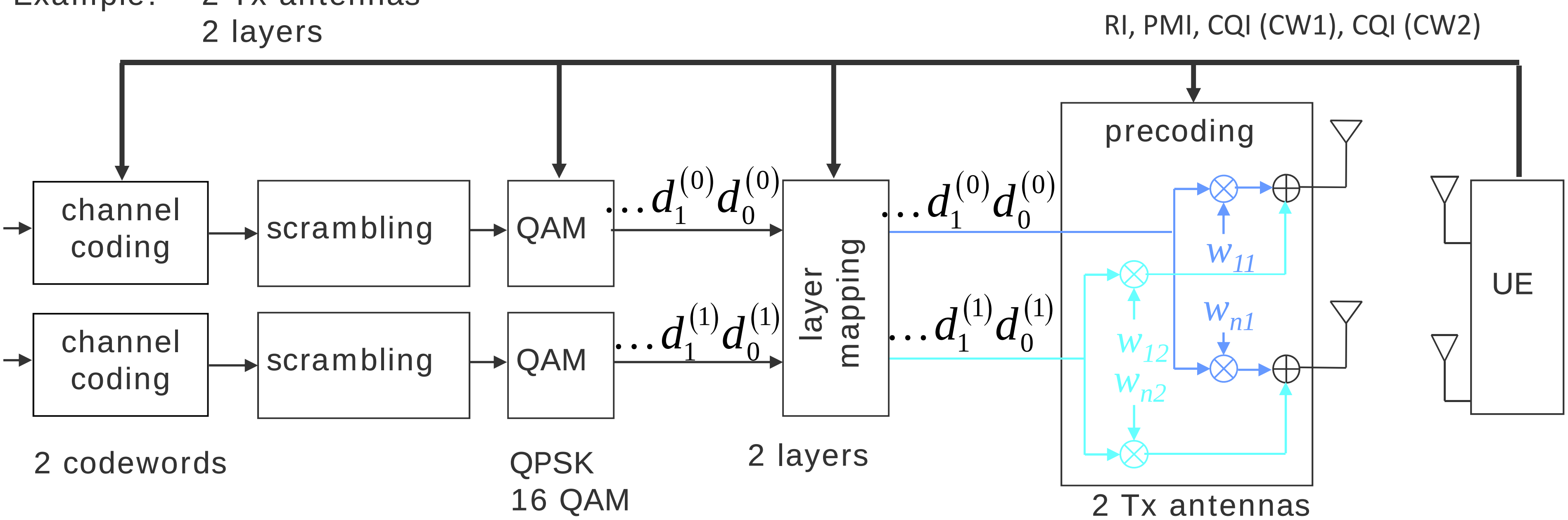


# Closed-Loop-MIMO

## Precoded Spatial Multiplexing: Rank 2

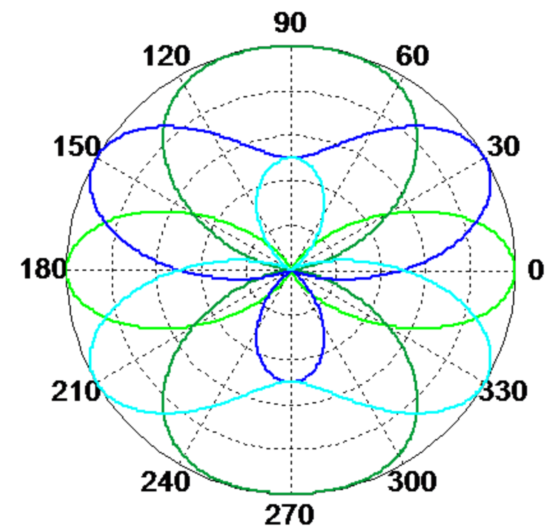


Example: 2 Tx antennas
2 layers




2 Tx antennas:
DFT-based precoder codebook

| index | 1 layer | 2 layers |
|---|---|---|
| 0 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ | $\frac{1}{2}\begin{bmatrix}1&1\\1&-1\end{bmatrix}$ |
| 1 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ | $\frac{1}{2}\begin{bmatrix}1&1\\j&-j\end{bmatrix}$ |
| 2 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix}$ | |
| 3 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix}$ | |



# Closed-Loop-MIMO

## Precoding Beam Patterns: 4 Tx Antennas and Rank 4

### Householder codebook

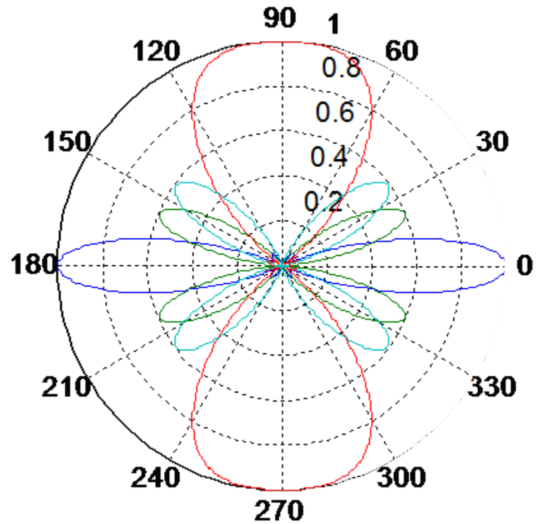
$$\mathbf{W}_n = \mathbf{I}_4 - 2\mathbf{u}_n\mathbf{u}_n^H / \mathbf{u}_n^H\mathbf{u}_n$$

For L layers: Use L columns of $\mathbf{W}_n / \sqrt{L}$

| index | $u_n$ |
|---|---|
| 0 | $u_0 = [1 \quad -1 \quad -1 \quad -1]^T$ |
| 1 | $u_1 = [1 \quad -j \quad 1 \quad j]^T$ |
| 2 | $u_2 = [1 \quad 1 \quad -1 \quad 1]^T$ |
| 3 | $u_3 = [1 \quad j \quad 1 \quad -j]^T$ |
| 4 | $u_4 = [1 \quad (-1-j)/\sqrt{2} \quad -j \quad (1-j)/\sqrt{2}]^T$ |
| 5 | $u_5 = [1 \quad (1-j)/\sqrt{2} \quad j \quad (-1-j)/\sqrt{2}]^T$ |
| 6 | $u_6 = [1 \quad (1+j)/\sqrt{2} \quad -j \quad (-1+j)/\sqrt{2}]^T$ |
| 7 | $u_7 = [1 \quad (-1+j)/\sqrt{2} \quad j \quad (1+j)/\sqrt{2}]^T$ |
| 8 | $u_8 = [1 \quad -1 \quad 1 \quad 1]^T$ |
| 9 | $u_9 = [1 \quad -j \quad -1 \quad -j]^T$ |
| 10 | $u_{10} = [1 \quad 1 \quad 1 \quad -1]^T$ |
| 11 | $u_{11} = [1 \quad j \quad -1 \quad j]^T$ |
| 12 | $u_{12} = [1 \quad -1 \quad -1 \quad 1]^T$ |
| 13 | $u_{13} = [1 \quad -1 \quad 1 \quad -1]^T$ |
| 14 | $u_{14} = [1 \quad 1 \quad -1 \quad -1]^T$ |
| 15 | $u_{15} = [1 \quad 1 \quad 1 \quad 1]^T$ |

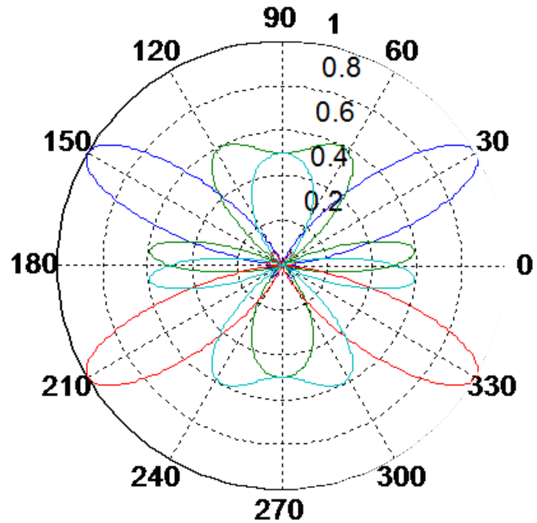
**Example:** $\mathbf{W}_0$:

$$\mathbf{W}_0 = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix}$$



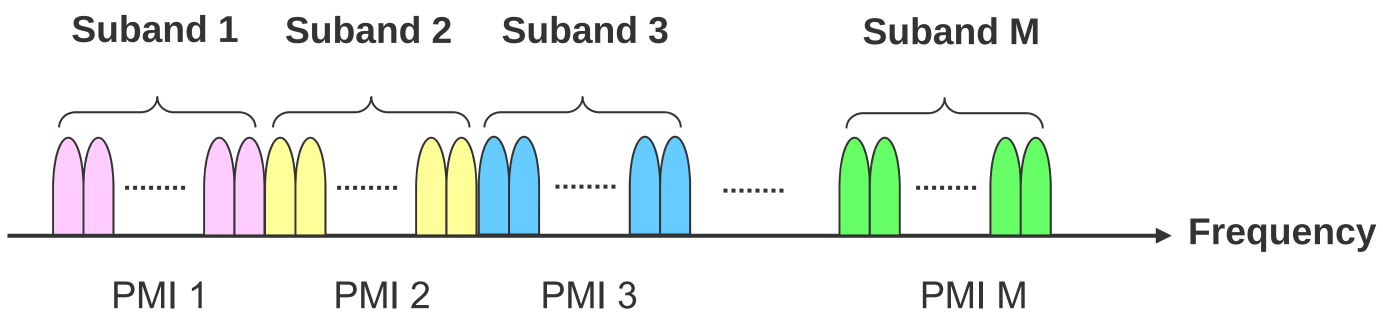

**Example:** $\mathbf{W}_1$:

$$\mathbf{W}_1 = \begin{bmatrix} 0.5 & -j0.5 & -0.5 & j0.5 \\ j0.5 & 0.5 & j0.5 & 0.5 \\ -0.5 & -j0.5 & 0.5 & j0.5 \\ -j0.5 & 0.5 & -j0.5 & 0.5 \end{bmatrix}$$

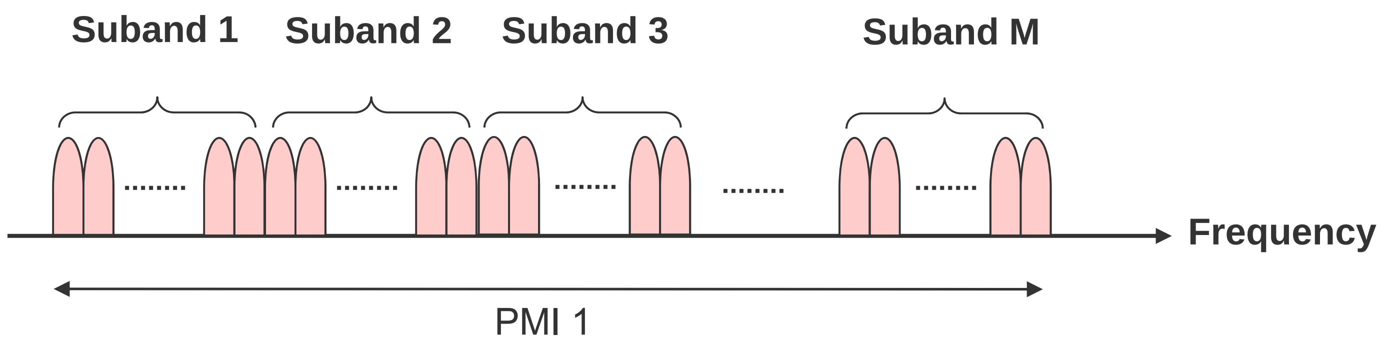


# Closed-Loop MIMO: Precoding Granularity

- **Narrowband precoding**




- **Wideband precoding**

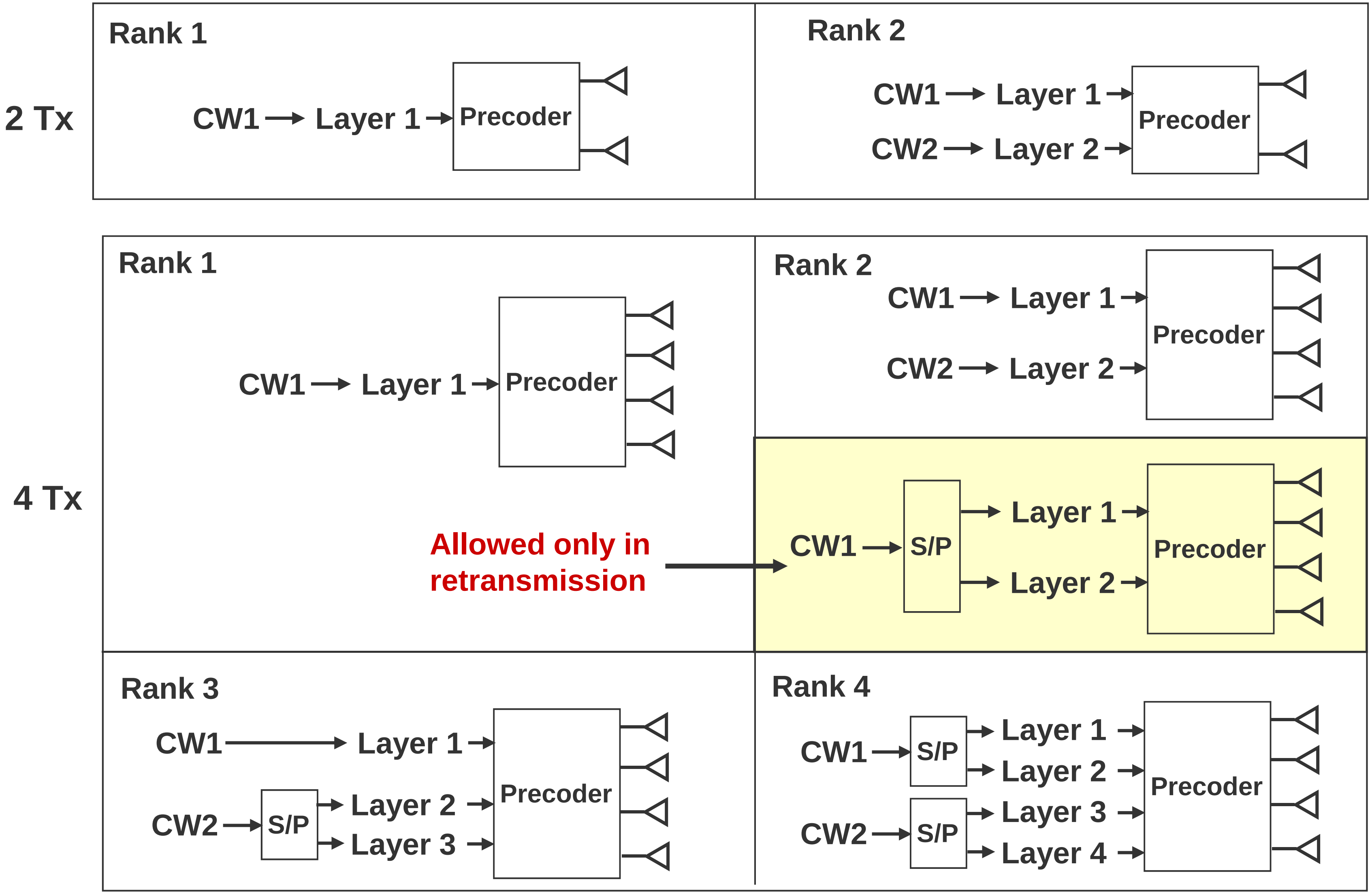



**Both wideband / narrow band precoding are supported**

# Closed-Loop-MIMO: Codeword to Layer Mapping



## LTE CL-MIMO supports maximum 2 codewords (CW)



# Single-User MIMO in 3GPP LTE

- Closed-Loop Downlink MIMO
- Open-Loop Downlink MIMO

# Open-Loop MIMO in LTE



## ◆ Rank = 1 : Transmit diversity

**2 Tx: Space frequency block coding (SFBC)**

Tx antennas → ; Subcarriers ↓

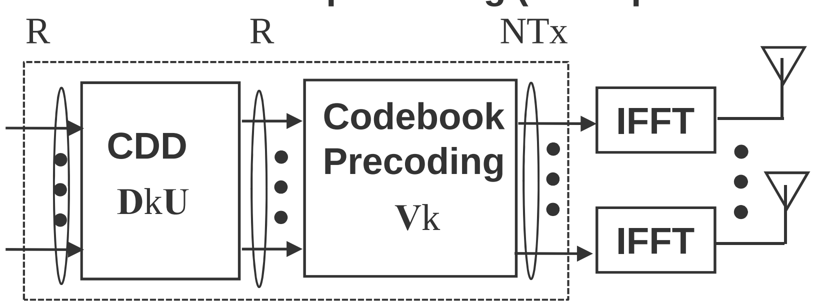
$$\begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$

**4 Tx: SFBC+Frequency-shifted transmit diversity (SFTD)**

Tx antennas → ; Subcarriers ↓

$$\begin{bmatrix} s_1 & 0 & -s_2^* & 0 \\ s_2 & 0 & s_1^* & 0 \\ 0 & s_3 & 0 & -s_4^* \\ 0 & s_4 & 0 & s_3^* \end{bmatrix}$$

## ◆ Rank > 1 : PMI-free precoding (CDD+precoder cycling)




**CDD among virtual antennas**

$$\mathbf{D}_k = \mathrm{diag}\left(1 \quad e^{-j\frac{2\pi}{R}k} \quad \cdots \quad e^{-j\frac{2\pi(R-1)}{R}k}\right) \in R \times R$$

$$\mathbf{U} = \frac{1}{\sqrt{R}}\left[ e^{-j\frac{2\pi}{R}(m-1)(n-1)} \right] \in R \times R \ \ (n,m = 1,\ldots,R)$$

**Precoder cycling**

Precoding matrix at k-th data symbol is

$$\mathbf{V}_k = \mathbf{F}_n \qquad n = \mathrm{mod}\left(\left\lceil \frac{k}{R} \right\rceil - 1, N\right) + 1$$

where,

$\{\mathbf{F}_1 \cdots \mathbf{F}_N\}$ : subset of the codebook $\{\mathrm{Cm}\}$

$$\begin{cases} \{\mathbf{F}_1 \cdots \mathbf{F}_4\} = \{\mathbf{C}_{12} \cdots \mathbf{C}_{15}\} & \text{(4-Tx)} \\ \{\mathbf{F}_1\} = \{\mathbf{C}_0\} & \text{(2-Tx)} \end{cases}$$

- **CDD for inter-layer diversity (CQI averaging)**
- **Precoder cycling for random beamforming**

⇨

- **Increased diversity gain**
- **No PMI feedback**
- **Only one CQI**

# Layer Mapping and Precoding for Rank 1 Transmit Diversity

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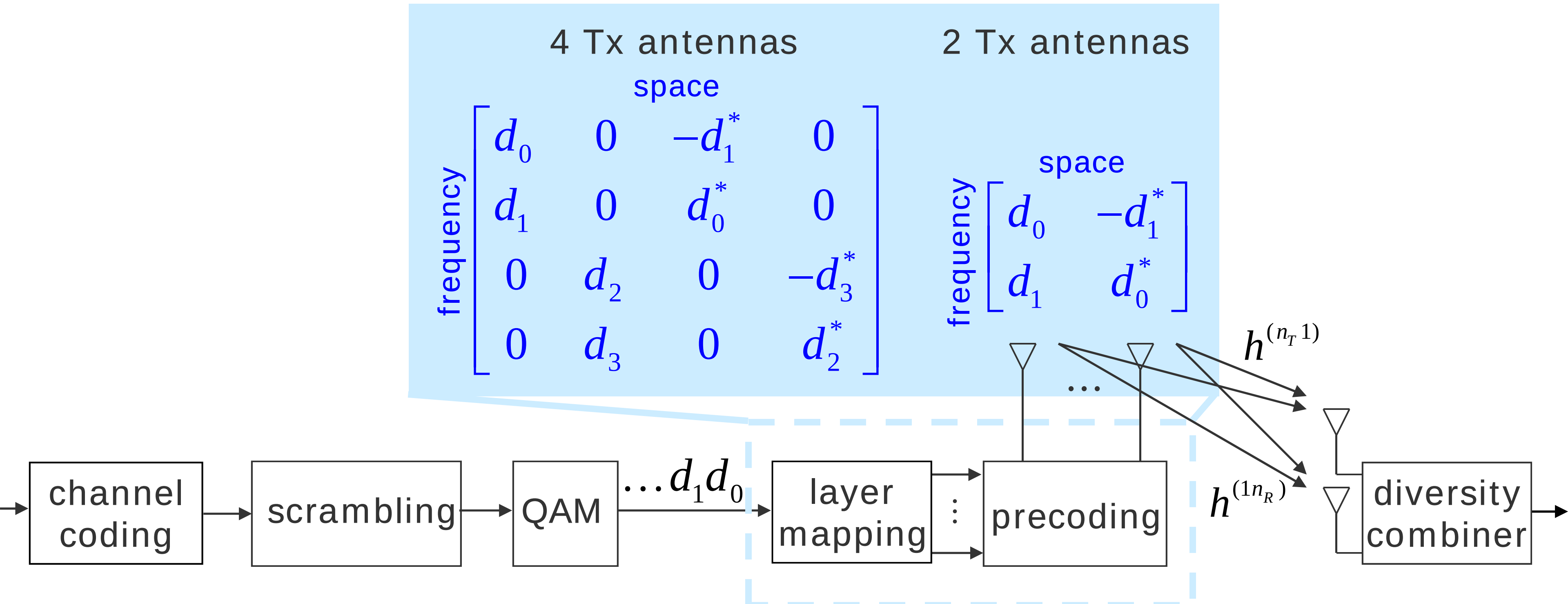
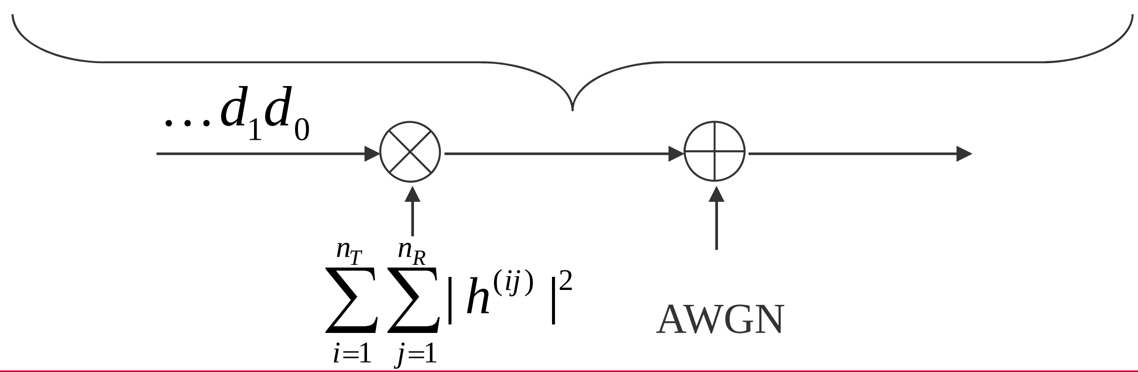
## Alamouti Space-Frequency Transmit Diversity



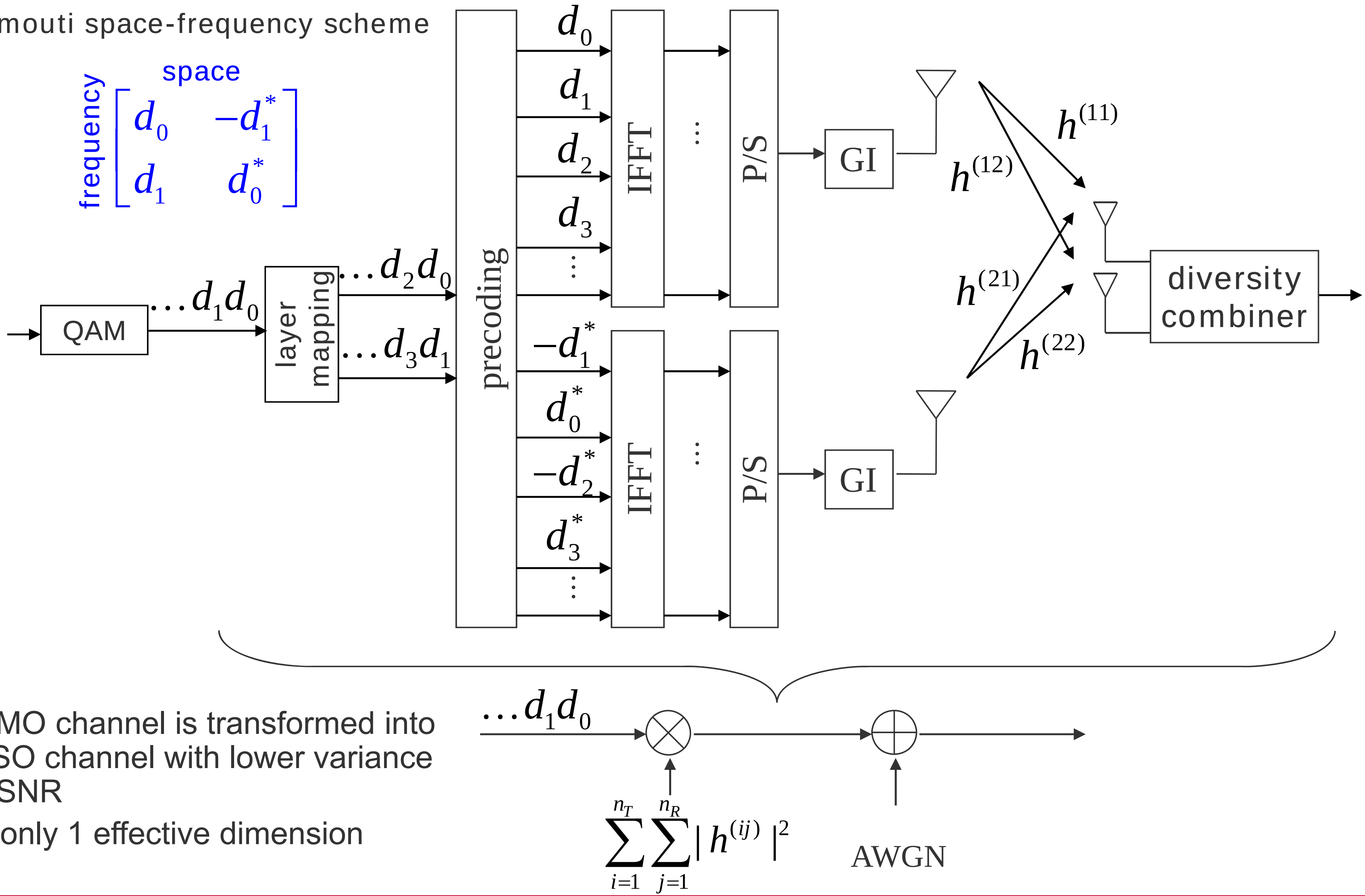




MIMO channel is transformed into SISO channel with lower variance of SNR

→ only 1 effective dimension

# Precoding for Transmit Diversity
## 2 Tx Antennas

Alamouti space-frequency scheme




MIMO channel is transformed into SISO channel with lower variance of SNR

→ only 1 effective dimension

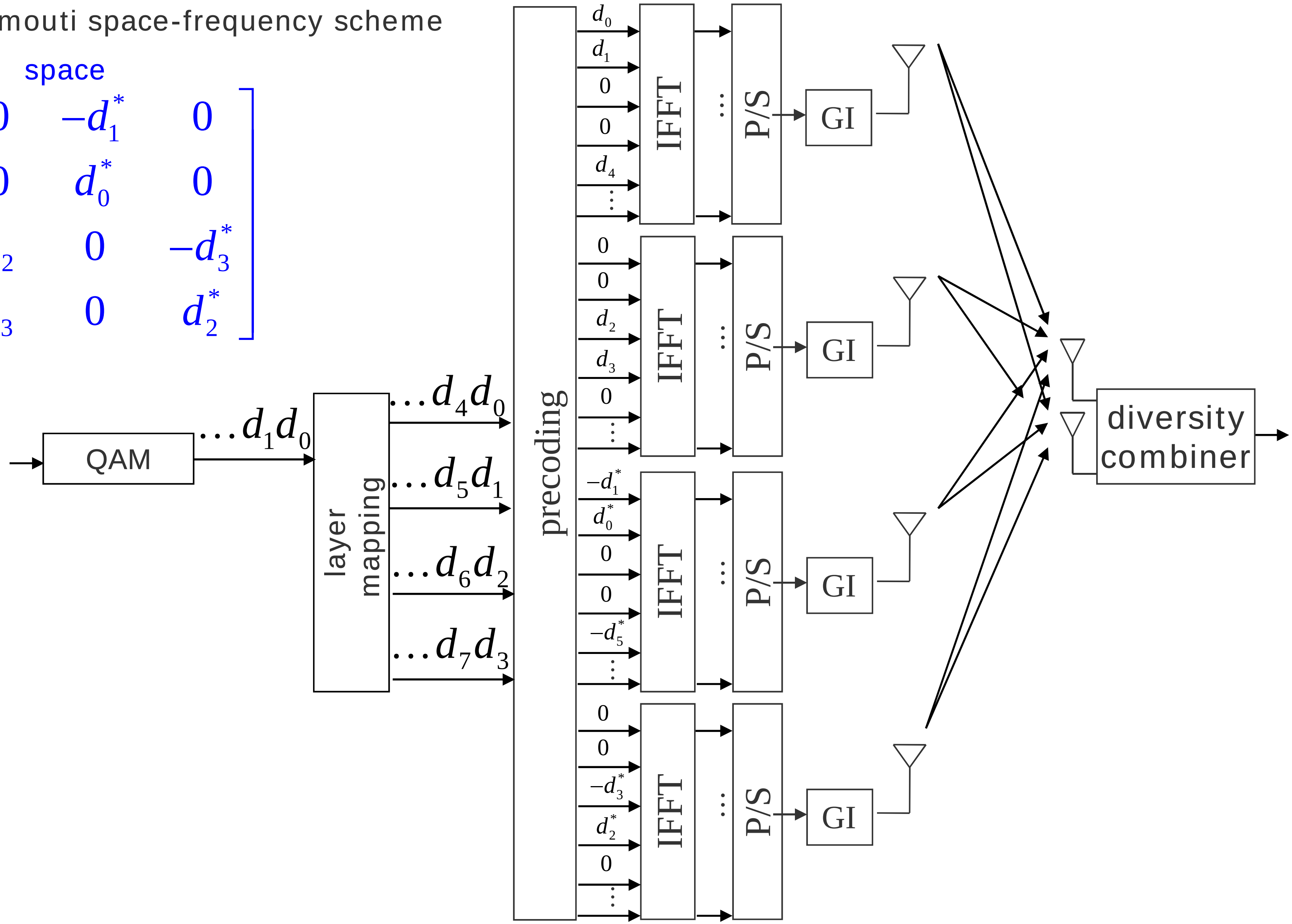
# Precoding for Transmit Diversity
# 4 Tx Antennas

Double Alamouti space-frequency scheme

space

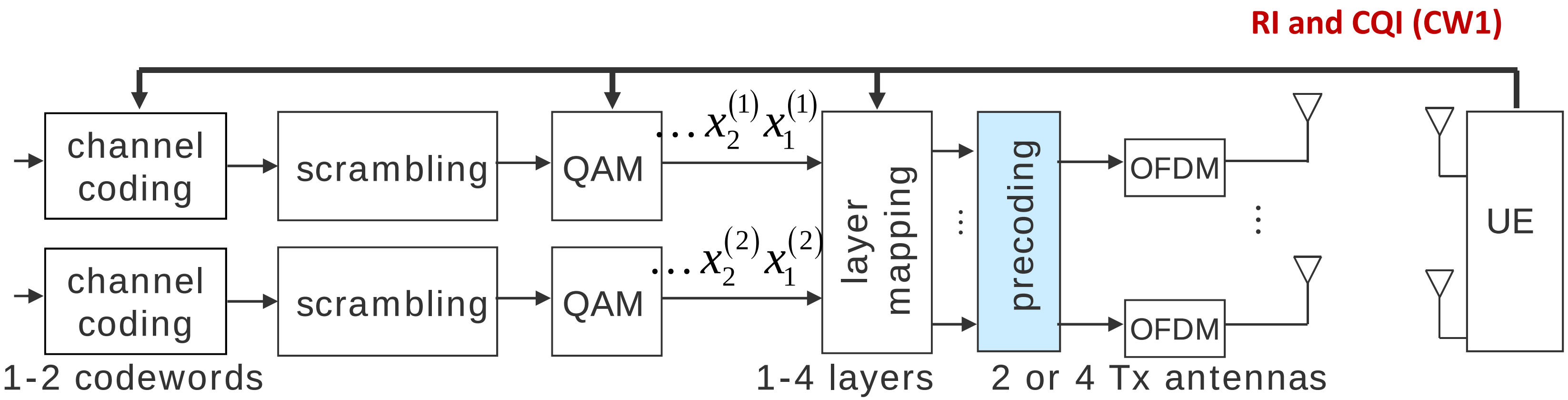
frequency

$$\begin{bmatrix} d_0 & 0 & -d_1^* & 0 \\ d_1 & 0 & d_0^* & 0 \\ 0 & d_2 & 0 & -d_3^* \\ 0 & d_3 & 0 & d_2^* \end{bmatrix}$$



# Open-Loop MIMO: Structure (Rank > 1)

CDD: Cyclic delay diversity



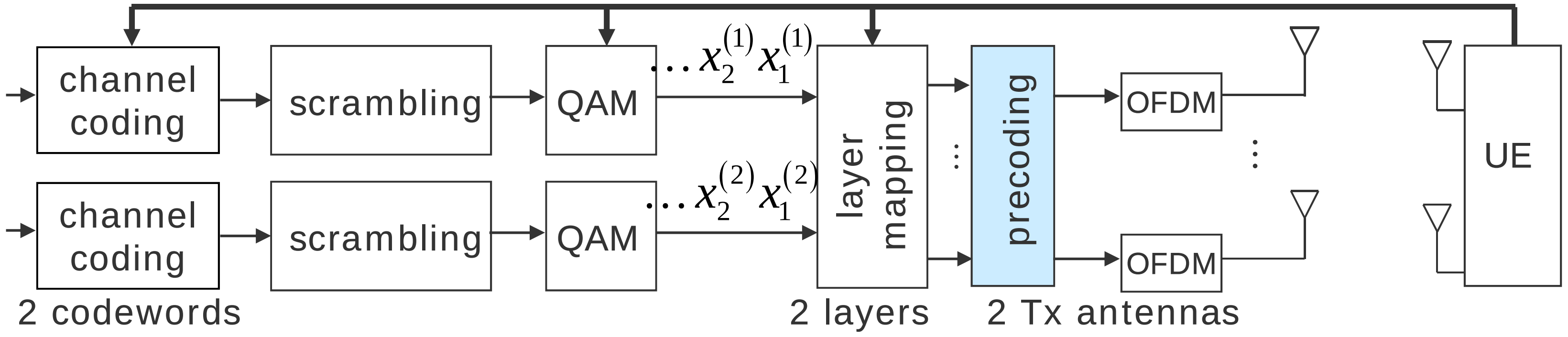

Precoder: $\mathbf{W}(i)\mathbf{D}(i)\mathbf{U}$ , where i corresponds to symbol index

| layers | $\mathbf{U}$ | $\mathbf{D}(i)$ | $\mathbf{W}(i)$ |
|---|---|---|---|
| 2 | $\begin{bmatrix}1 & 1\\ 1 & e^{-j2\pi/2}\end{bmatrix}$ | $\begin{bmatrix}1 & 0\\ 0 & e^{-j2\pi i/2}\end{bmatrix}$ | 2 Tx: $\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ \| 4Tx: $\mathbf{W}_n = \mathbf{I}_4 - 2\mathbf{u}_n\mathbf{u}_n^H / \mathbf{u}_n^H\mathbf{u}_n,\; n = 12,...,15$ |
| 3 | $\begin{bmatrix}1 & 1 & 1\\ 1 & e^{-j2\pi/3} & e^{-j4\pi/3}\\ 1 & e^{-j4\pi/3} & e^{-j8\pi/3}\end{bmatrix}$ | $\begin{bmatrix}1 & 0 & 0\\ 0 & e^{-j2\pi i/3} & 0\\ 0 & 0 & e^{-j4\pi i/3}\end{bmatrix}$ | $\mathbf{W}_n = \mathbf{I}_4 - 2\mathbf{u}_n\mathbf{u}_n^H / \mathbf{u}_n^H\mathbf{u}_n,\; n = 12,...,15$ Householder codebook |
| 4 | $\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & e^{-j2\pi/4} & e^{-j4\pi/4} & e^{-j6\pi/4}\\ 1 & e^{-j4\pi/4} & e^{-j8\pi/4} & e^{-j12\pi/4}\\ 1 & e^{-j6\pi/4} & e^{-j12\pi/4} & e^{-j18\pi/4}\end{bmatrix}$ | $\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & e^{-j2\pi i/4} & 0 & 0\\ 0 & 0 & e^{-j4\pi i/4} & 0\\ 0 & 0 & 0 & e^{-j6\pi i/4}\end{bmatrix}$ | $\mathbf{W}_n = \mathbf{I}_4 - 2\mathbf{u}_n\mathbf{u}_n^H / \mathbf{u}_n^H\mathbf{u}_n,\; n = 12,...,15$ Householder codebook |

# Open-Loop MIMO: Structure: 2Tx and Rank=2



CDD: Cyclic delay diversity

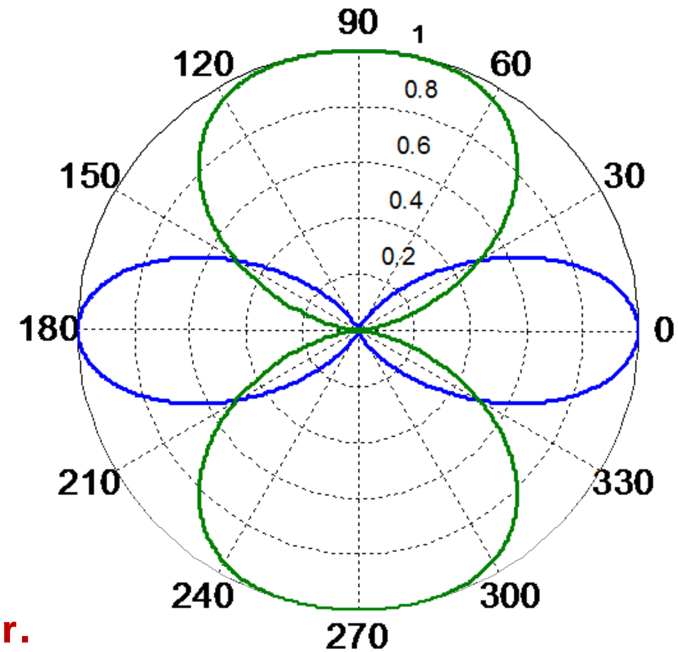



Precoder: $\mathbf{W}(i)\mathbf{D}(i)\mathbf{U}$ , where i corresponds to subcarrier index

**Example: 2 Tx antennas, 2 layers:**

| layers | $\mathbf{U}$ | $\mathbf{D}(i)$ | $\mathbf{W}(i)$ |
|---|---|---|---|
| 2 | $\begin{bmatrix} 1 & 1 \\ 1 & e^{-j2\pi/2} \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 \\ 0 & e^{-j2\pi i/2} \end{bmatrix}$ | $\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ |

$$\mathbf{D}(i)\mathbf{U} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j2\pi i/2} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & e^{-j2\pi/2} \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & \text{for even } i \\ \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} & \text{for odd } i \end{cases}$$

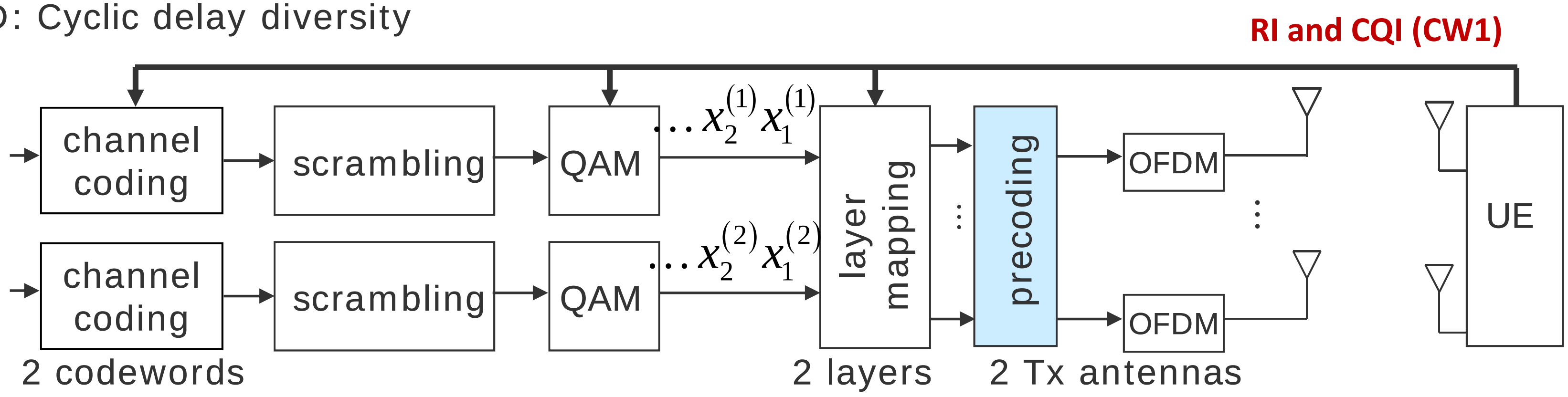



$\Rightarrow$ **Beams are switched on every other subcarrier.**

# Open-Loop MIMO: Structure: 2Tx and Rank=2



CDD: Cyclic delay diversity



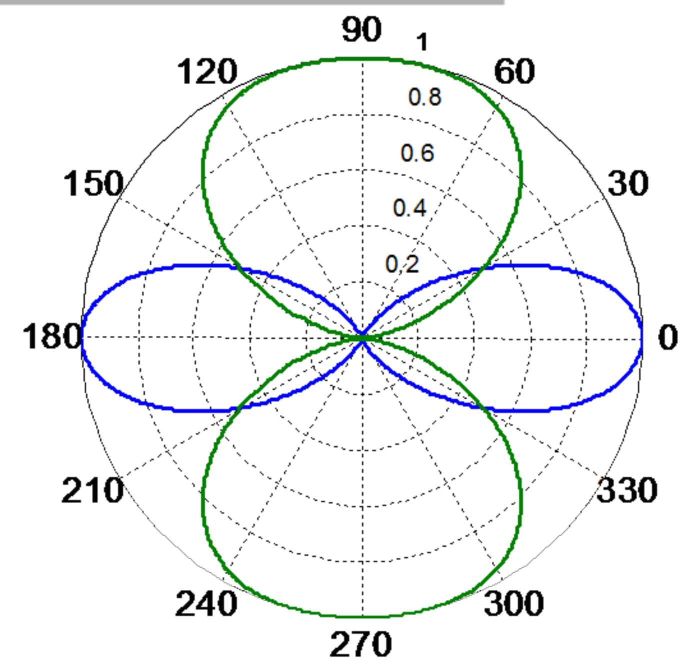

Precoder: $\mathbf{W}(i)\mathbf{D}(i)\mathbf{U}$, where i corresponds to subcarrier index

**Example: 2 Tx antennas, 2 layers:**

$$\begin{bmatrix} y_k^{(1)}(i) \\ \vdots \\ y_k^{(n_T)}(i) \end{bmatrix} = \underbrace{\begin{bmatrix} H_k^{(11)}(i) & H_k^{(1n_R)}(i) \\ H_k^{(1n_R)}(i) & H_k^{(2n_R)}(i) \end{bmatrix} \mathbf{W}(i)\mathbf{D}(i)\mathbf{U}} \begin{bmatrix} x_k^{(1)}(i) \\ x_k^{(2)}(i) \end{bmatrix}$$

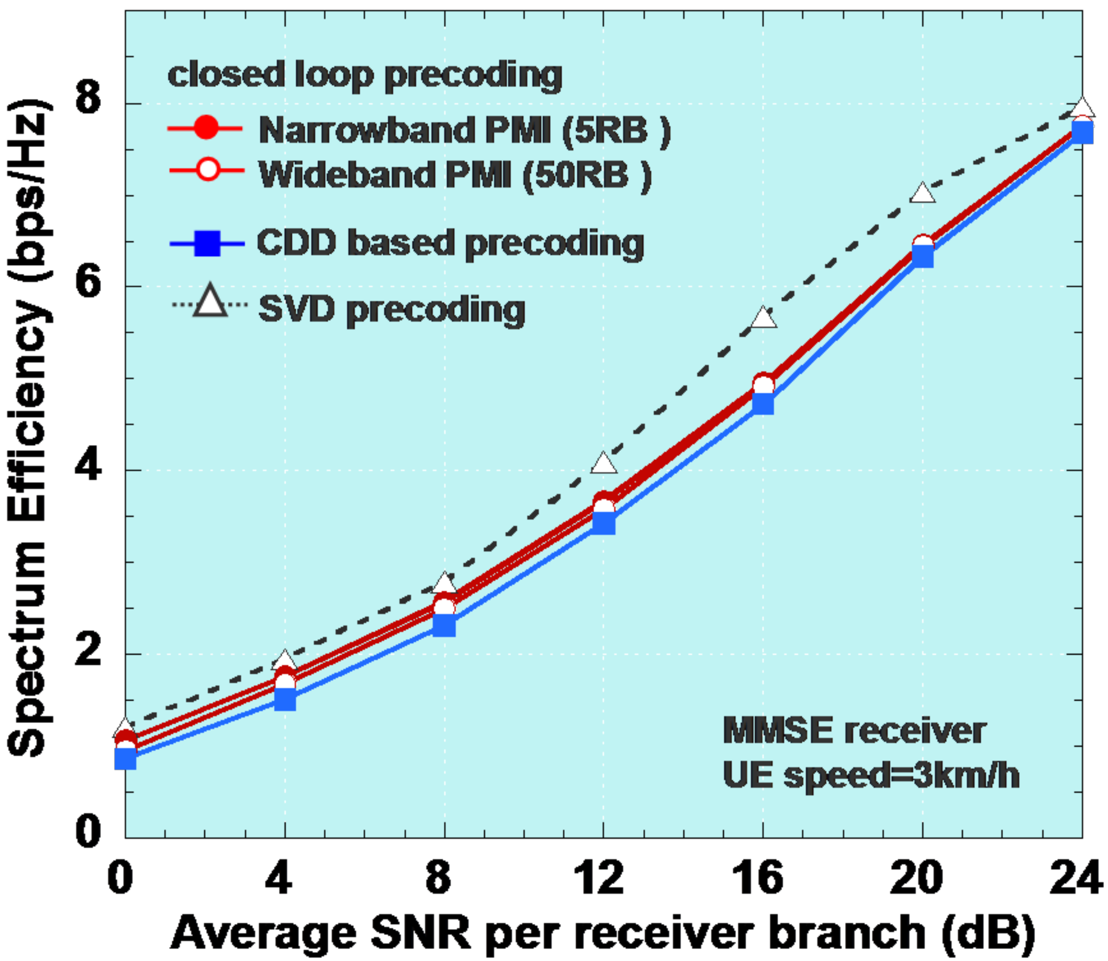
$$\begin{bmatrix} H_k^{(11)}(i)+H_k^{(21)}(i) & H_k^{(11)}(i)-H_k^{(21)}(i) \\ \vdots & \vdots \\ H_k^{(1n_R)}(i)+H_k^{(2n_R)}(i) & H_k^{(1n_R)}(i)-H_k^{(2n_R)}(i) \end{bmatrix} \quad \text{for even } i$$

$$\begin{bmatrix} H_k^{(11)}(i)-H_k^{(21)}(i) & H_k^{(11)}(i)+H_k^{(21)}(i) \\ \vdots & \vdots \\ H_k^{(1n_R)}(i)-H_k^{(2n_R)}(i) & H_k^{(1n_R)}(i)+H_k^{(2n_R)}(i) \end{bmatrix} \quad \text{for odd } i$$



# Open-Loop vs. Closed-Loop Precoding in LTE

- 2 Tx, 2 Rx antennas
- Bandwidth: 10 MHz
- Carrier frequency: 2 GHz
- Subcarrier spacing: 15 kHz
- FFT size: 1024
- 12 subcarriers/resource block
- 10 resource blocks/user
- Channel model:
- spatially uncorrelated six-ray typical urban
- Adaptive coding and modulation:
- R=1/16-12/13
- QPSK, 16-QAM, 64-QAM



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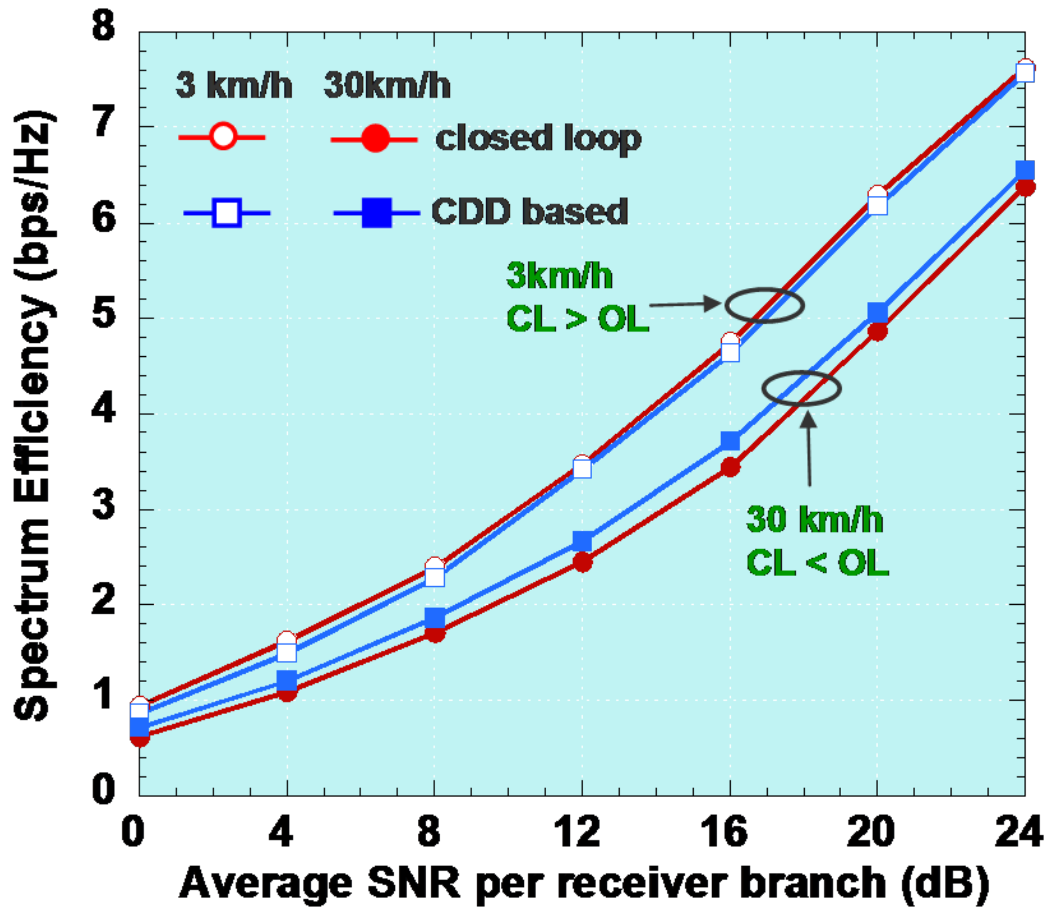
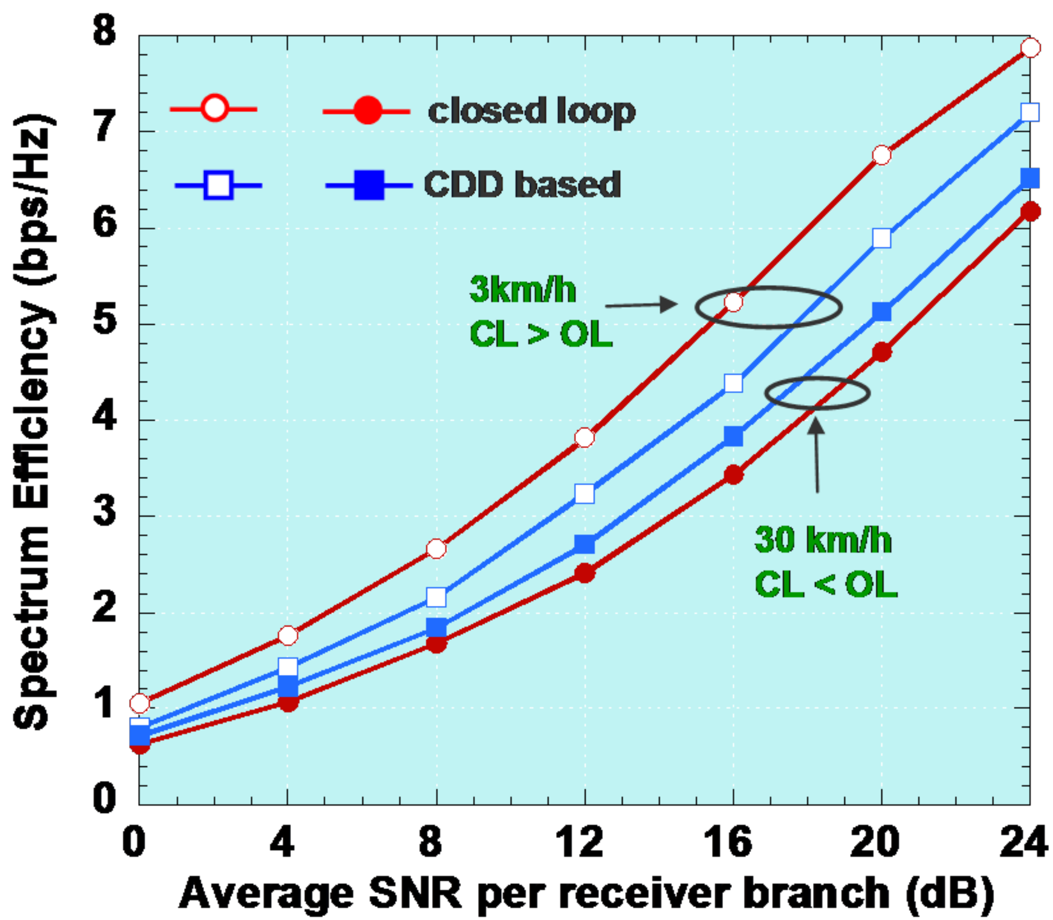


# Open-Loop vs. Closed-Loop Precoding in LTE

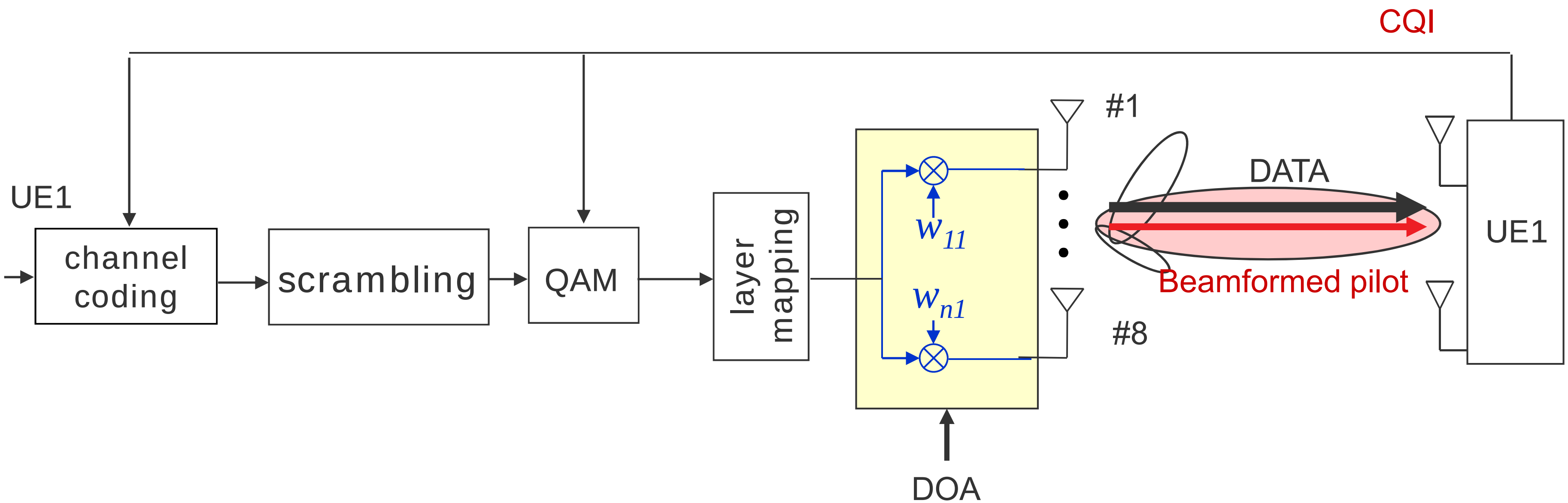


- 4 Tx, 2 Rx antennas
- Bandwidth: 10 MHz
- Carrier frequency: 2 GHz
- Subcarrier spacing: 15 kHz
- FFT size: 1024
- 12 subcarriers/resource block
- 10 resource blocks/user
- Channel model:
- spatially uncorrelated six-ray typical urban
- Adaptive coding and modulation:
- R=1/16-12/13
- QPSK, 16-QAM, 64-QAM



# Downlink Rank1-beamforming






- Rank1
- Only CQI feedback is needed.
- Specification is only UE specific reference.

  ⇒ Design freedom in transmitter

  One good example is DOA-based non-codebook beamforming with more than 4 Tx antennas

# Outline (1)

DOCOMO Euro-Labs

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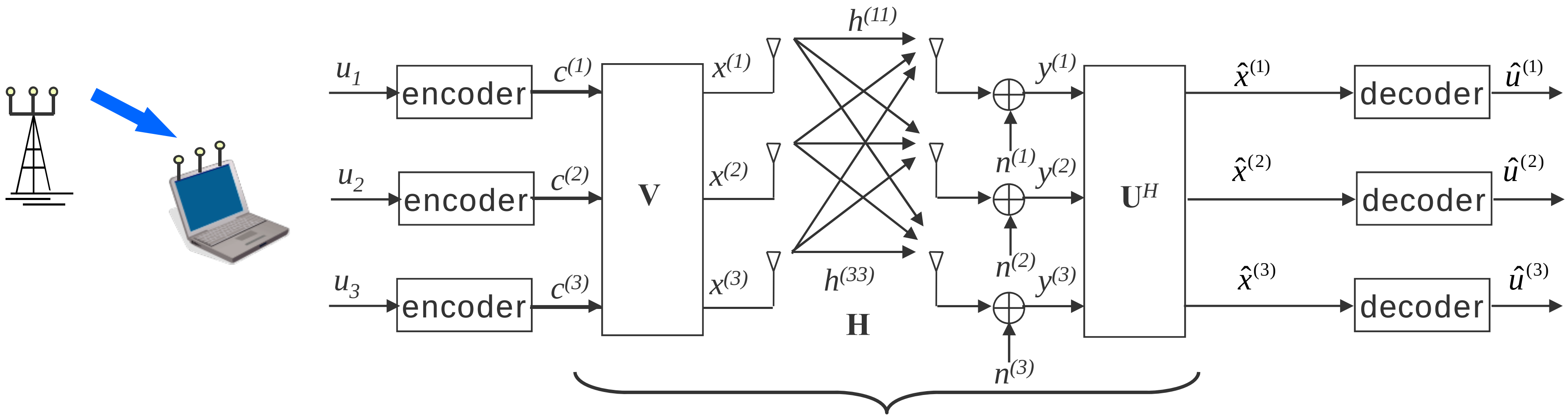
- Introduction
  - MIMO capacity
  - Single-user (SU) MIMO versus multi-user (MU) MIMO
  - Uplink MU-MIMO versus downlink MU-MIMO
  - Linear versus non-linear MU-MIMO
  - Multiuser diversity

- Single-user MIMO
  - Single-user MIMO in 3GPP Long-Term-Evolution (LTE)
  - Spatial multiplexing with Rx and Tx processing

- Theoretical fundamentals
  - Introduction to Dirty Paper Coding (DPC)
  - Tomlinson-Harashima precoding (THP)
  - Precoding for the MIMO broadcast channel

# Spatial Multiplexing with Singular Value Decomposition





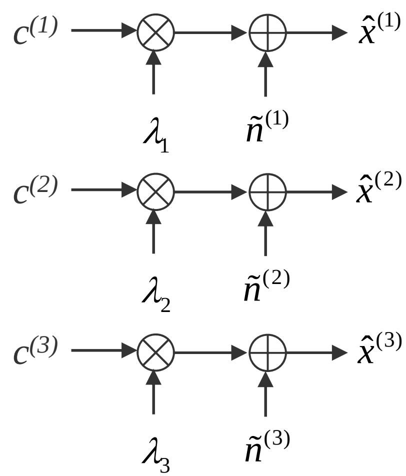

$$\begin{bmatrix} y^{(1)} \\ y^{(2)} \\ y^{(3)} \end{bmatrix} = \begin{bmatrix} h^{(11)} & h^{(21)} & h^{(31)} \\ h^{(12)} & h^{(22)} & h^{(32)} \\ h^{(13)} & h^{(23)} & h^{(33)} \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix} + \begin{bmatrix} n^{(1)} \\ n^{(2)} \\ n^{(3)} \end{bmatrix}$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

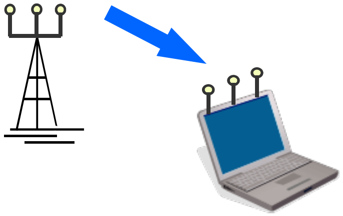
$$\mathbf{H} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^H$$

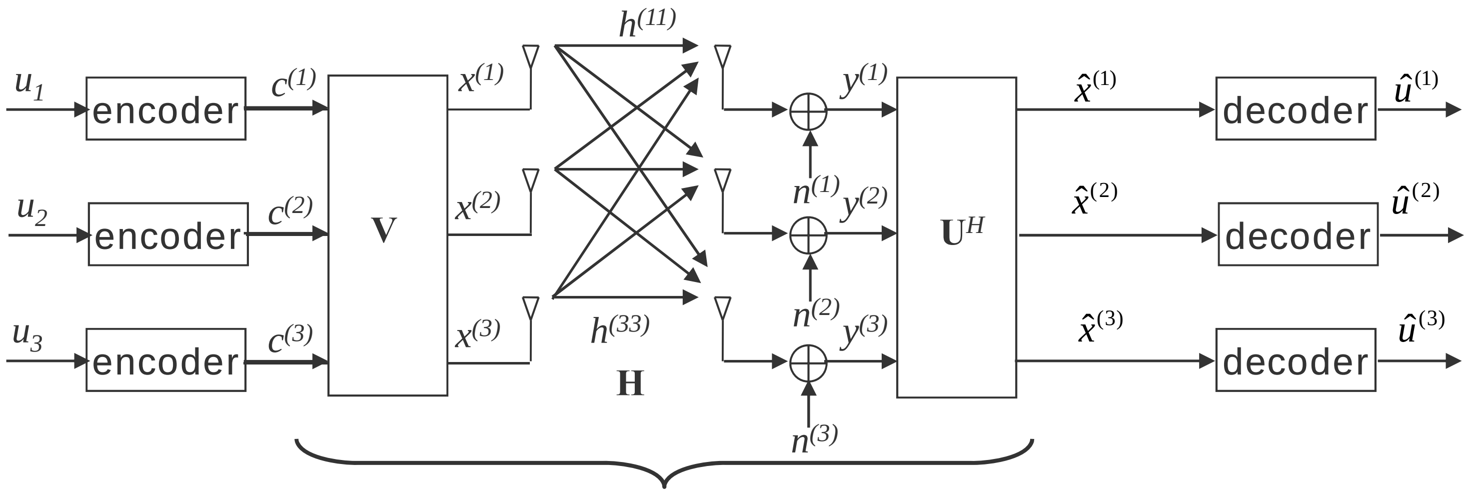
$$\mathbf{U}^H\mathbf{y} = \mathbf{U}^H\mathbf{H}\mathbf{V}\mathbf{c} + \mathbf{U}^H\mathbf{n} == \underbrace{\mathbf{U}^H\mathbf{U}}_{\mathbf{I}_{n_R}}\boldsymbol{\Lambda}\underbrace{\mathbf{V}^H\mathbf{V}}_{\mathbf{I}_{n_T}}\mathbf{c} + \tilde{\mathbf{n}}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

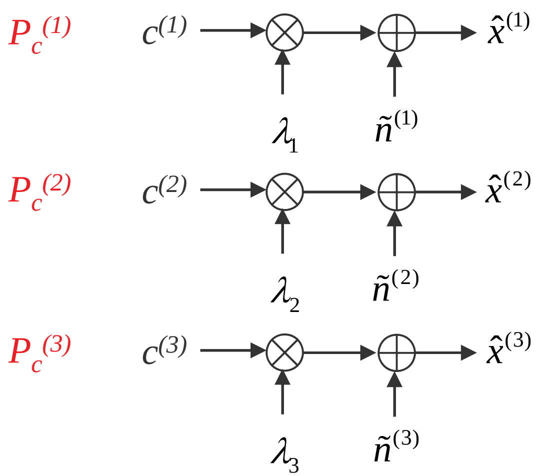


# Spatial Multiplexing with Singular Value Decomposition






Optimum power allocation to parallel subchannels: Waterfilling



# Waterfilling



Capacity of $M$ parallel AWGN channels with channel knowledge at the transmitter:

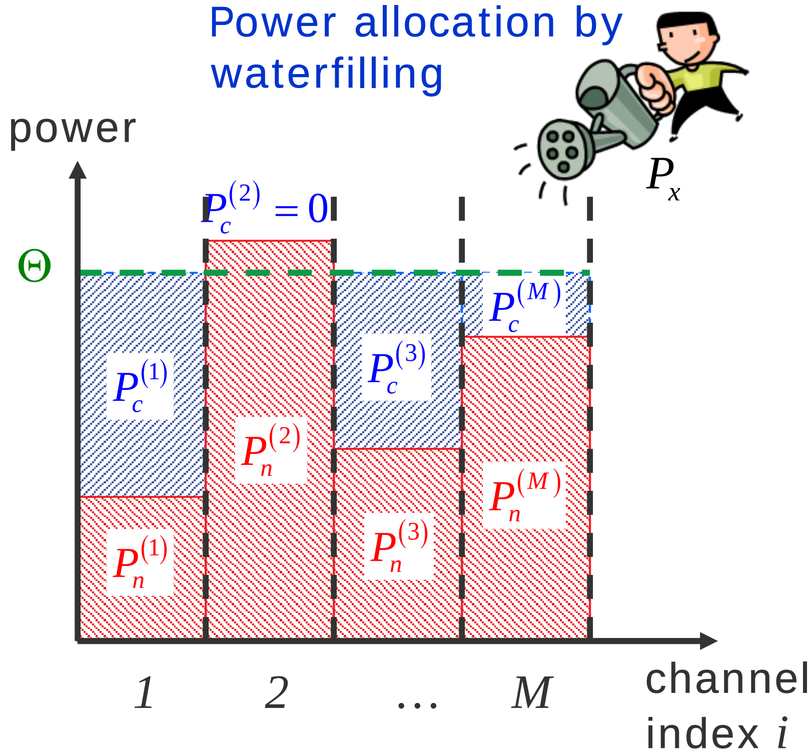
$$C = \sum_{i=1}^{M} \log_2\left(1 + \frac{P_c^{(i)}}{P_n^{(i)}}\right)$$

where

$$P_c^{(i)} = \left(\Theta - P_n^{(i)}\right)^+ = \begin{cases} \Theta - P_n^{(i)}, & \text{for } \Theta - P_n^{(i)} > 0 \\ 0, & \text{otherwise} \end{cases}$$

and $\Theta$ is the solution of the waterfilling problem

$$\sum_{i=1}^{M} \left(\Theta - P_n^{(i)}\right)^+ = P_c$$




$P_n^{(i)}$ : Noise power in channel $i$.

$P_c^{(i)}$ : Transmit power which is allocated to channel $i$ according to a waterfilling solution.

$P_c = \sum_{i=1}^{M} P_c^{(i)}$ : Total transmit power.

# Bell-Labs Layered Space-Time Architecture: V-BLAST with QR-Decomposition

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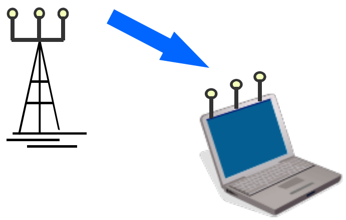
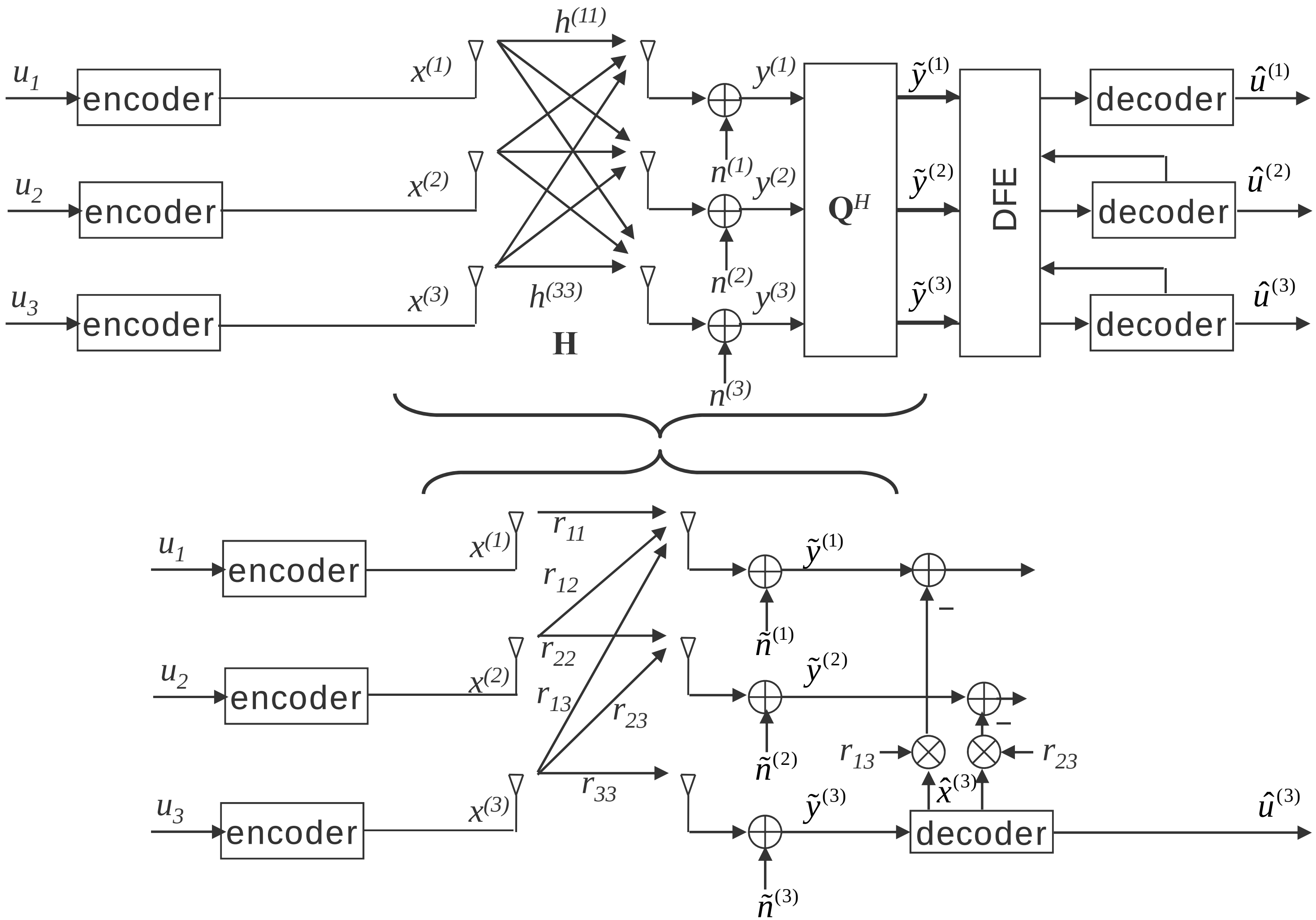





$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\mathbf{H} = \mathbf{Q}\mathbf{R}$$

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$$

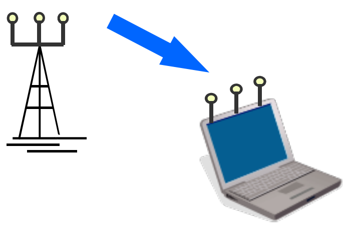
$$\tilde{\mathbf{y}} = \mathbf{Q}^H\mathbf{H}\mathbf{x} + \mathbf{Q}^H\mathbf{n}$$

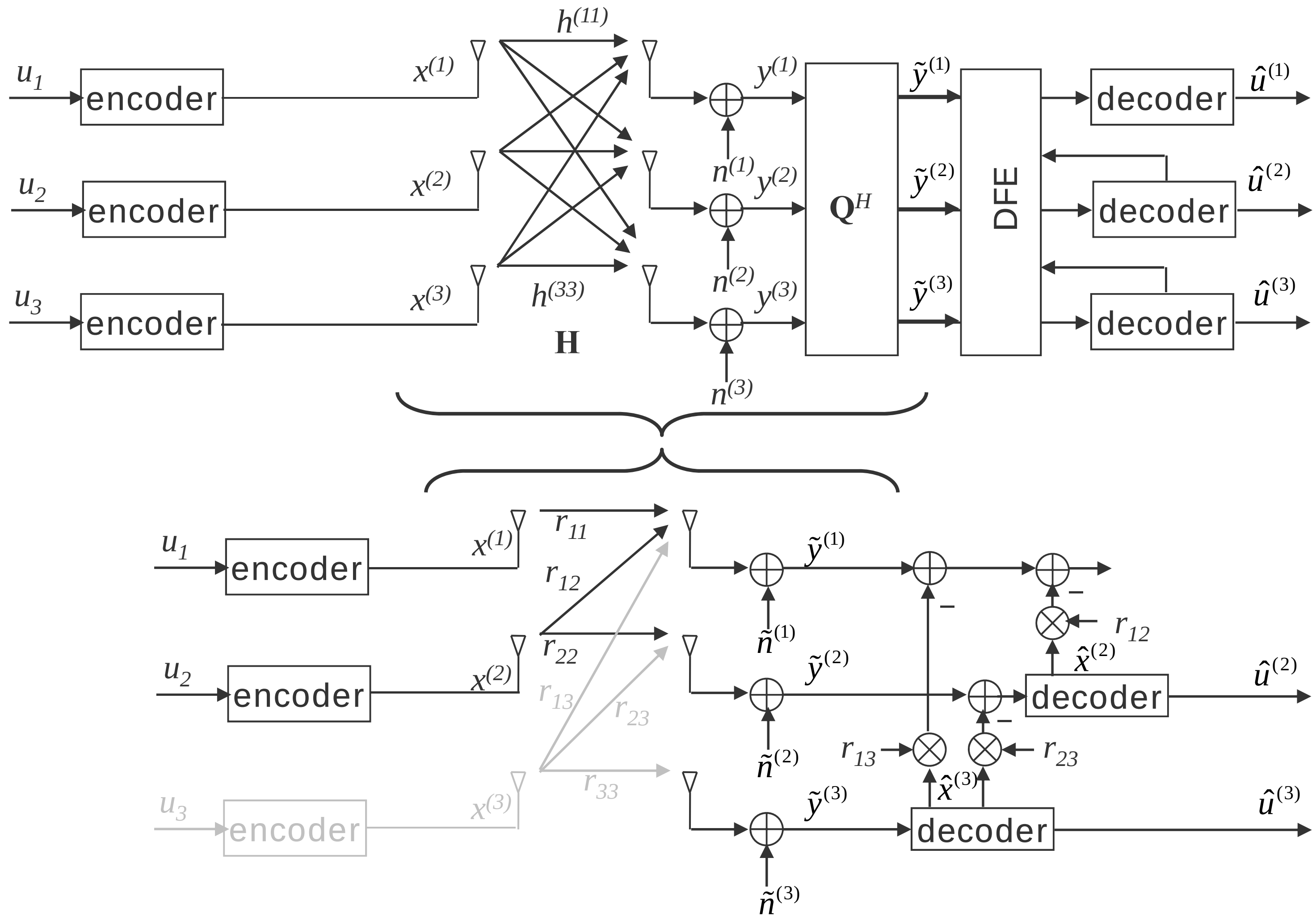
$$= \underbrace{\mathbf{Q}^H\mathbf{Q}}_{\mathbf{I}_{n_R}}\mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$$

$$= \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$$

# Bell-Labs Layered Space-Time Architecture: V-BLAST with QR-Decomposition








$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

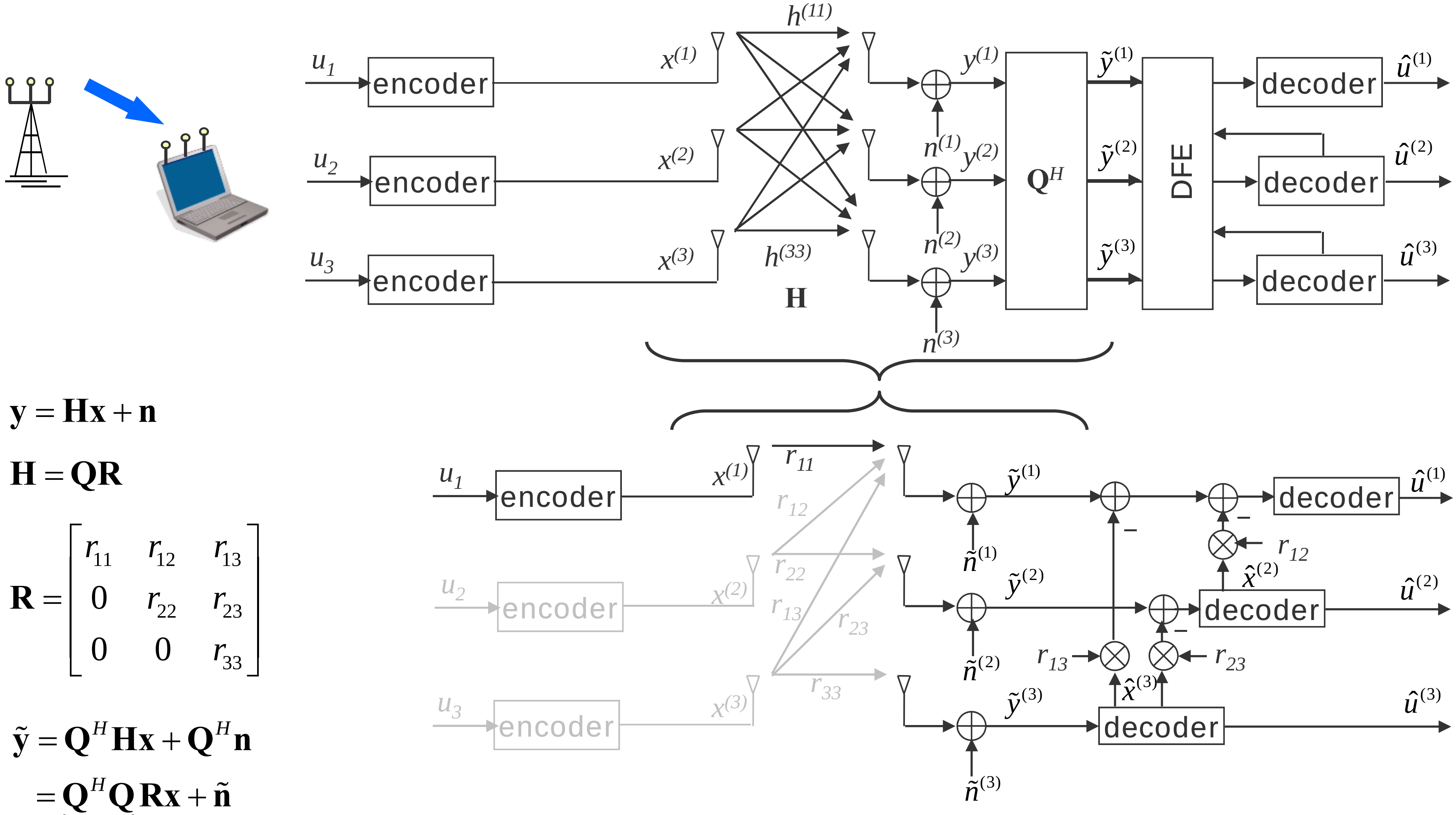
$$\mathbf{H} = \mathbf{Q}\mathbf{R}$$

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$$

$$\begin{aligned} \tilde{\mathbf{y}} &= \mathbf{Q}^H \mathbf{H}\mathbf{x} + \mathbf{Q}^H \mathbf{n} \\ &= \underbrace{\mathbf{Q}^H \mathbf{Q}}_{\mathbf{I}_{n_R}} \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}} \\ &= \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}} \end{aligned}$$

# Bell-Labs Layered Space-Time Architecture: V-BLAST with QR-Decomposition






$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

$$\mathbf{H} = \mathbf{Q}\mathbf{R}$$

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$$

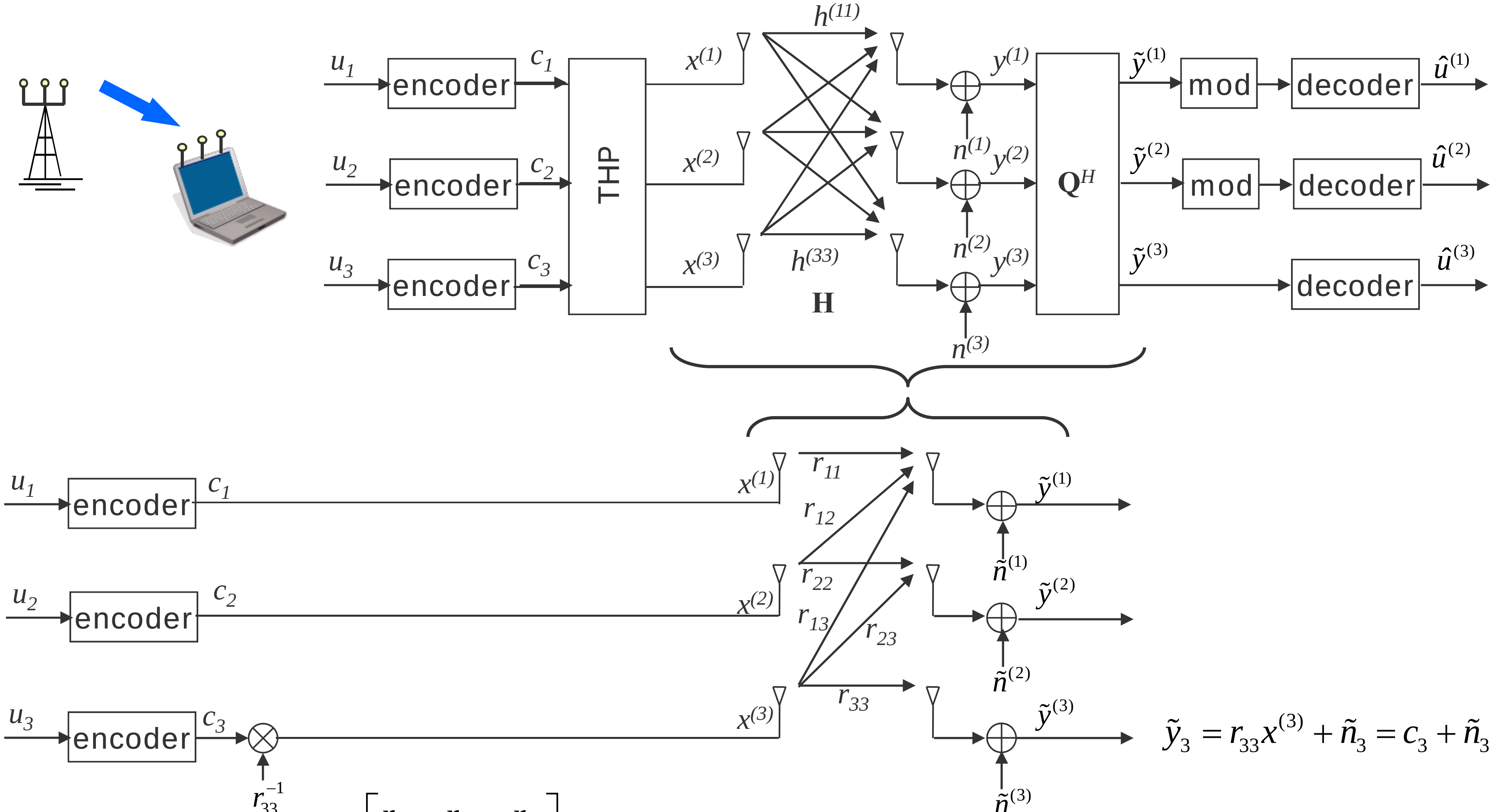
$$\tilde{\mathbf{y}} = \mathbf{Q}^H\mathbf{H}\mathbf{x} + \mathbf{Q}^H\mathbf{n}$$

$$= \underbrace{\mathbf{Q}^H\mathbf{Q}}_{\mathbf{I}_{n_R}}\mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$$

$$= \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$$

# Spatial Multiplexing with Tomlinson-Harashima Precoding

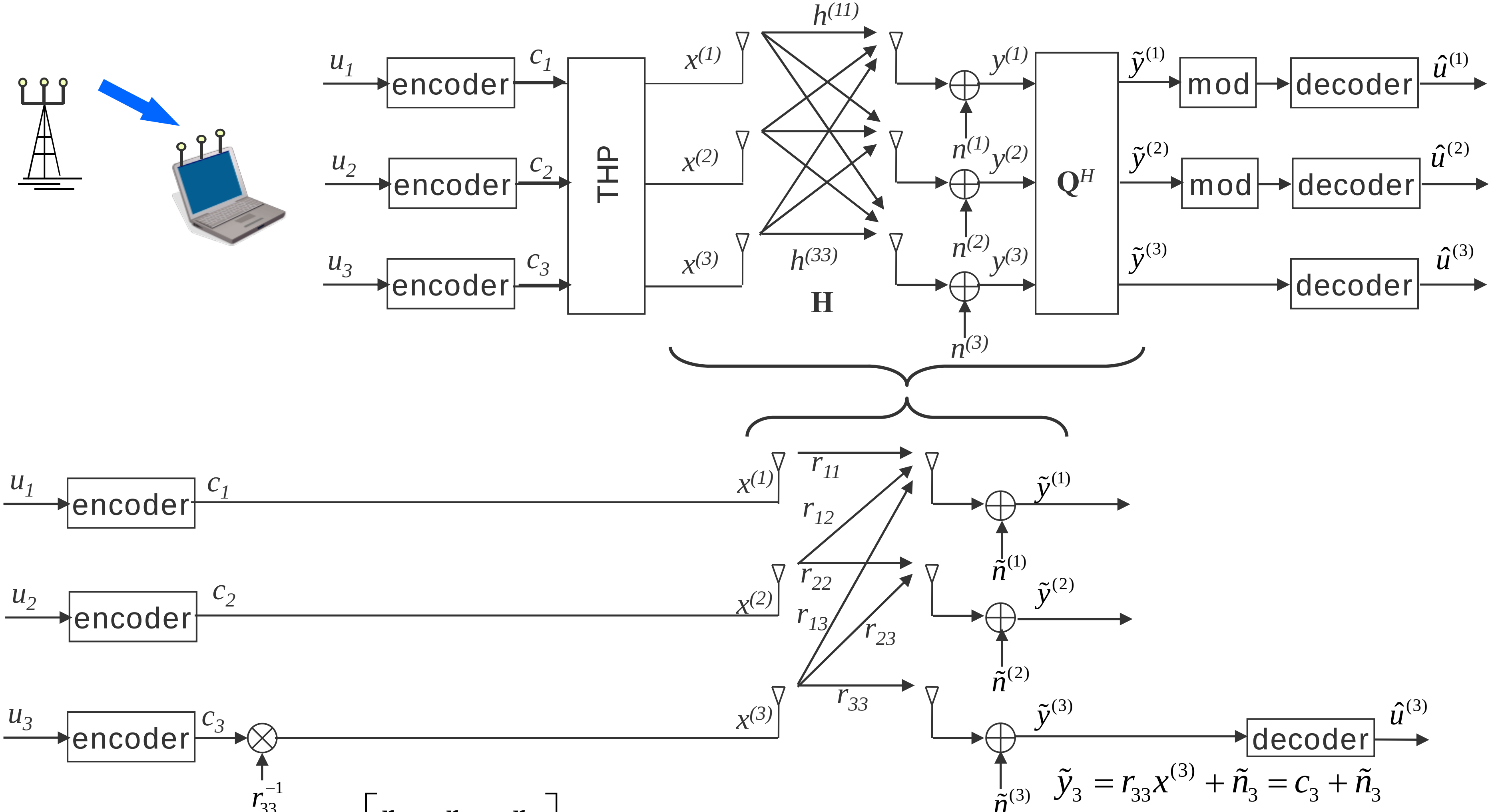





$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}} \quad \text{where} \quad \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$$

# Spatial Multiplexing with Tomlinson-Harashima Precoding






$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}} \quad \text{where} \quad \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$$

# Spatial Multiplexing with Tomlinson-Harashima Precoding

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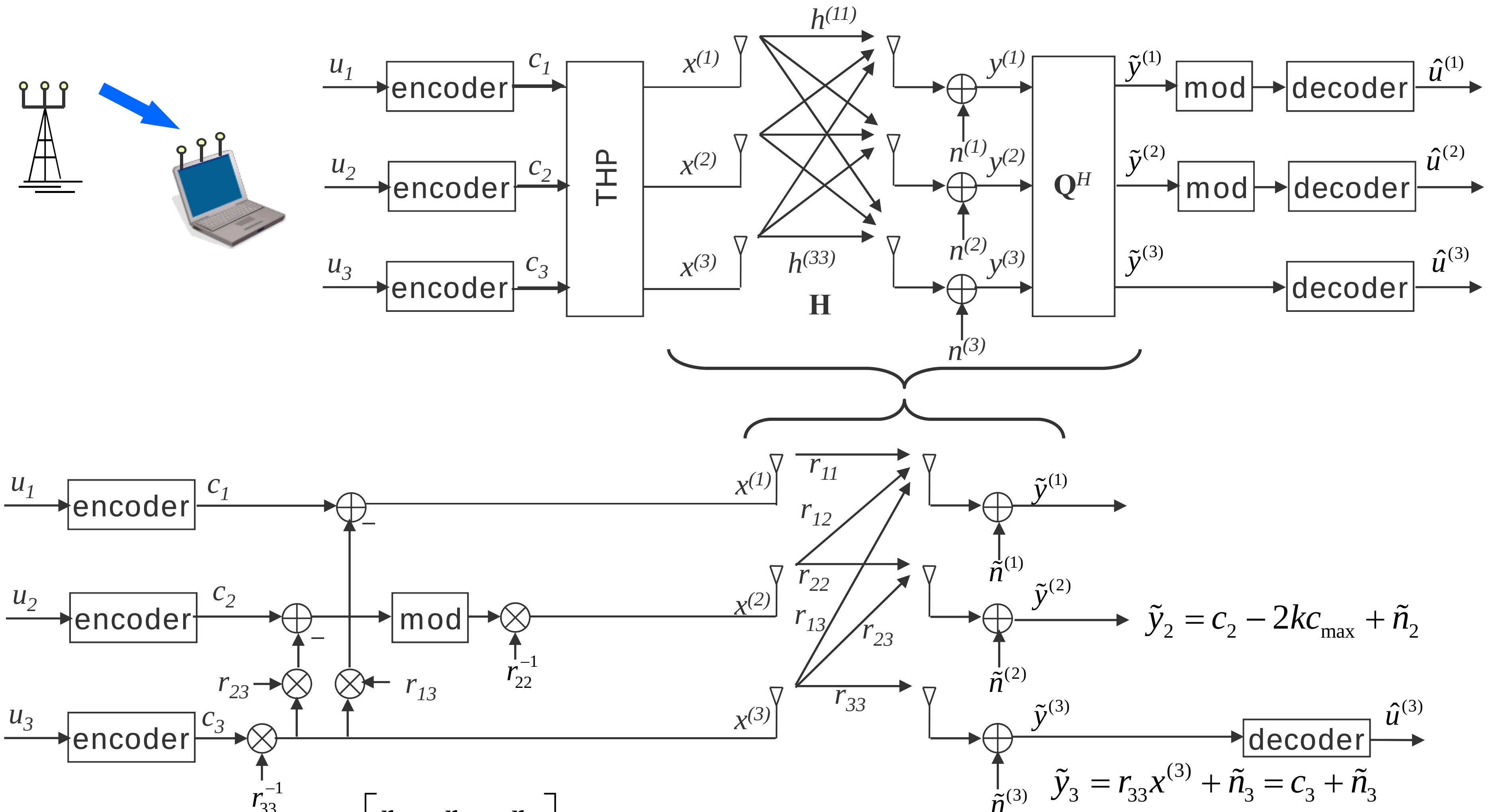


$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}} \quad \text{where} \quad \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$$

$$\tilde{y}_2 = r_{22}x^{(2)} + \tilde{n}_2 = \left(c_2 - r_{23}x^{(3)} - 2kc_{\max}\right) r_{22}^{-1} r_{22} + r_{23}x^{(3)} + \tilde{n}_2$$
$$= c_2 - 2kc_{\max} + \tilde{n}_2$$

# Spatial Multiplexing with Tomlinson-Harashima Precoding

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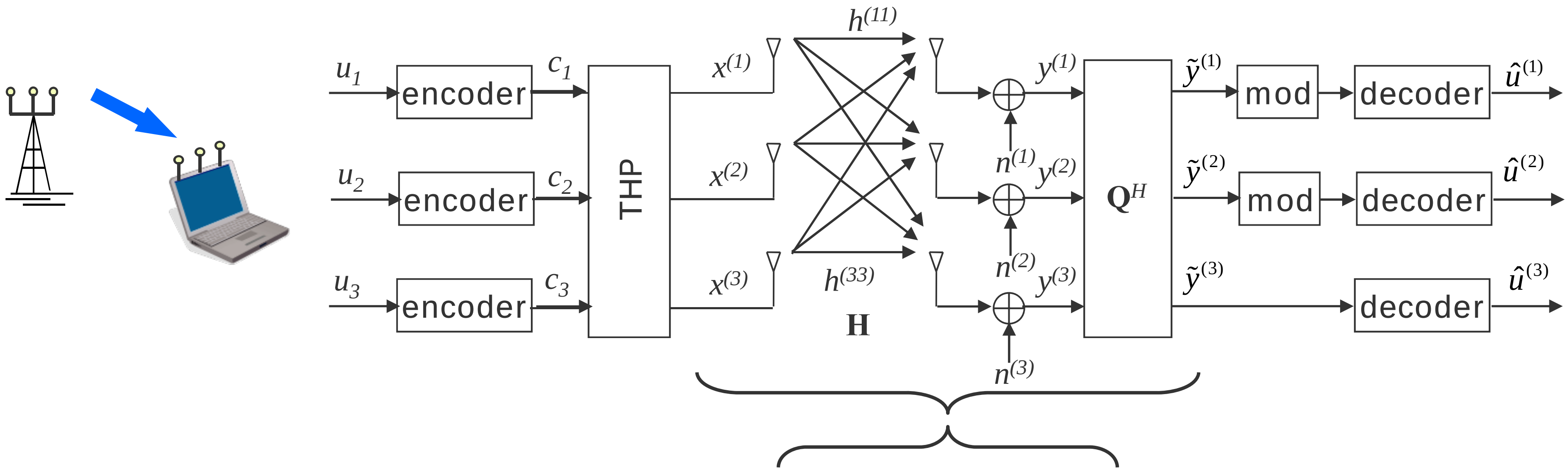


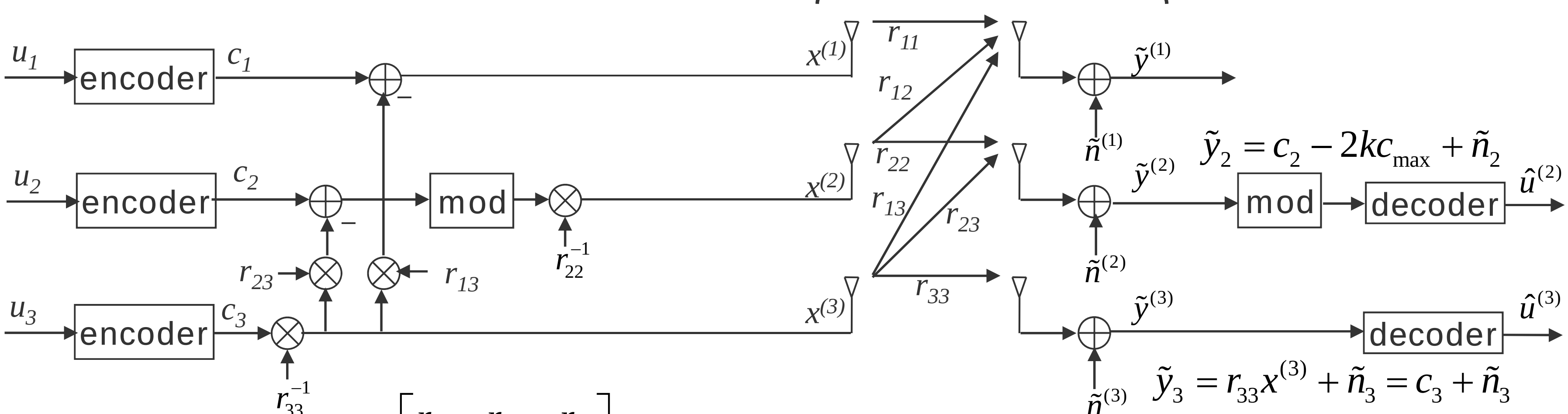


$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}} \quad \text{where} \quad \mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{bmatrix}$$

$$\tilde{y}_2 = r_{22}x^{(2)} + \tilde{n}_2 = \left(c_2 - r_{23}x^{(3)} - 2kc_{\max}\right) r_{22}^{-1} r_{22} + r_{23}x^{(3)} + \tilde{n}_2$$
$$= c_2 - 2kc_{\max} + \tilde{n}_2$$

# Spatial Multiplexing with Tomlinson-Harashima Precoding

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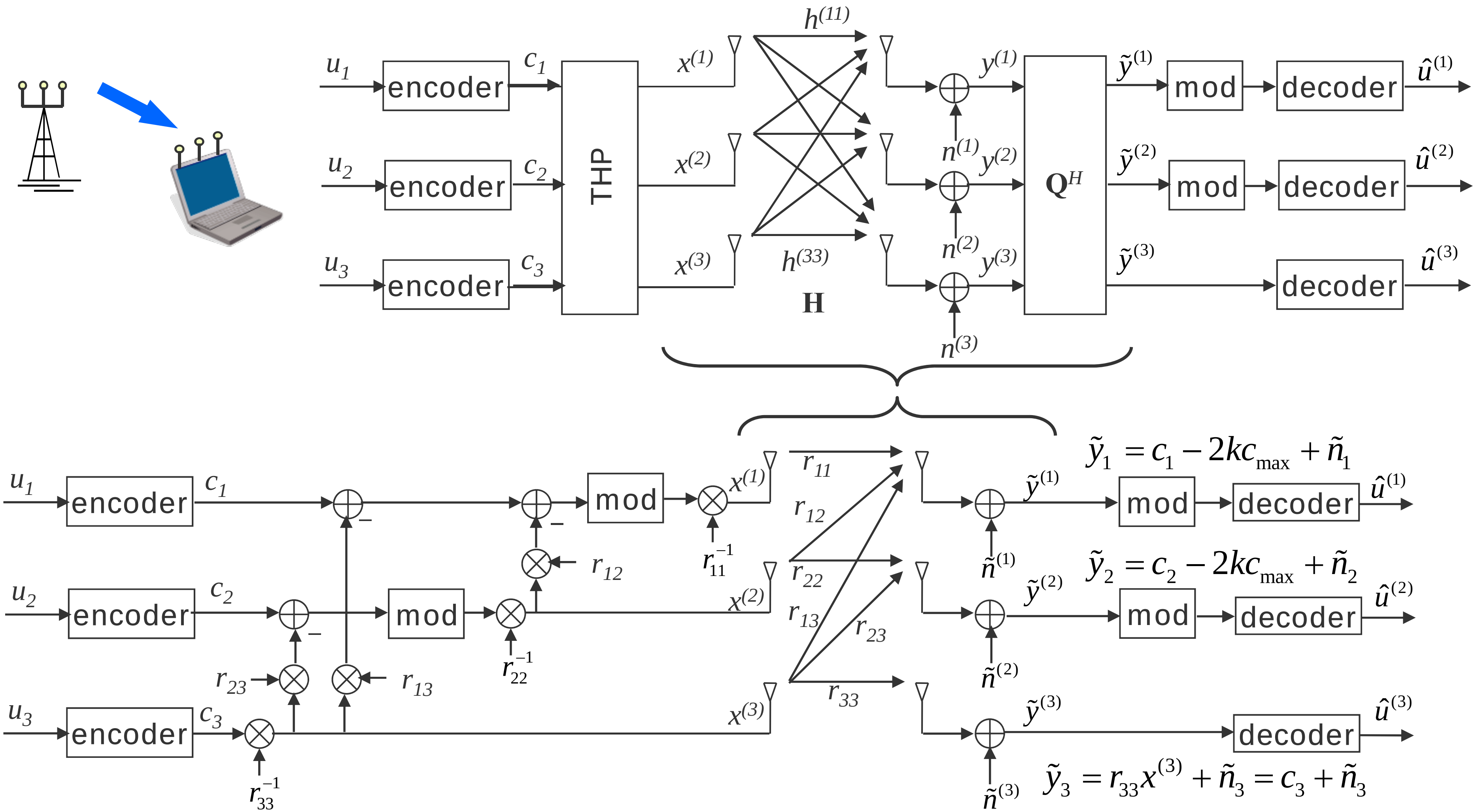
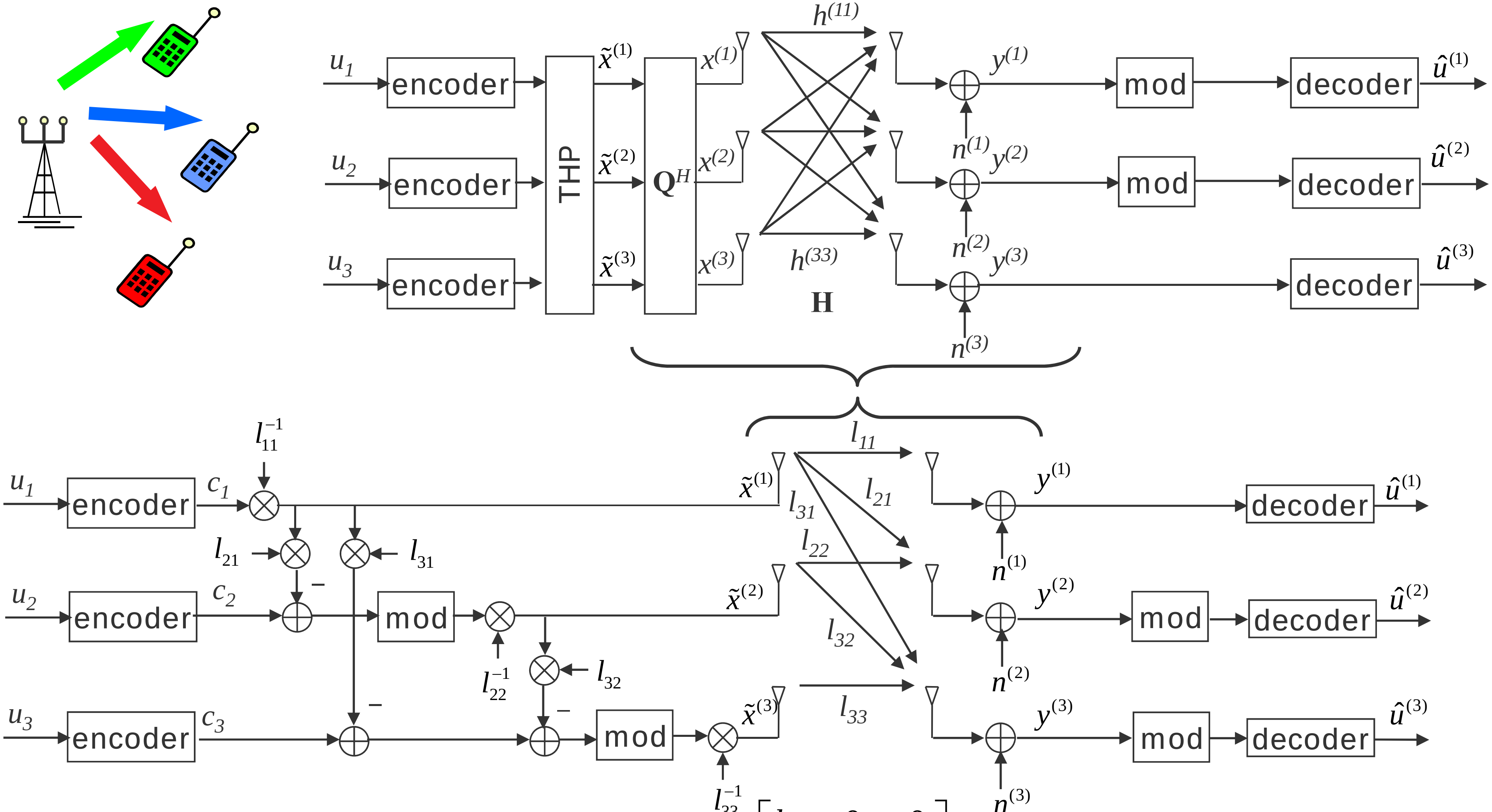



$$\tilde{y}_1 = r_{11}x^{(1)} + \tilde{n}_1 = \left(c_1 - r_{13}x^{(3)} - r_{12}x^{(2)} - 2kc_{\max}\right) r_{11}^{-1} r_{11} + r_{13}x^{(3)} + r_{12}x^{(2)} + \tilde{n}_1$$

$$= c_1 - 2kc_{\max} + \tilde{n}_1$$

# Spatial Multiplexing with Tomlinson-Harashima Precoding and Linear Transmit Filter






$$\mathbf{H} = \mathbf{L}\mathbf{Q}$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \mathbf{L}\underbrace{\mathbf{Q}\mathbf{Q}^H}_{\mathbf{I}_{n_T}}\tilde{\mathbf{x}} + \mathbf{n} = \mathbf{L}\tilde{\mathbf{x}} + \mathbf{n} \quad \text{where} \quad \mathbf{L} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

# Outline (1)

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- Introduction
  - MIMO capacity
  - Single-user (SU) MIMO versus multi-user (MU) MIMO
  - Uplink MU-MIMO versus downlink MU-MIMO
  - Linear versus non-linear MU-MIMO
  - Multiuser diversity

- Single-user MIMO
  - Spatial multiplexing with Rx and Tx processing

- Theoretical fundamentals
  - Introduction to Dirty Paper Coding (DPC)
  - Tomlinson-Harashima precoding (THP)
  - Precoding for the MIMO broadcast channel

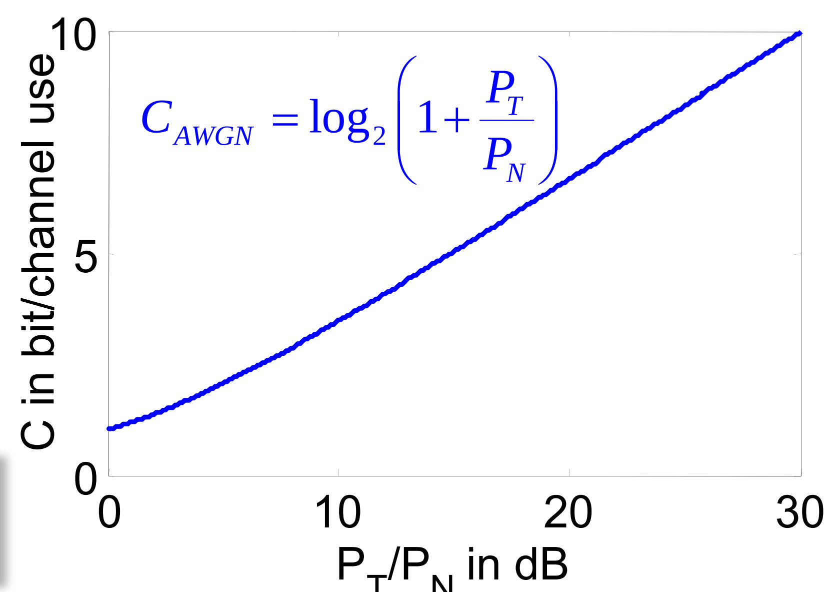
# Channel Capacity of the AWGN Channel


Power constraint: $P_X = E\left\{\left|X\right|^2\right\} \le P_T$

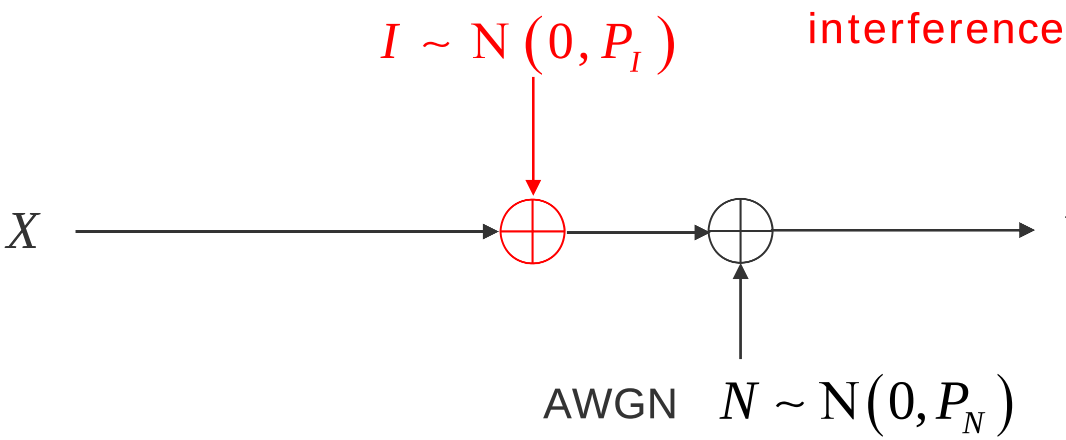
$$Y = X + N$$

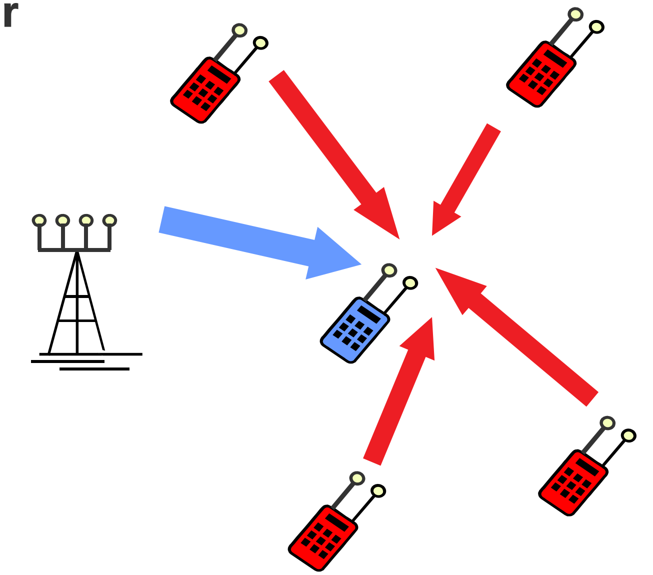
$$C = \log_2\left(1 + \frac{P_X}{P_N}\right) = \log_2\left(1 + \frac{P_T}{P_N}\right)$$

The optimum transmit symbols $X$ are Gaussian distributed with zero mean and $\sigma^2 = P_X$.



## Interference unknown to transmitter and receiver

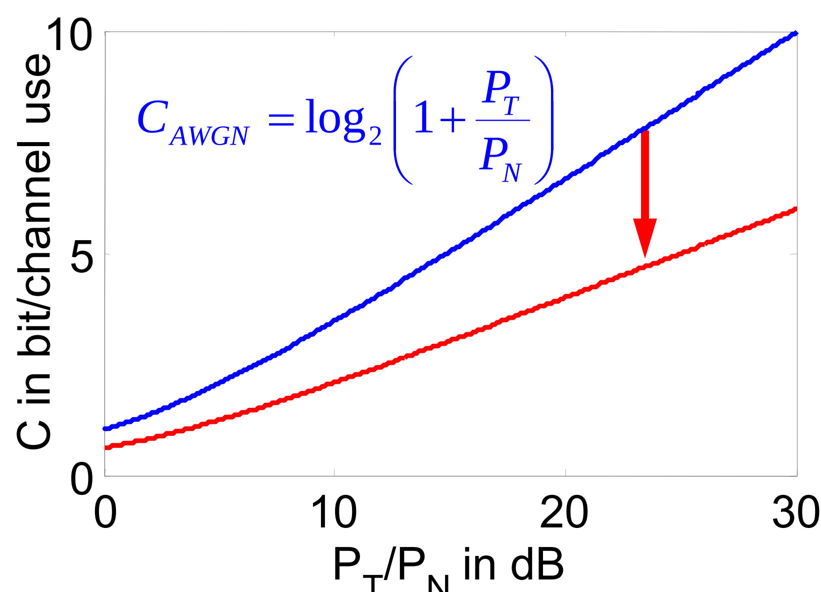


Power constraint: $P_X = E\left\{|X|^2\right\} \le P_T$

$$Y = X + I + N$$

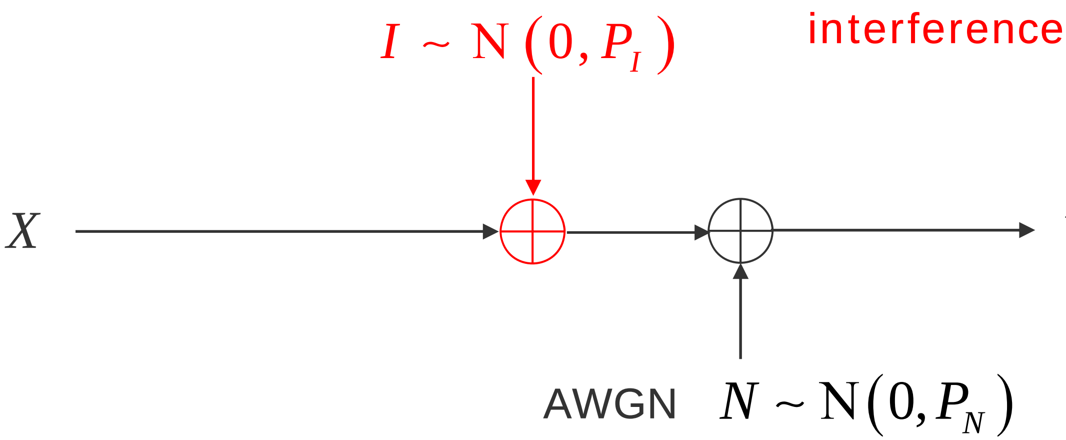
$$C = \log_2\left(1 + \frac{P_X}{P_I + P_N}\right) = \log_2\left(1 + \frac{P_T}{P_I + P_N}\right)$$

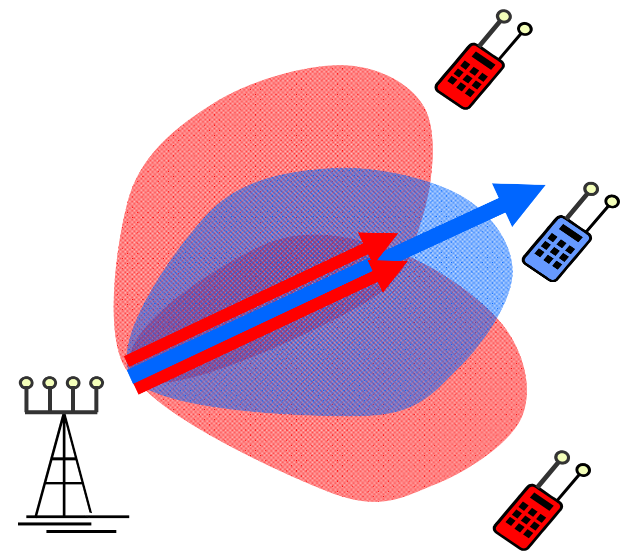


# Channel Capacity of the AWGN Channel with Gaussian Interference



## Interference unknown to transmitter and receiver

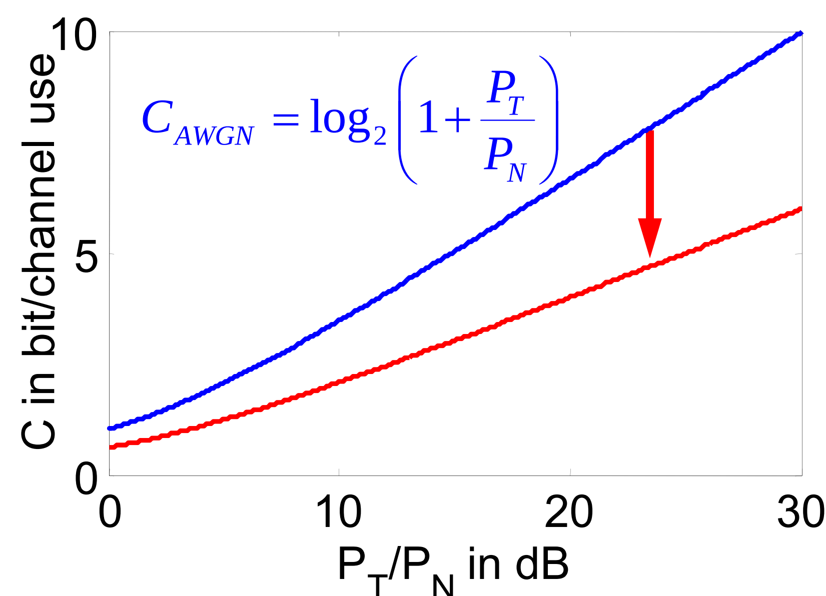


Power constraint: $P_X = E\{|X|^2\} \le P_T$

$$Y = X + I + N$$

$$C = \log_2\left(1 + \frac{P_X}{P_I + P_N}\right) = \log_2\left(1 + \frac{P_T}{P_I + P_N}\right)$$



# Channel Capacity of the AWGN Channel with Gaussian Interference

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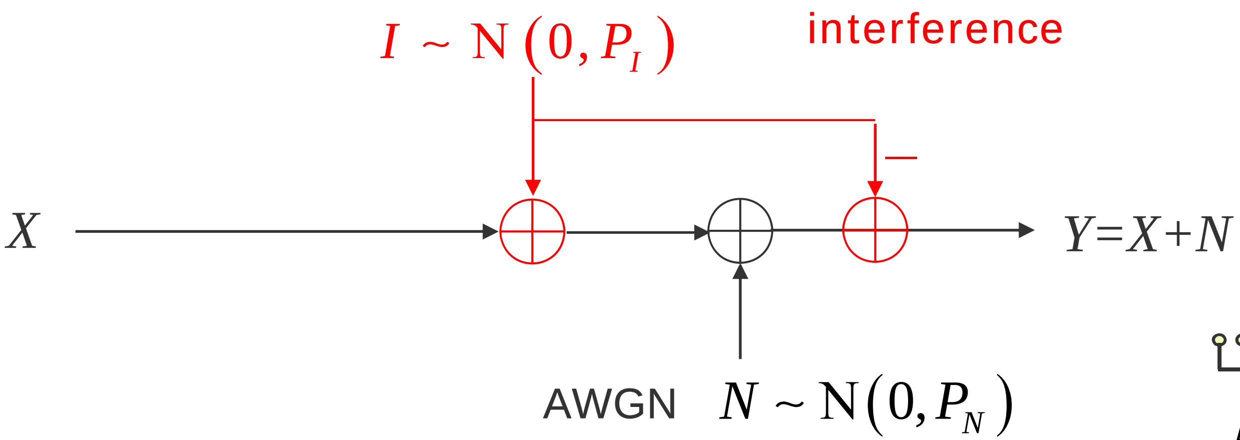
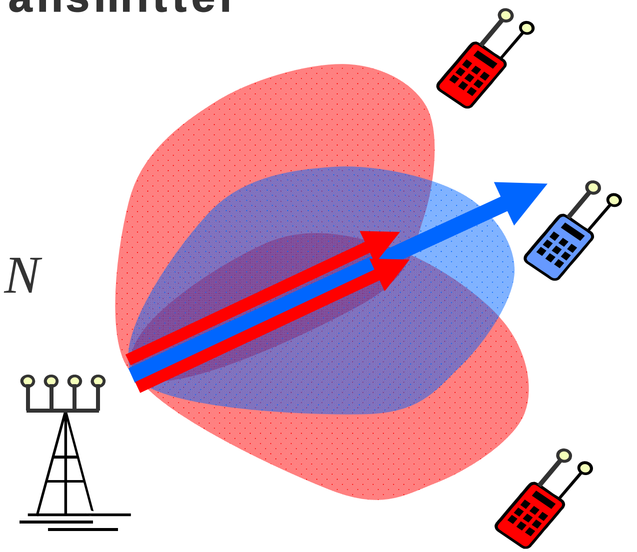
## Interference known to receiver but unknown to transmitter

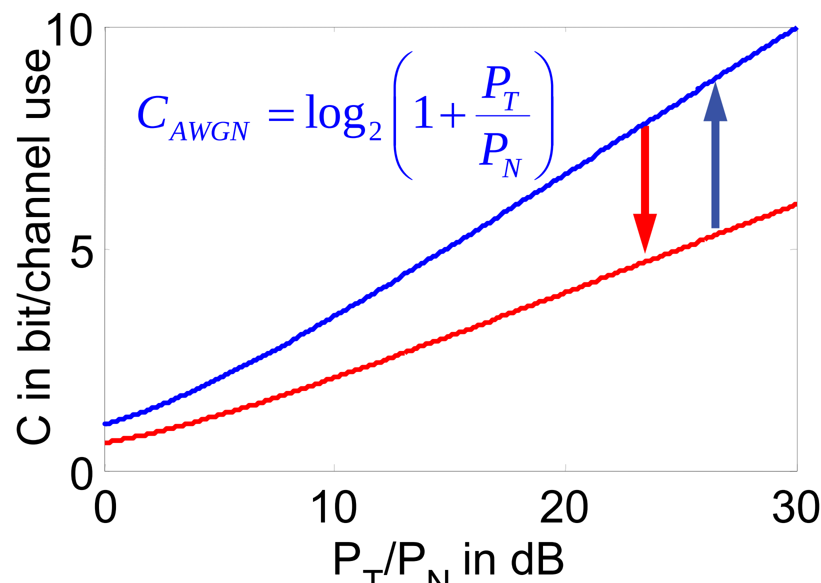


Power constraint: $P_X = E\left\{|X|^2\right\} \le P_T$

$$Y = X + I + N - I = X + N$$

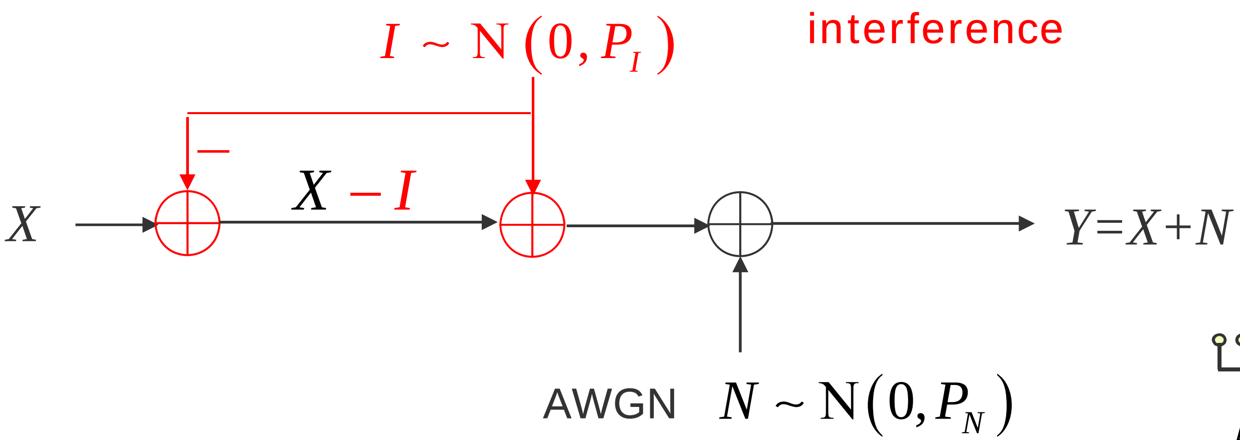
$$C = \log_2\left(1 + \frac{P_X}{P_N}\right) = \log_2\left(1 + \frac{P_T}{P_N}\right)$$

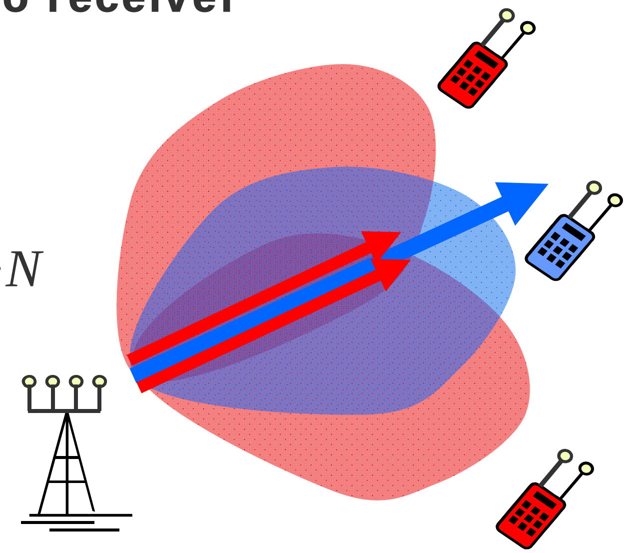



Interference which is known to the receiver does not cause any loss in terms of capacity.

# Channel Capacity of the AWGN Channel with Gaussian Interference

## Interference known to transmitter but unknown to receiver



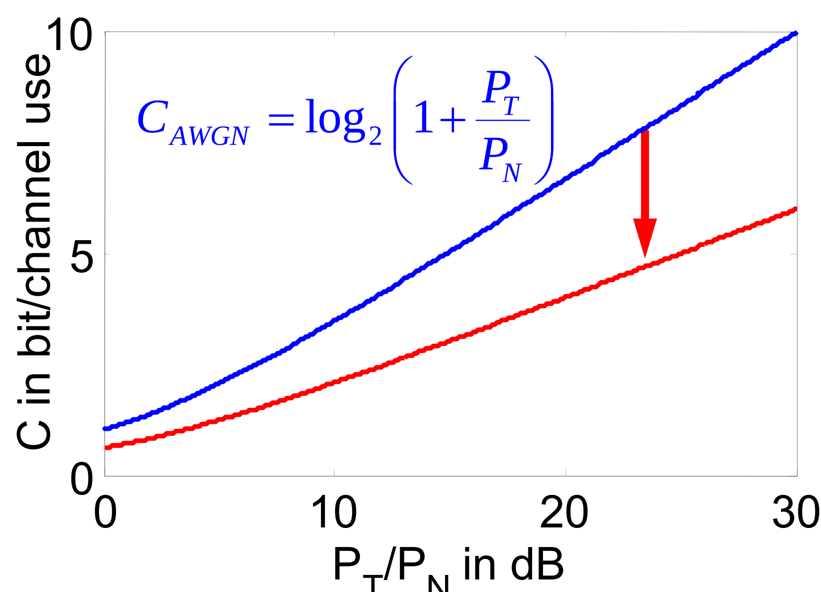



Power constraint:

$$P_{X-I} = E\left\{ |X - I|^2 \right\} = P_X + P_I \le P_T$$

$$Y = X + I + N - I = X + N$$

$$C = \log_2\left(1 + \frac{P_X}{P_N}\right) = \log_2\left(1 + \frac{P_T - P_I}{P_N}\right)$$



# Channel Capacity with Dirty Paper Coding

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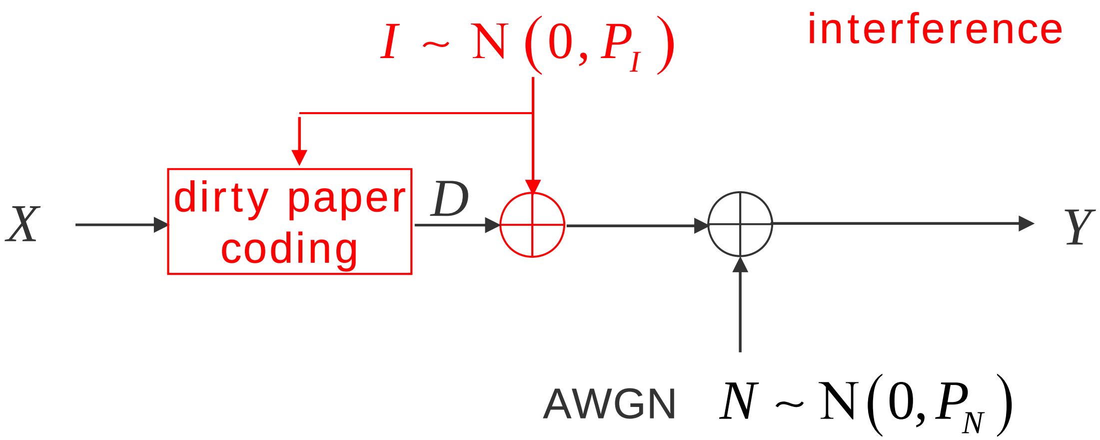
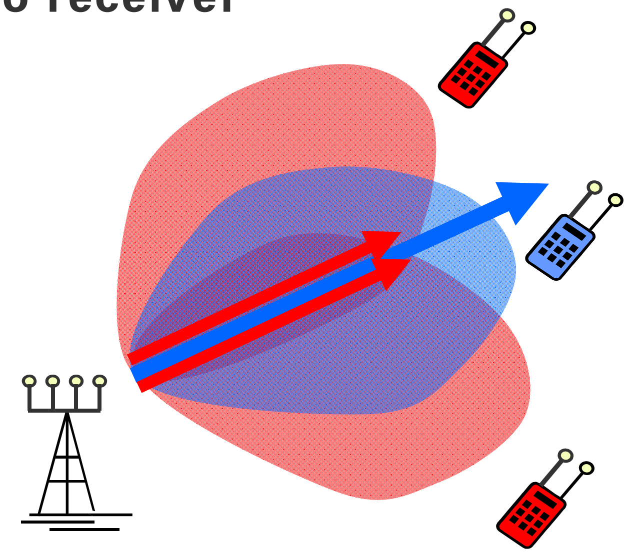
## Interference known to transmitter but unknown to receiver



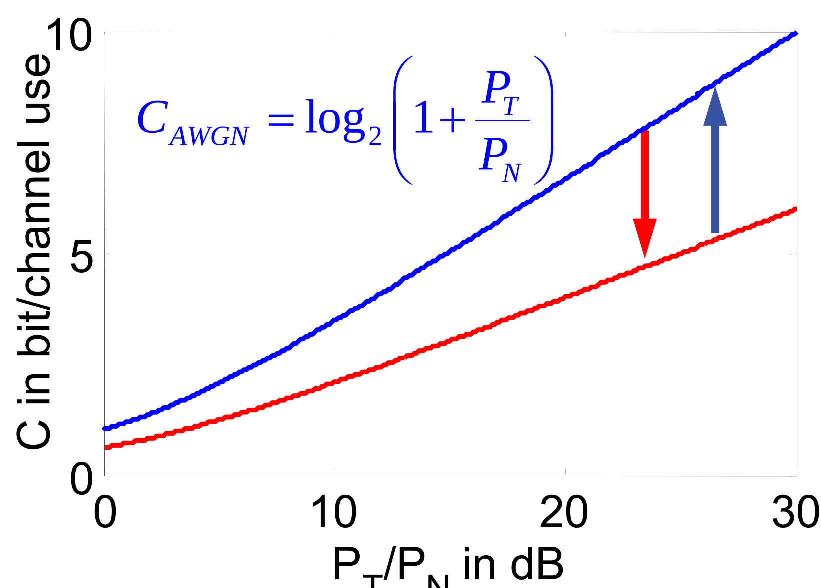



Power constraint: $P_D = E\left\{|D|^2\right\} \leq P_T$

$$Y = D + I + N$$

$$C = \log_2\left(1 + \frac{P_T}{P_N}\right)$$

Costa 1983:
*Writing on dirty paper*

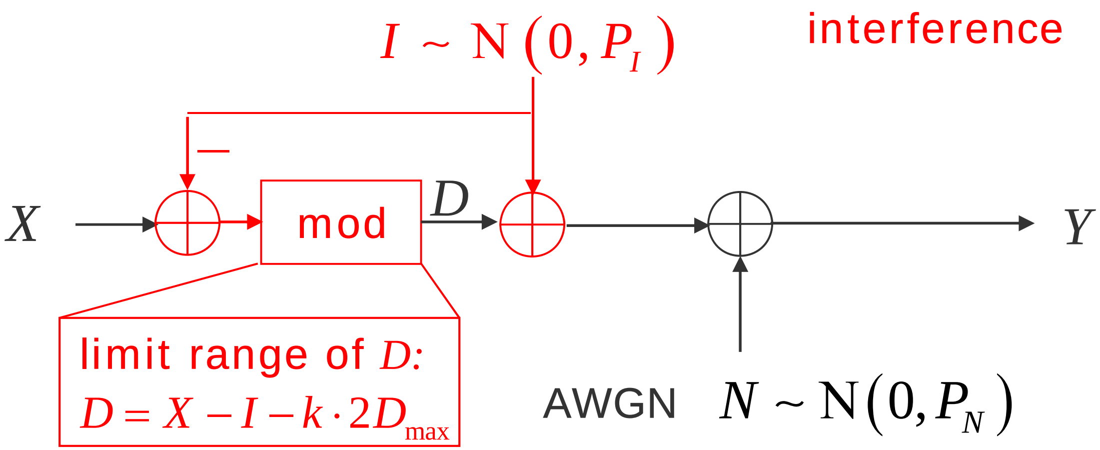


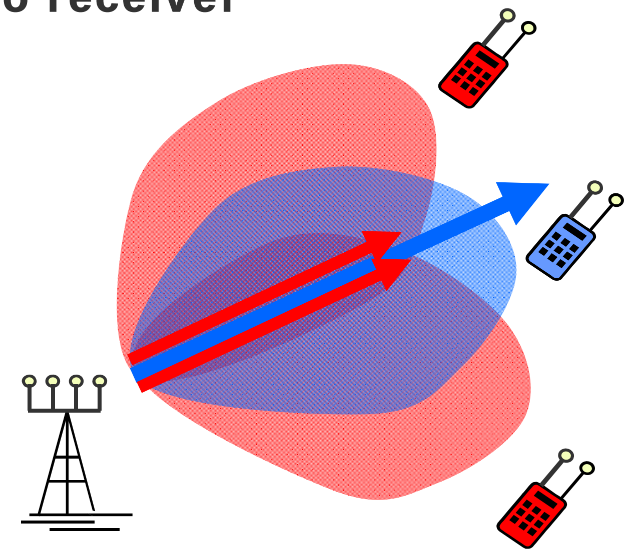

Interference which is known to the transmitter does not cause any loss in terms of capacity.

# Tomlinson-Harashima Precoding



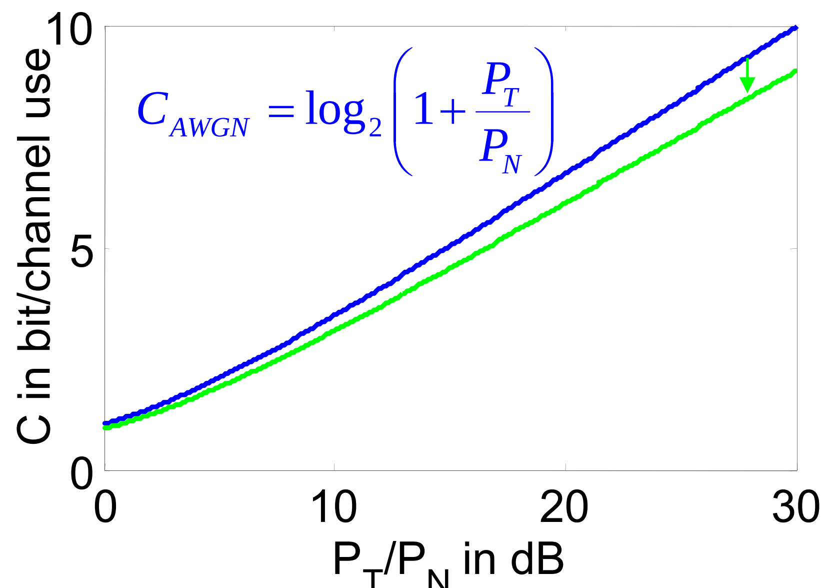
## Interference known to transmitter but unknown to receiver



Power constraint: $P_D = E\{|D|^2\} \leq P_T$

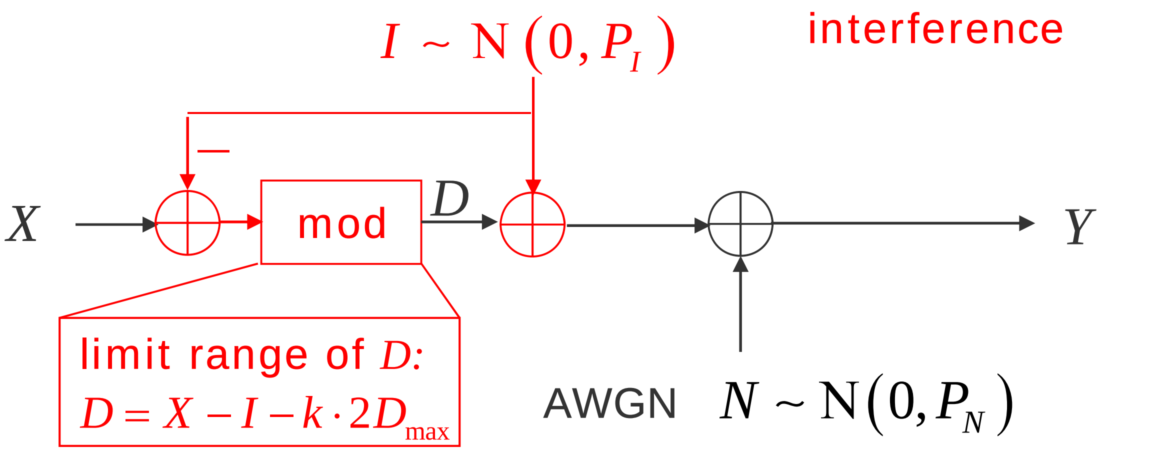
$$Y = D + I + N$$



# Tomlinson-Harashima Precoding

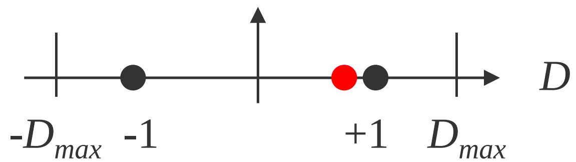


## Interference known to transmitter but unknown to receiver




Example:

$D_{\max} = 1.5$




| | | | | |
|---|---|---|---|---|
| $X \in \{-1, +1\}$ | +1 | -1 | -1 | +1 |
| $I$ | 0.1 | 2.5 | -0.2 | -5 |
| $X - I$ | 0.9 | | | |
| $D = X - I - k \cdot 2D_{\max}$ | 0.9 | | | |
| $k$ | 0 | | | |

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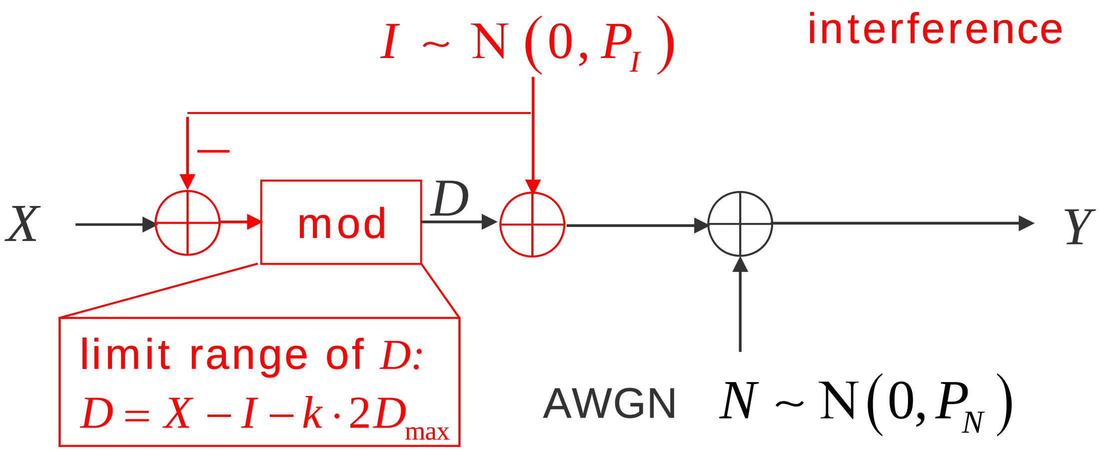
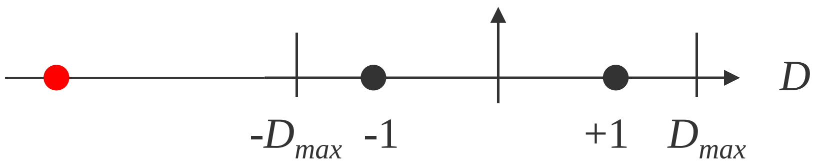
## Interference known to transmitter but unknown to receiver



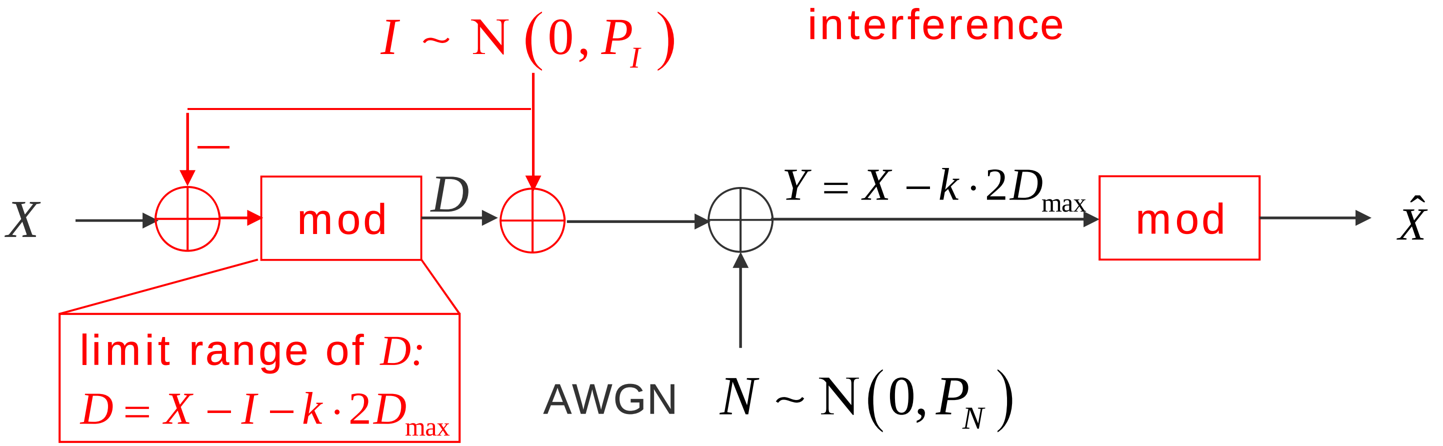

Example:

$D_{\max} = 1.5$

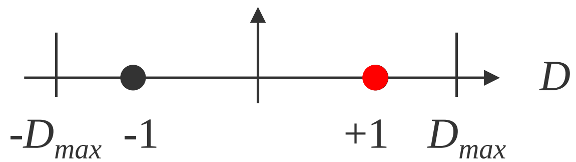



| $X \in \{-1, +1\}$ | +1 | -1 | -1 | +1 |
|---|---|---|---|---|
| $I$ | 0.1 | 2.5 | -0.2 | -5 |
| $X - I$ | 0.9 | -3.5 | -0.8 | 6 |
| $D = X - I - k \cdot 2D_{\max}$ | 0.9 | -0.5 | -0.8 | 0 |
| $k$ | 0 | -1 | 0 | 2 |

## Interference known to transmitter but unknown to receiver



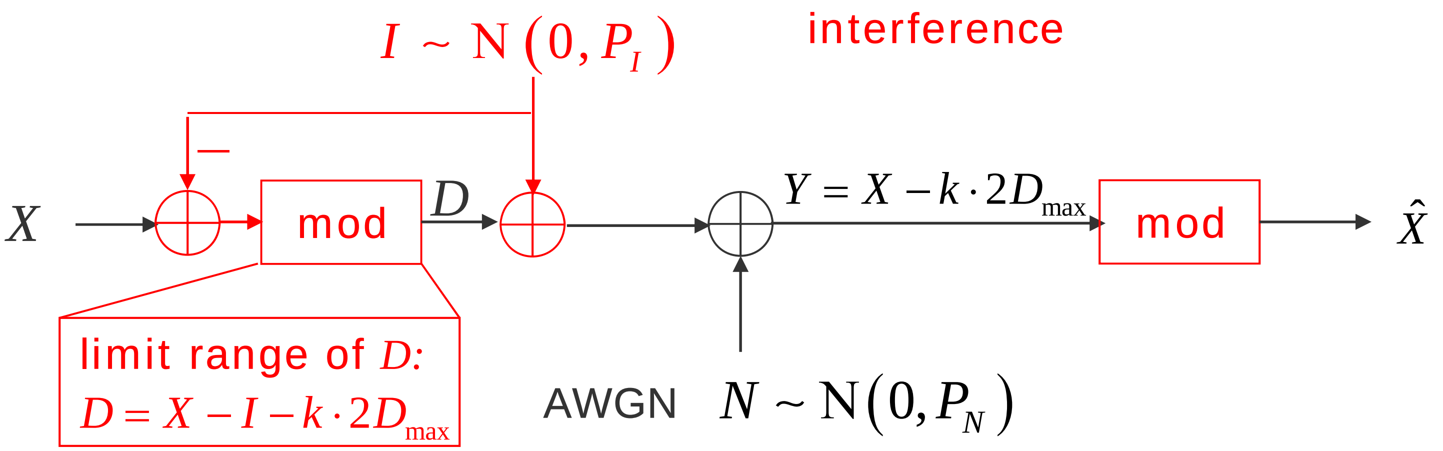

Example:




$D_{max} = 1.5$

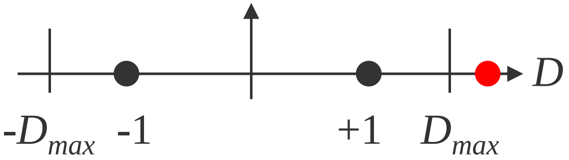
| | | | | |
|---|---|---|---|---|
| $X \in \{-1, +1\}$ | +1 | -1 | -1 | +1 |
| $I$ | 0.1 | 2.5 | -0.2 | -5 |
| $X - I$ | 0.9 | -3.5 | -0.8 | 6 |
| $D = X - I - k \cdot 2D_{max}$ | 0.9 | -0.5 | -0.8 | 0 |
| $k$ | 0 | -1 | 0 | 2 |
| $Y = X - I + I - k \cdot 2D_{max} = X - k \cdot 2D_{max}$ | +1 | | | |
| $\hat{X} = Y + k \cdot 2D_{max}$ | +1 | | | |

## Interference known to transmitter but unknown to receiver



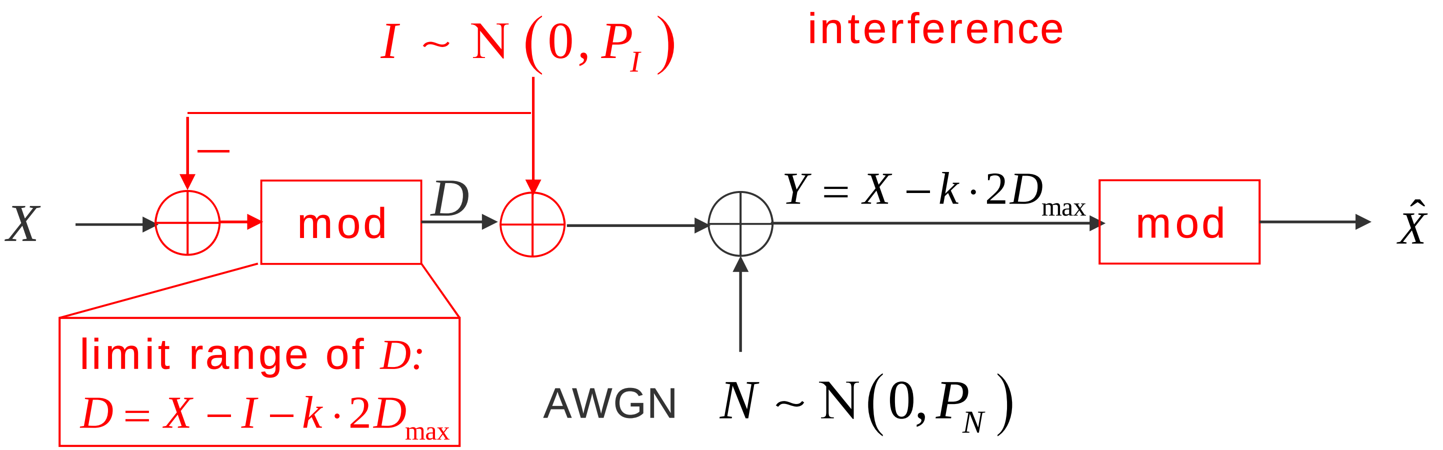

Example:

$D_{\max} = 1.5$

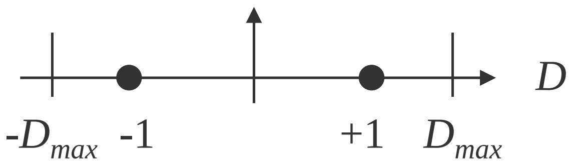



| | | | | |
|---|---|---|---|---|
| $X \in \{-1, +1\}$ | +1 | -1 | -1 | +1 |
| $I$ | 0.1 | 2.5 | -0.2 | -5 |
| $X - I$ | 0.9 | -3.5 | -0.8 | 6 |
| $D = X - I - k \cdot 2D_{\max}$ | 0.9 | -0.5 | -0.8 | 0 |
| $k$ | 0 | -1 | 0 | 2 |
| $Y = X - I + I - k \cdot 2D_{\max} = X - k \cdot 2D_{\max}$ | +1 | 2 | -1 | -5 |
| $\hat{X} = Y + k \cdot 2D_{\max}$ | +1 | -1 | -1 | +1 |

# Interference known to transmitter but unknown to receiver




Example:




$D_{\max} = 1.5$

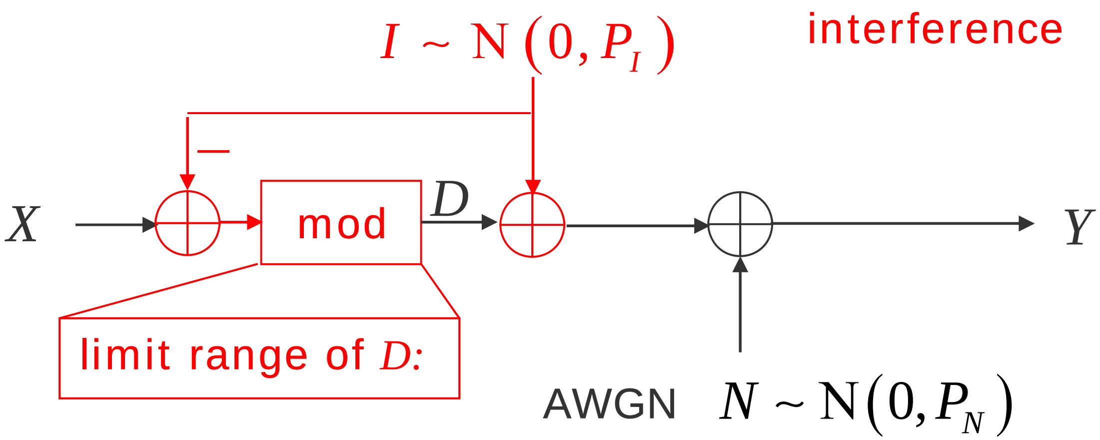
| | | | | |
|---|---|---|---|---|
| $X \in \{-1, +1\}$ | +1 | -1 | -1 | +1 |
| $I$ | 0.1 | 2.5 | -0.2 | -5 |
| $X - I$ | 0.9 | -3.5 | -0.8 | 6 |
| $D = X - I - k \cdot 2D_{\max}$ | 0.9 | -0.5 | -0.8 | 0 |
| $k$ | 0 | -1 | 0 | 2 |
| $Y = X - I + I - k \cdot 2D_{\max} = X - k \cdot 2D_{\max}$ | +1 | 2 | -1 | -5 |
| $\hat{X} = Y + k \cdot 2D_{\max}$ | +1 | -1 | -1 | +1 |

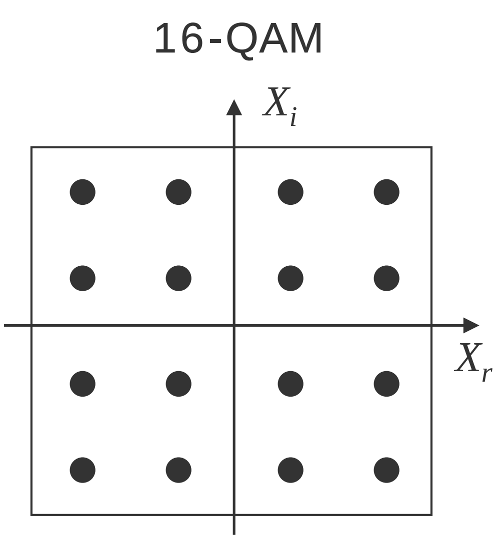
$$D = (X - I + D_{\max}) \bmod (2D_{\max}) - D_{\max}$$

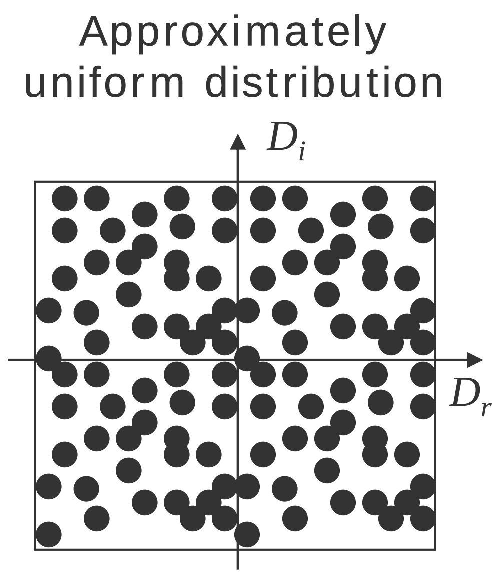
$$\hat{X} = (Y + D_{\max}) \bmod (2D_{\max}) - D_{\max}$$

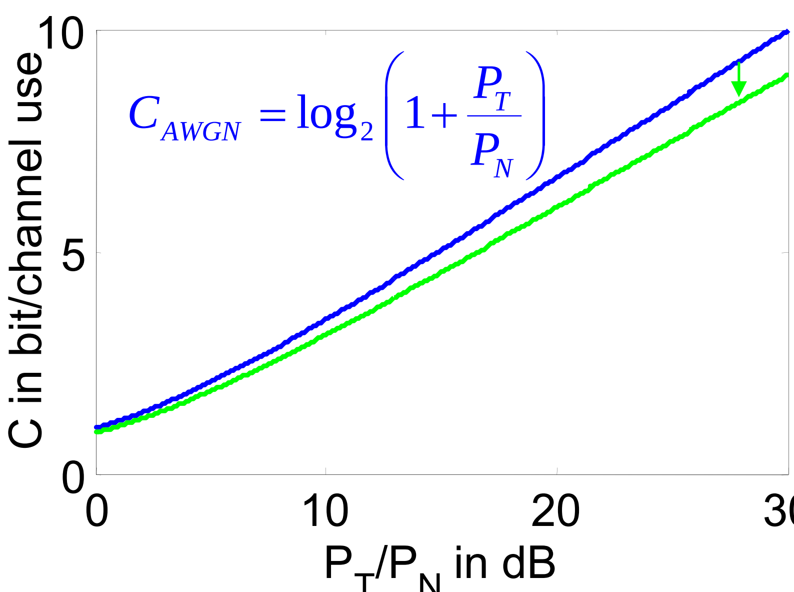
# Tomlinson-Harashima Precoding

## Interference known to transmitter but unknown to receiver



# Example: MIMO Broadcast Channel

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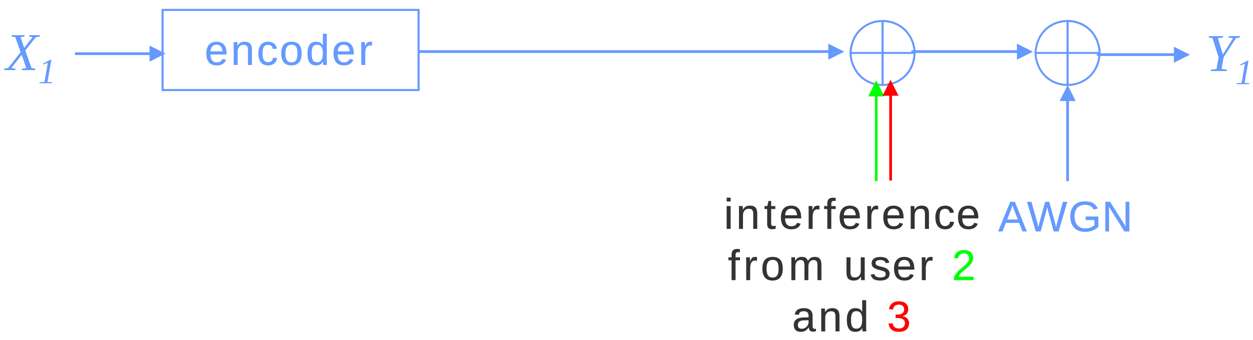
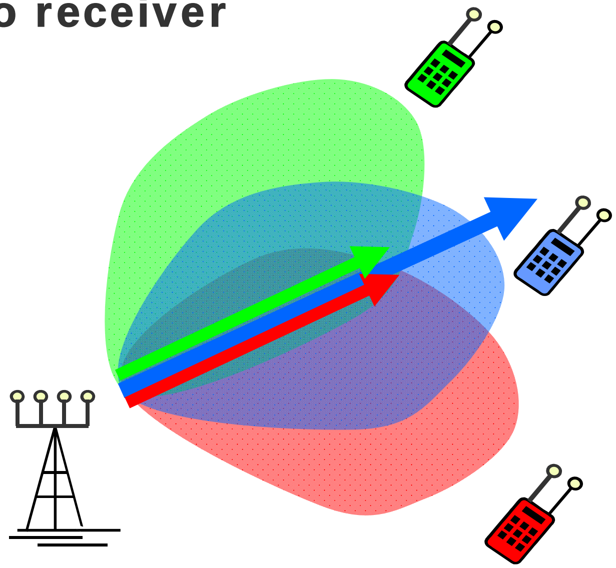
**Interference known to transmitter but unknown to receiver**

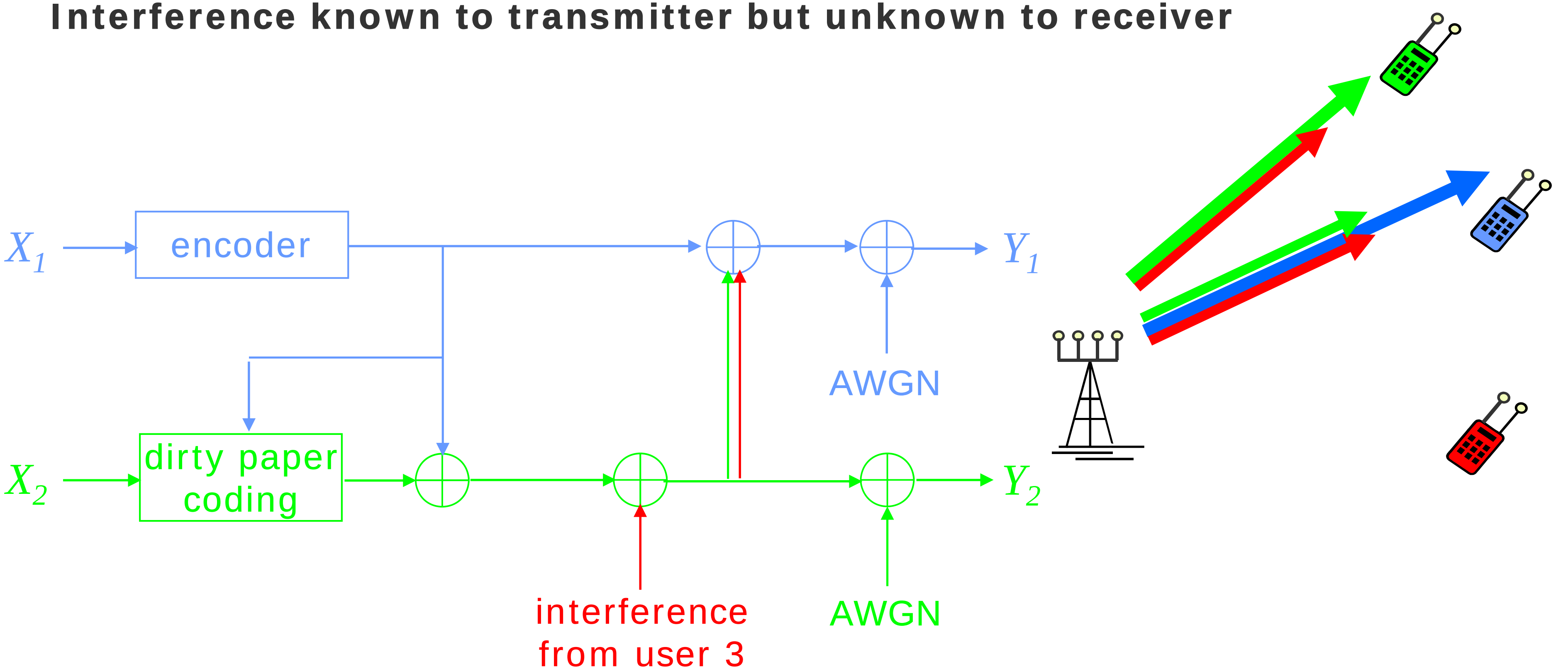


# Example: MIMO Broadcast Channel

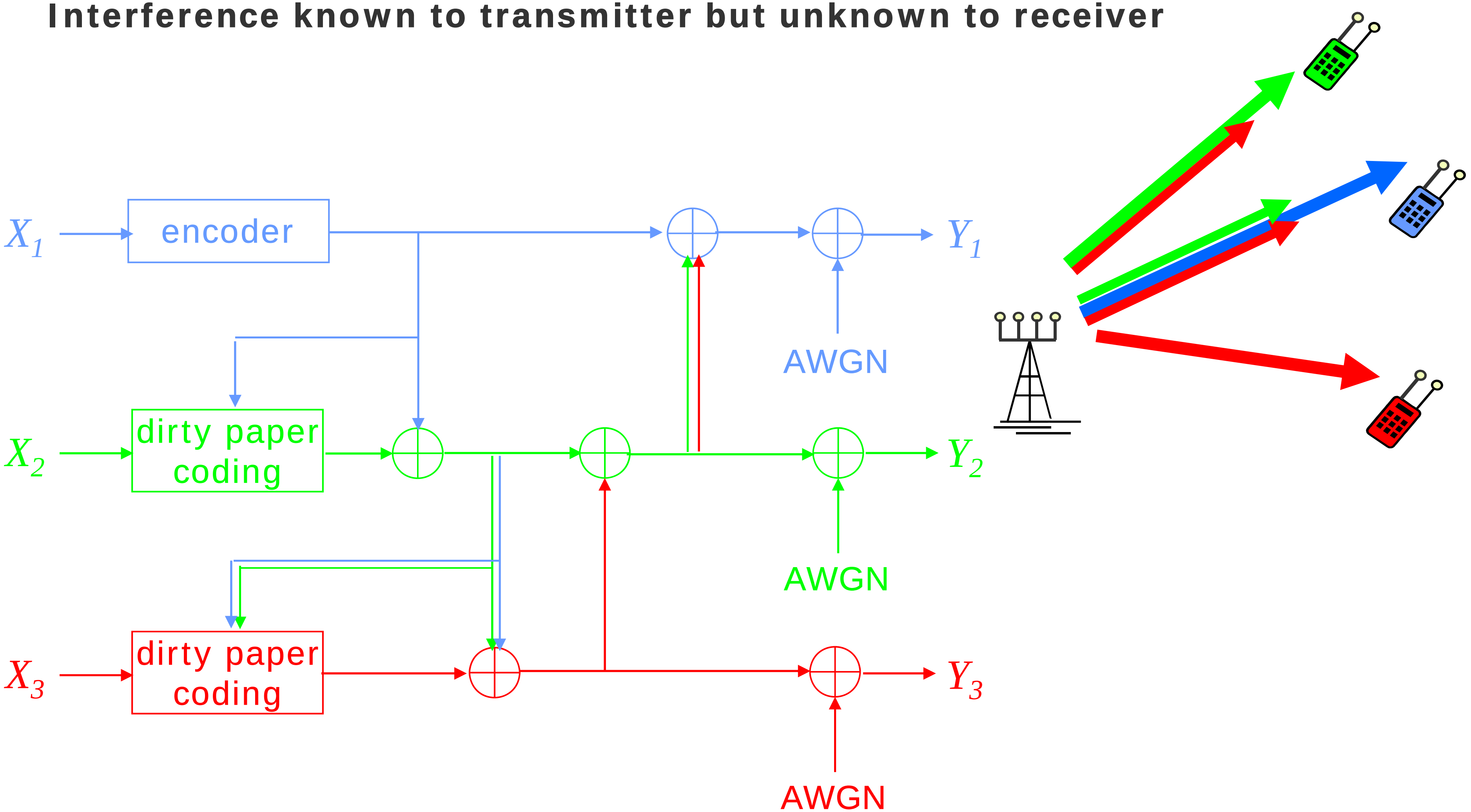


**Interference known to transmitter but unknown to receiver**



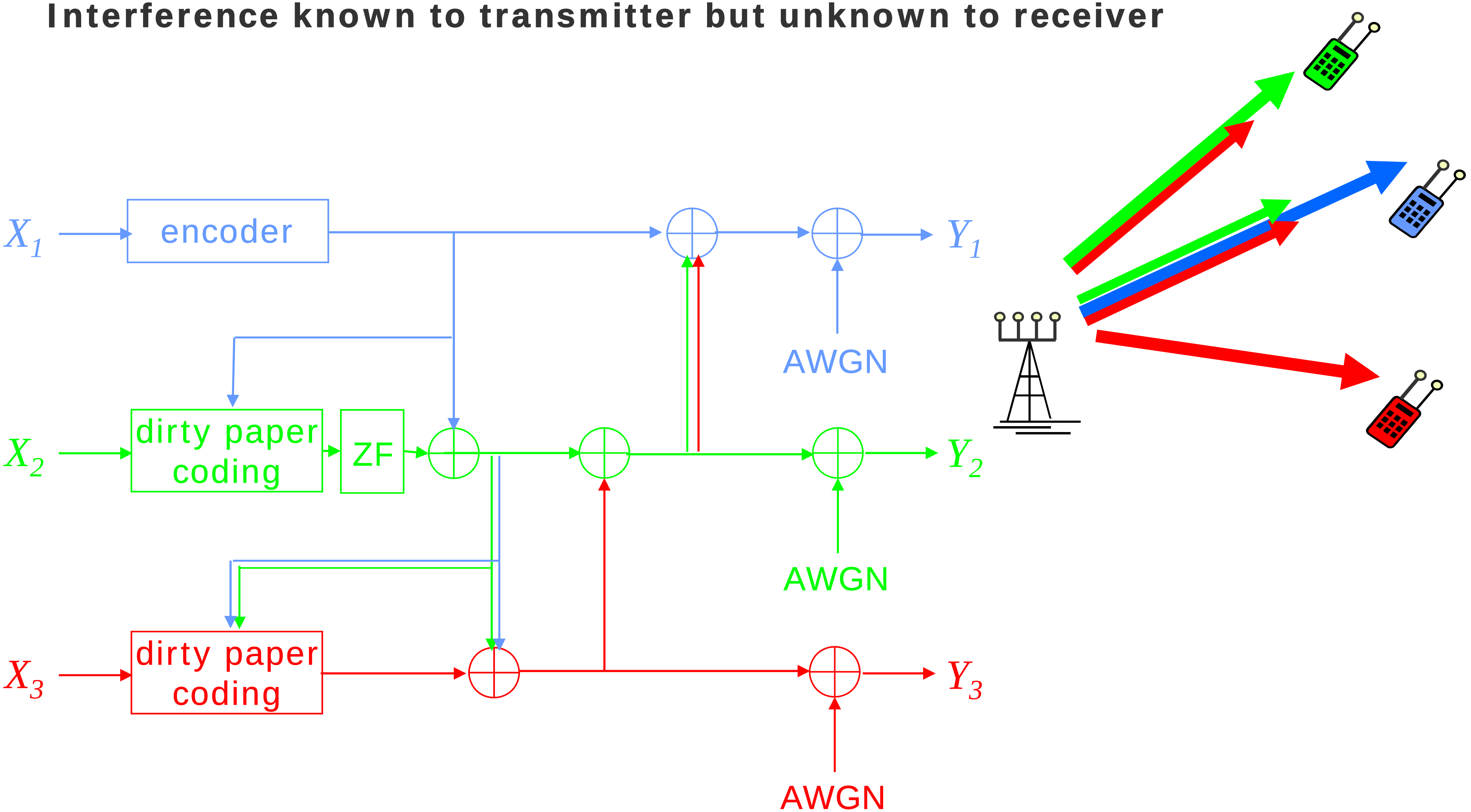
# Example: MIMO Broadcast Channel

**Interference known to transmitter but unknown to receiver**



# Example: MIMO Broadcast Channel with Sequential Encoding and Zero-Forcing (ZF)

**Interference known to transmitter but unknown to receiver**



# Example: MIMO Broadcast Channel with Sequential Encoding and Zero-Forcing (ZF)

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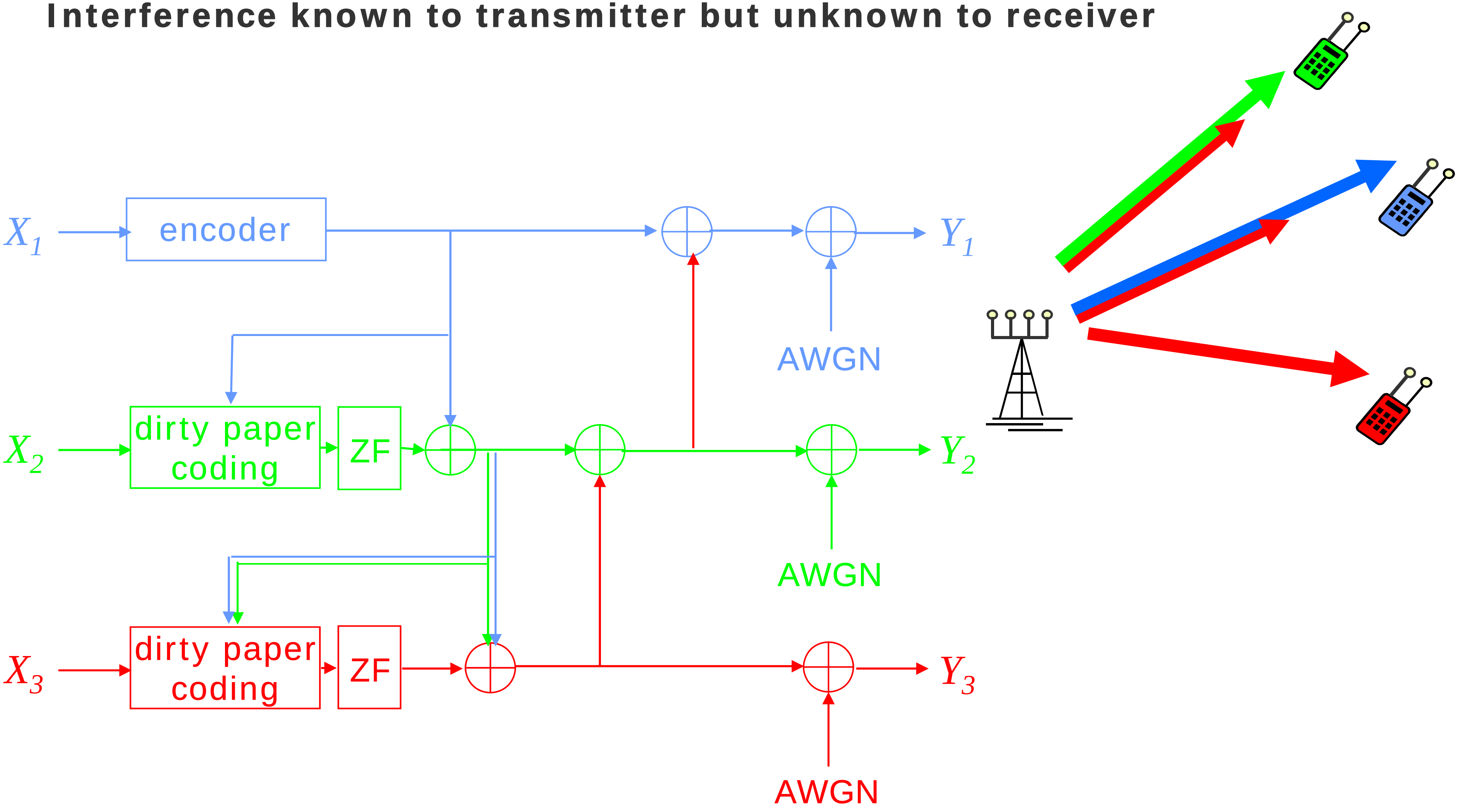
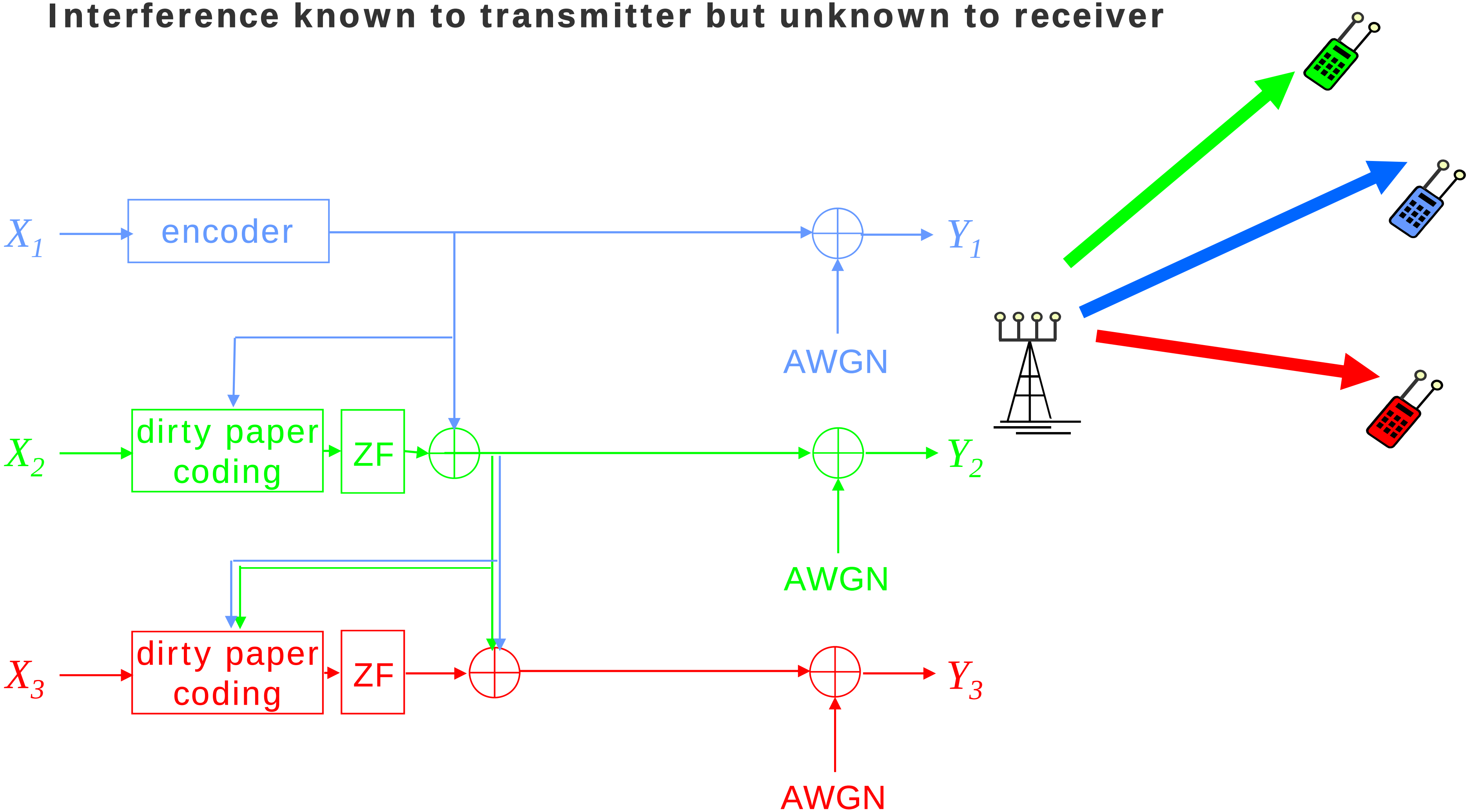
**Interference known to transmitter but unknown to receiver**



# Example: MIMO Broadcast Channel with Sequential Encoding and Zero-Forcing (ZF)



## Interference known to transmitter but unknown to receiver



# Outline (2)

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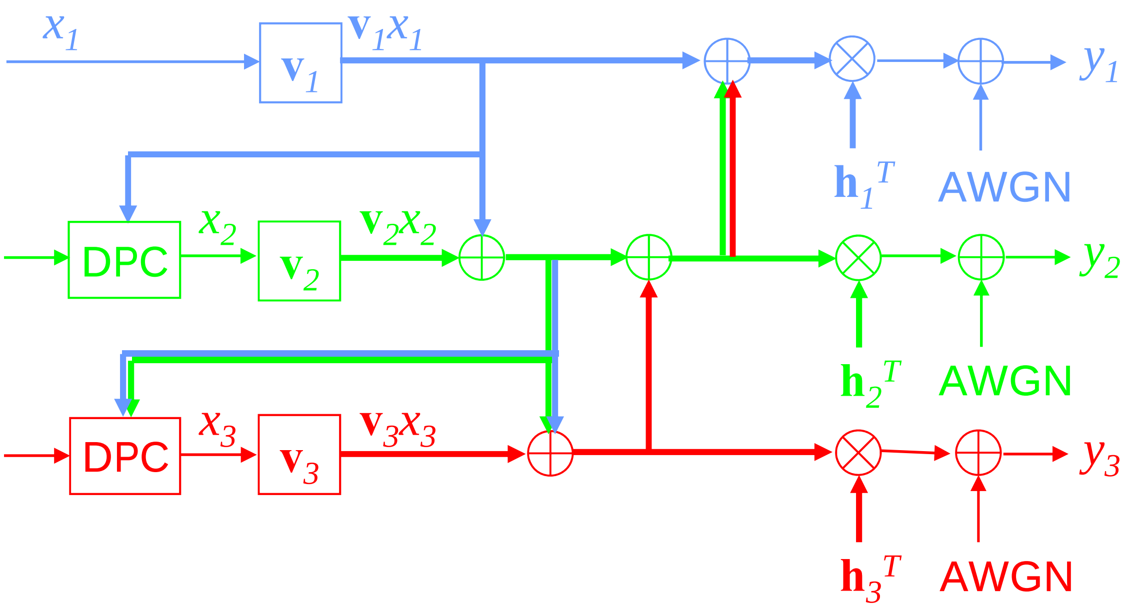
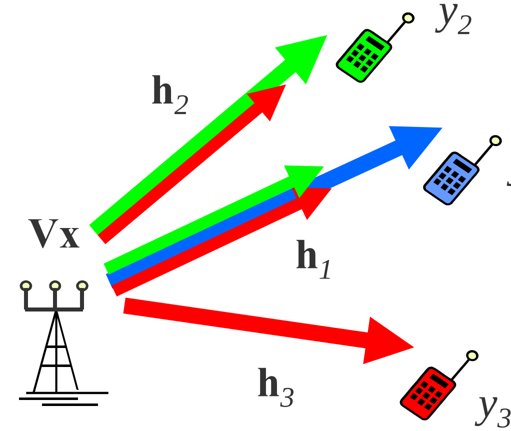
- Non-linear MU-MIMO algorithms based on Dirty Paper Coding (DPC) and zero-forcing (ZF)
  - Sequential encoding with DPC and ZF for single receive antennas
  - Sequential encoding with DPC and block zero-forcing (block ZF)
  - SESAM: A capacity approaching algorithm
  - Comparison of achievable rates

- Theoretical limits
  - Capacity of the SU-MIMO channel
  - Capacity region of the MIMO multiple-access channel (MAC)
  - Sum capacity of the MIMO broadcast channel (Sato bound)
  - DPC and dual MAC region of the MIMO broadcast channel
  - Capacity region of the MIMO broadcast channel

## Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)




1 Rx antenna per user




$$y_k = \underbrace{\sum_{i=1}^{k-1} \mathbf{h}_k^T \mathbf{v}_i x_i}_{\text{eliminated by DPC}} + \underbrace{\mathbf{h}_k^T \mathbf{v}_k x_k}_{\text{desired signal}} + \underbrace{\sum_{j=k+1}^{K} \mathbf{h}_k^T \mathbf{v}_j x_j}_{\text{eliminated by ZF}} + n_k$$

$$\mathbf{v}_k = \begin{bmatrix} v_{1k} \\ \vdots \\ v_{n_T k} \end{bmatrix} \qquad \mathbf{h}_k = \begin{bmatrix} h_k^{(1)} \\ \vdots \\ h_k^{(n_T)} \end{bmatrix}$$

**Gram-Schmidt Procedure:**

Design $\mathbf{v}_k$ such that it is orthonormal to all $\mathbf{v}_l$, $l<k$ by subtracting a linear combination of all previous $\mathbf{v}_l$ from $\mathbf{h}_k^*$:

$$\mathbf{v}_k = \frac{\mathbf{P}_k \mathbf{h}_k^*}{\left\| \mathbf{P}_k \mathbf{h}_k^* \right\|}, \text{where} \quad \mathbf{P}_1 = \mathbf{I}_{n_T}, \quad \mathbf{P}_{k+1} = \mathbf{P}_k \left( \mathbf{I}_{n_T} - \mathbf{v}_k \mathbf{v}_k^H \right)$$

Goal:

$\mathbf{v}_l^H \mathbf{v}_k = 0$, for $l < k$ $\qquad \mathbf{v}_k^H \mathbf{v}_k = 1$

$\mathbf{h}_k^T \mathbf{v}_j = 0$, for $k < j$

# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)

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$\mathbf{v}_l^H \mathbf{v}_k = 0$, for $l < k$

**Proof:**

$$\mathbf{v}_1^H\mathbf{v}_2 = \frac{1}{\left\|\mathbf{P}_2\mathbf{h}_2^*\right\|}\mathbf{v}_1^H\left(\mathbf{I}_{n_T}-\mathbf{v}_1\mathbf{v}_1^H\right)\mathbf{h}_2^* = \frac{1}{\left\|\mathbf{P}_2\mathbf{h}_2^*\right\|}\Big(\underbrace{\mathbf{v}_1^H - \overbrace{\mathbf{v}_1^H\mathbf{v}_1}^{1}\mathbf{v}_1^H}_{0}\Big)\mathbf{h}_2^* = 0$$

$$\mathbf{v}_1^H\mathbf{v}_3 = \frac{1}{\left\|\mathbf{P}_3\mathbf{h}_3^*\right\|}\mathbf{v}_1^H\left(\mathbf{I}_{n_T}-\mathbf{v}_1\mathbf{v}_1^H\right)\left(\mathbf{I}_{n_T}-\mathbf{v}_2\mathbf{v}_2^H\right)\mathbf{h}_3^*$$

$$= \frac{1}{\left\|\mathbf{P}_3\mathbf{h}_3^*\right\|}\Big(\underbrace{\mathbf{v}_1^H - \overbrace{\mathbf{v}_1^H\mathbf{v}_1}^{1}\mathbf{v}_1^H}_{0}\Big)\left(\mathbf{I}_{n_T}-\mathbf{v}_2\mathbf{v}_2^H\right)\mathbf{h}_3^* = 0$$

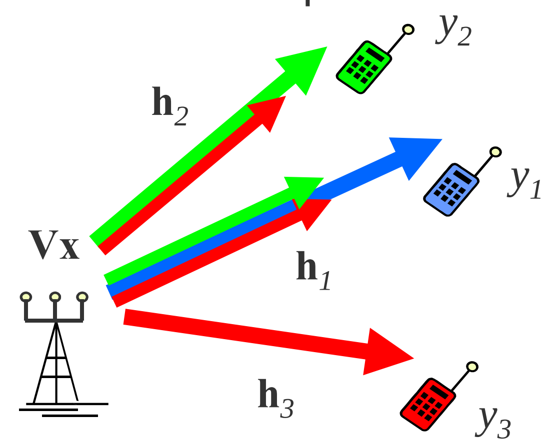
$$\mathbf{v}_l^H\mathbf{v}_k = \frac{1}{\left\|\mathbf{P}_k\mathbf{h}_k^*\right\|}\mathbf{v}_l^H\prod_{i=1}^{k-1}\left(\mathbf{I}_{n_T}-\mathbf{v}_i\mathbf{v}_i^H\right)\mathbf{h}_k^*$$

$$= \frac{1}{\left\|\mathbf{P}_k\mathbf{h}_k^*\right\|}\mathbf{v}_l^H\prod_{j=1}^{l-1}\left(\mathbf{I}_{n_T}-\mathbf{v}_j\mathbf{v}_j^H\right)\prod_{i=l}^{k-1}\left(\mathbf{I}_{n_T}-\mathbf{v}_i\mathbf{v}_i^H\right)\mathbf{h}_k^*$$

$$= \frac{1}{\left\|\mathbf{P}_k\mathbf{h}_k^*\right\|}\underbrace{\Big(\underbrace{\mathbf{v}_l^H - \overbrace{\mathbf{v}_l^H\mathbf{v}_1}^{0}\mathbf{v}_1^H}_{\mathbf{v}_l^H}\Big)\prod_{j=2}^{l-1}\left(\mathbf{I}_{n_T}-\mathbf{v}_j\mathbf{v}_j^H\right)}_{\mathbf{v}_l^H}\prod_{i=l}^{k-1}\left(\mathbf{I}_{n_T}-\mathbf{v}_i\mathbf{v}_i^H\right)\mathbf{h}_k^*$$

$$= \frac{1}{\left\|\mathbf{P}_k\mathbf{h}_k^*\right\|}\underbrace{\Big(\mathbf{v}_l^H - \underbrace{\mathbf{v}_l^H\mathbf{v}_l}_{1}\mathbf{v}_l^H\Big)}_{0}\prod_{i=l+1}^{k-1}\left(\mathbf{I}_{n_T}-\mathbf{v}_i\mathbf{v}_i^H\right)\mathbf{h}_k^* = 0$$

1 Rx antenna per user

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Design $\mathbf{v}_k$ such that it is orthonormal to all $\mathbf{v}_l$, $l<k$ by subtracting a linear combination of all previous $\mathbf{v}_l$ from $\mathbf{h}_k^*$:

$$\mathbf{v}_k = \frac{\mathbf{P}_k\mathbf{h}_k^*}{\left\|\mathbf{P}_k\mathbf{h}_k^*\right\|}, \text{where} \quad \begin{array}{l}\mathbf{P}_1 = \mathbf{I}_{n_T} \\ \mathbf{P}_{k+1} = \mathbf{P}_k\left(\mathbf{I}_{n_T}-\mathbf{v}_k\mathbf{v}_k^H\right)\end{array}$$

Goal:

$\mathbf{v}_l^H\mathbf{v}_k = 0$, for $l < k$ $\qquad$ $\mathbf{v}_k^H\mathbf{v}_k = 1$

$\mathbf{h}_k^T\mathbf{v}_j = 0$, for $k < j$

# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)

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Projector matrix:

$$\mathbf{P}_k = \mathbf{P}_k^H$$

$$\mathbf{P}_k \mathbf{P}_k = \mathbf{P}_k$$

Proof:

$$\mathbf{P}_{k+1} = \left(\mathbf{I}_{n_T} - \mathbf{v}_1\mathbf{v}_1^H\right)\left(\mathbf{I}_{n_T} - \mathbf{v}_2\mathbf{v}_2^H\right)\prod_{i=3}^{k}\left(\mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H\right)$$

$$= \left(\mathbf{I}_{n_T} - \mathbf{v}_1\mathbf{v}_1^H - \mathbf{v}_2\mathbf{v}_2^H + \mathbf{v}_1 \underbrace{\mathbf{v}_1^H\mathbf{v}_2}_{0} \mathbf{v}_2^H\right)\prod_{i=3}^{k}\left(\mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H\right)$$

$$= \mathbf{I}_{n_T} - \sum_{i=1}^{k}\mathbf{v}_i\mathbf{v}_i^H = \mathbf{P}_{k+1}^H$$

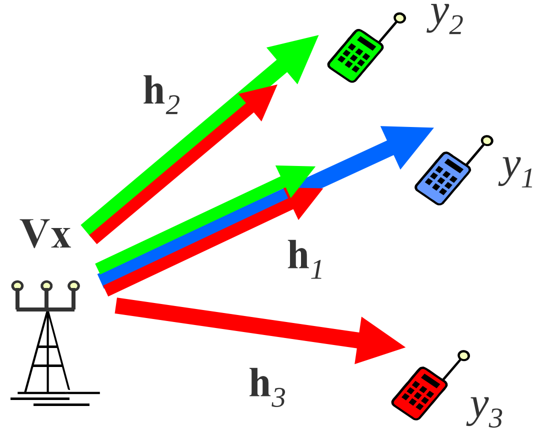
$$\mathbf{P}_k\mathbf{P}_k = \prod_{i=1}^{k-1}\left[\left(\mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H\right)\left(\mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H\right)\right]$$

$$= \prod_{i=1}^{k-1}\left(\mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H \underbrace{- \mathbf{v}_i\mathbf{v}_i^H + \mathbf{v}_i \overbrace{\mathbf{v}_i^H\mathbf{v}_i}^{1} \mathbf{v}_i^H}_{\mathbf{0}}\right) = \mathbf{P}_k$$

since $\left(\mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H\right)\left(\mathbf{I}_{n_T} - \mathbf{v}_j\mathbf{v}_j^H\right) = \mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H - \mathbf{v}_j\mathbf{v}_j^H + \mathbf{v}_i \underbrace{\mathbf{v}_i^H\mathbf{v}_j}_{0 \text{ for } i \neq j} \mathbf{v}_j^H$

$$= \left(\mathbf{I}_{n_T} - \mathbf{v}_j\mathbf{v}_j^H\right)\left(\mathbf{I}_{n_T} - \mathbf{v}_i\mathbf{v}_i^H\right)$$

1 Rx antenna per user

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# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)

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$$\mathbf{h}_k^T \mathbf{v}_j = 0, \text{ for } k < j$$

**Proof:**

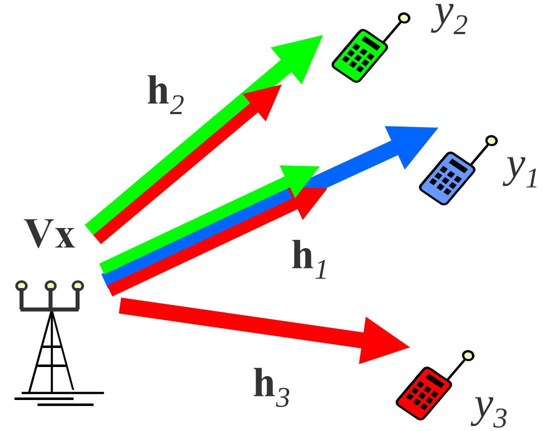
$$\mathbf{P}_k = \mathbf{P}_k^H$$

$$\mathbf{h}_k^T \mathbf{v}_j = \frac{1}{\left\|\mathbf{P}_j \mathbf{h}_j^*\right\|} \mathbf{h}_k^T \mathbf{P}_j \mathbf{h}_j^*$$

$$= \frac{1}{\left\|\mathbf{P}_j \mathbf{h}_j^*\right\|} \underbrace{\mathbf{h}_k^T \mathbf{P}_k}_{\left\|\mathbf{P}_k \mathbf{h}_k^*\right\| \mathbf{v}_k^H} \left(\mathbf{I}_{n_T} - \mathbf{v}_k \mathbf{v}_k^H\right) \prod_{i=k+1}^{j-1} \left(\mathbf{I}_{n_T} - \mathbf{v}_i \mathbf{v}_i^H\right) \mathbf{h}_j^*$$

$$= \frac{\left\|\mathbf{P}_k \mathbf{h}_k^*\right\|}{\left\|\mathbf{P}_j \mathbf{h}_j^*\right\|} \underbrace{\left(\mathbf{v}_k^H - \underbrace{\mathbf{v}_k^H \mathbf{v}_k}_{1} \mathbf{v}_k^H\right)}_{0} \prod_{i=k+1}^{j-1} \left(\mathbf{I}_{n_T} - \mathbf{v}_i \mathbf{v}_i^H\right) \mathbf{h}_j^* = 0$$

1 Rx antenna per user

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# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)


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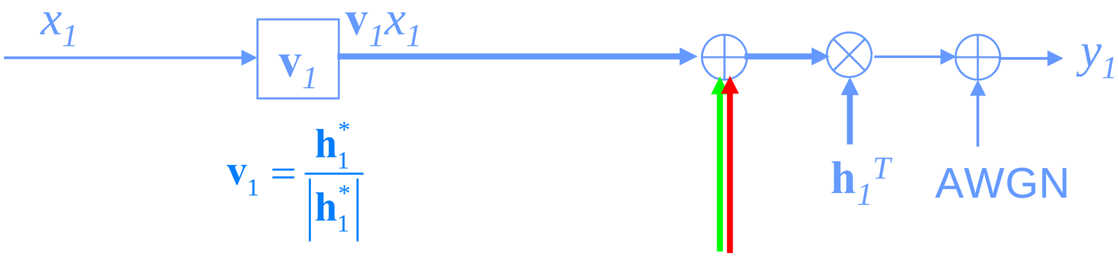
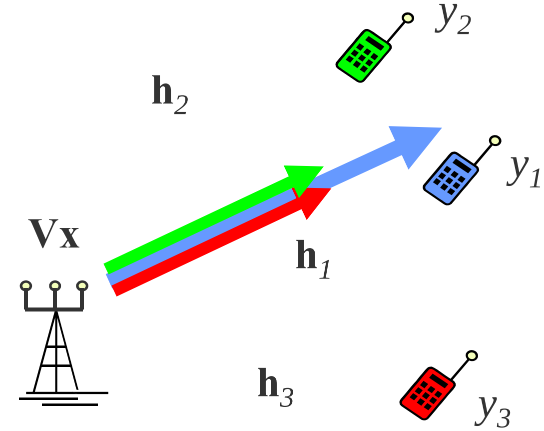



$$\mathbf{v}_1 = \frac{\mathbf{h}_1^*}{|\mathbf{h}_1^*|}$$

Tx matched filter

1 Rx antenna per user




$$y_k = \underbrace{\sum_{i=1}^{k-1} \mathbf{h}_k^T \mathbf{v}_i x_i}_{\text{eliminated by DPC}} + \underbrace{\mathbf{h}_k^T \mathbf{v}_k x_k}_{\text{desired signal}} + \underbrace{\sum_{j=k+1}^{K} \mathbf{h}_k^T \mathbf{v}_j x_j}_{\text{eliminated by ZF}} + n_k$$

$$\mathbf{v}_k = \begin{bmatrix} v_{1k} \\ \vdots \\ v_{n_T k} \end{bmatrix} \qquad \mathbf{h}_k = \begin{bmatrix} h_k^{(1)} \\ \vdots \\ h_k^{(n_T)} \end{bmatrix}$$

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# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)

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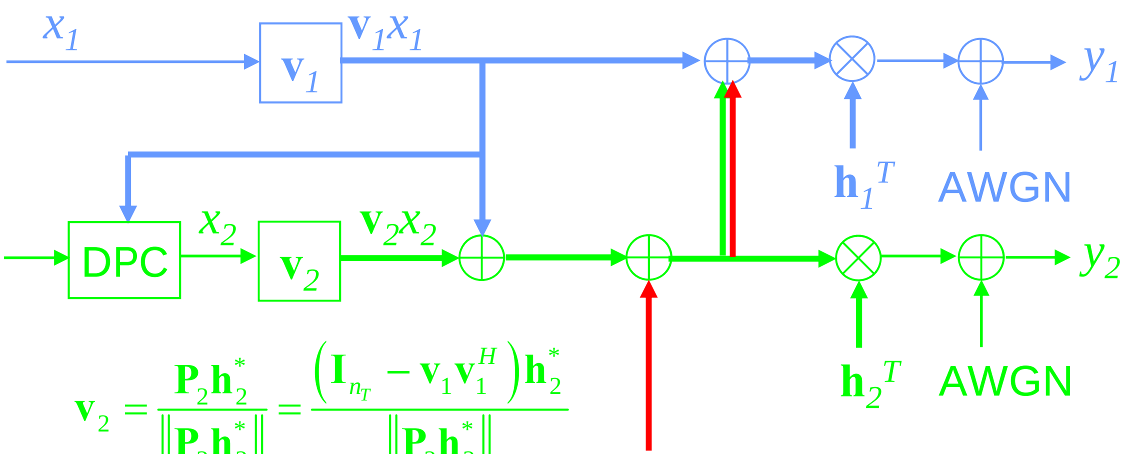
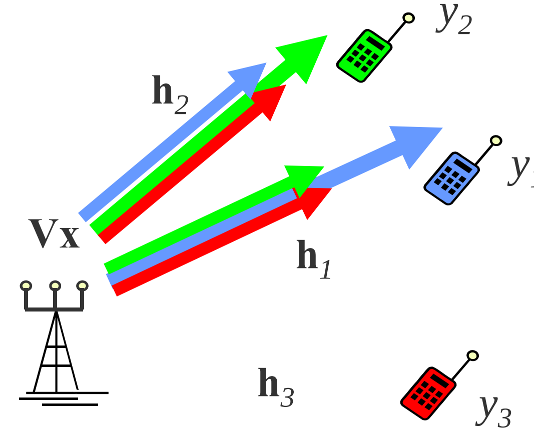



$$\mathbf{v}_2 = \frac{\mathbf{P}_2\mathbf{h}_2^*}{\left\|\mathbf{P}_2\mathbf{h}_2^*\right\|} = \frac{\left(\mathbf{I}_{n_T} - \mathbf{v}_1\mathbf{v}_1^H\right)\mathbf{h}_2^*}{\left\|\mathbf{P}_2\mathbf{h}_2^*\right\|}$$

1 Rx antenna per user




$$y_k = \underbrace{\sum_{i=1}^{k-1} \mathbf{h}_k^T \mathbf{v}_i x_i}_{\text{eliminated by DPC}} + \underbrace{\mathbf{h}_k^T \mathbf{v}_k x_k}_{\text{desired signal}} + \underbrace{\sum_{j=k+1}^{K} \mathbf{h}_k^T \mathbf{v}_j x_j}_{\text{eliminated by ZF}} + n_k$$

$$\mathbf{v}_k = \begin{bmatrix} v_{1k} \\ \vdots \\ v_{n_T k} \end{bmatrix} \qquad \mathbf{h}_k = \begin{bmatrix} h_k^{(1)} \\ \vdots \\ h_k^{(n_T)} \end{bmatrix}$$

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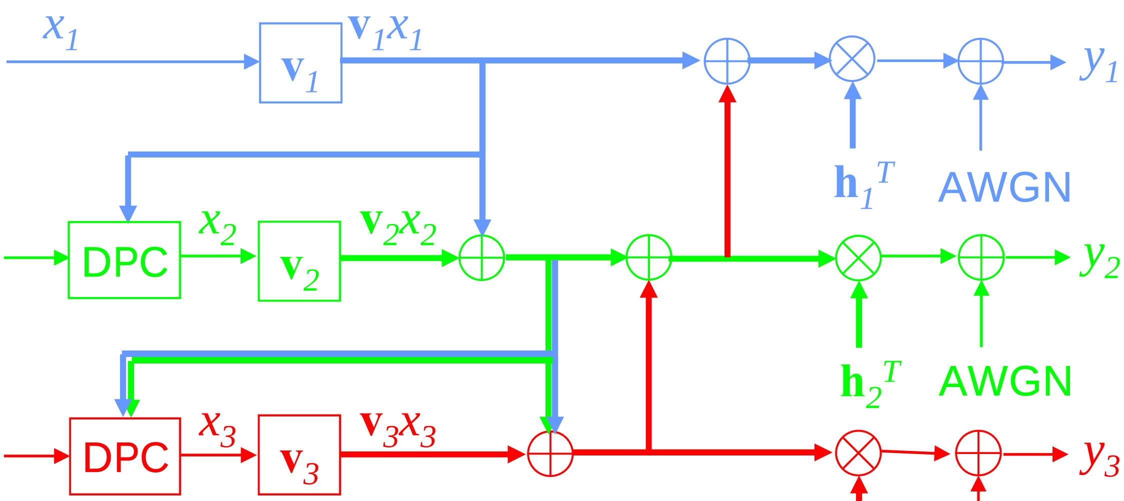
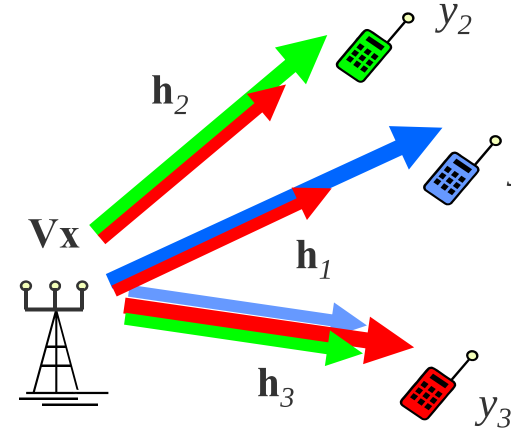
# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)






$$\mathbf{v}_3 = \frac{\mathbf{P}_3 \mathbf{h}_3^*}{\left\|\mathbf{P}_3 \mathbf{h}_3^*\right\|} = \frac{\mathbf{P}_2 \left(\mathbf{I}_{n_T} - \mathbf{v}_2 \mathbf{v}_2^H\right) \mathbf{h}_3^*}{\left\|\mathbf{P}_3 \mathbf{h}_3^*\right\|}$$

1 Rx antenna per user

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# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)

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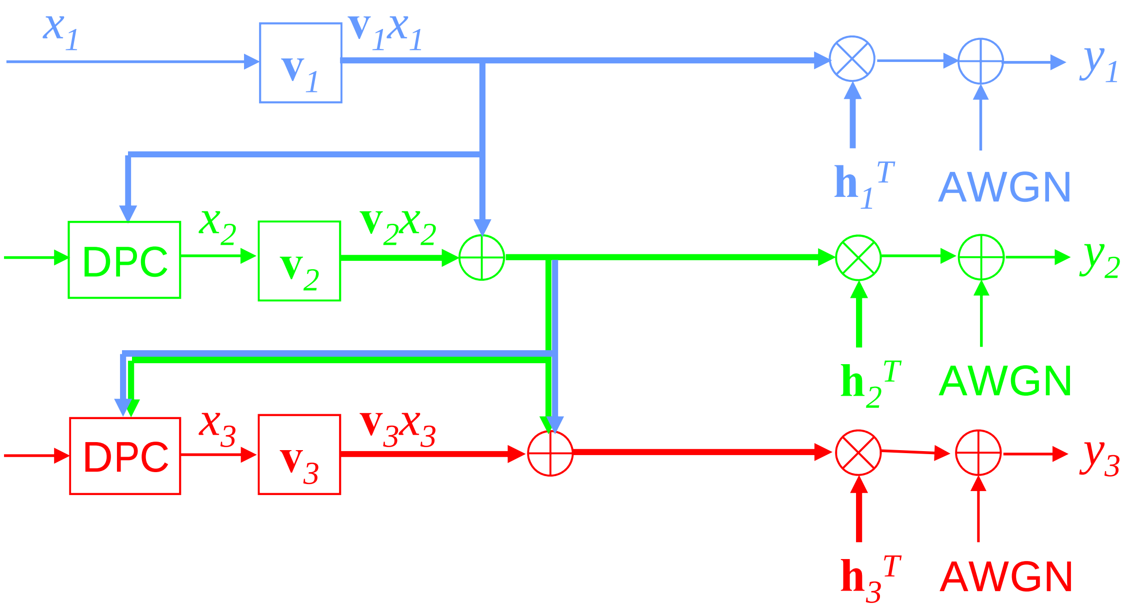
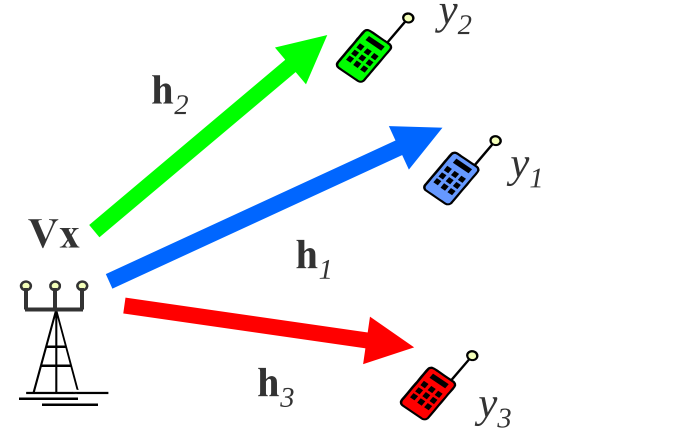



1 Rx antenna per user




$$y_k = \underbrace{\sum_{i=1}^{k-1} \mathbf{h}_k^T \mathbf{v}_i x_i}_{\text{eliminated by DPC}} + \underbrace{\mathbf{h}_k^T \mathbf{v}_k x_k}_{\text{desired signal}} + \underbrace{\sum_{j=k+1}^{K} \mathbf{h}_k^T \mathbf{v}_j x_j}_{\text{eliminated by ZF}} + n_k$$

$$\mathbf{v}_k = \begin{bmatrix} v_{1k} \\ \vdots \\ v_{n_T k} \end{bmatrix} \qquad \mathbf{h}_k = \begin{bmatrix} h_k^{(1)} \\ \vdots \\ h_k^{(n_T)} \end{bmatrix}$$

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Design $\mathbf{v}_k$ such that it is orthonormal to all $\mathbf{v}_l$, $l<k$ by subtracting a linear combination of all previous $\mathbf{v}_l$ from $\mathbf{h}_k^*$:

$$\mathbf{v}_k = \frac{\mathbf{P}_k \mathbf{h}_k^*}{\left\| \mathbf{P}_k \mathbf{h}_k^* \right\|}, \text{where} \quad \begin{aligned} \mathbf{P}_1 &= \mathbf{I}_{n_T} \\ \mathbf{P}_{k+1} &= \mathbf{P}_k \left( \mathbf{I}_{n_T} - \mathbf{v}_k \mathbf{v}_k^H \right) \end{aligned}$$

Goal:

$\mathbf{v}_l^H \mathbf{v}_k = 0$, for $l < k$ $\qquad \mathbf{v}_k^H \mathbf{v}_k = 1$

$\mathbf{h}_k^T \mathbf{v}_j = 0$, for $k < j$

# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)

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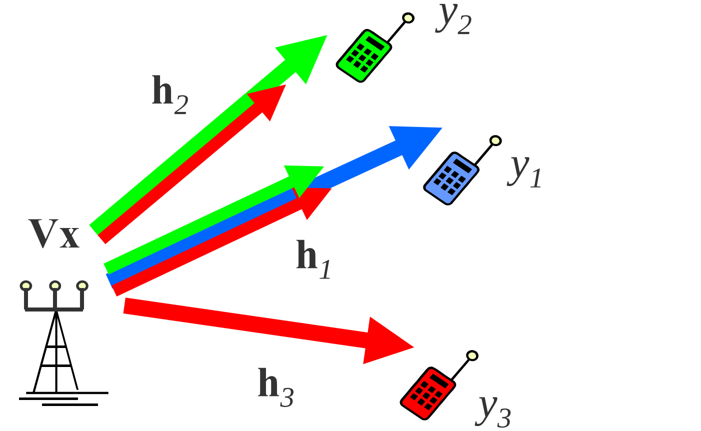
## LQ Decomposition:

$$\mathbf{H} = \mathbf{L}\mathbf{Q}$$

$$\mathbf{y} = \mathbf{H}\mathbf{Q}^H\mathbf{x} + \mathbf{n} = \mathbf{L}\underbrace{\mathbf{Q}\mathbf{Q}^H}_{\mathbf{I}_{n_T}}\mathbf{x} + \mathbf{n} = \mathbf{L}\mathbf{x} + \mathbf{n}$$

where $\mathbf{L} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$

1 Rx antenna per user




## Gram-Schmidt Procedure:

$$\mathbf{y} = \mathbf{H}\mathbf{V}\mathbf{x} + \mathbf{n} = \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \\ \vdots \\ \mathbf{h}_K^T \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_K \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix} + \mathbf{n}$$

$$= \begin{bmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & 0 \\ l_{K1} & l_{K2} & l_{K3} & \cdots & l_{KK} \end{bmatrix} \mathbf{x} + \mathbf{n} = \mathbf{L}\mathbf{x} + \mathbf{n}$$

$\mathbf{Q}^H \;\hat{=}\; \mathbf{V}$

**Gram-Schmidt Procedure:**
Design $\mathbf{v}_k$ such that it is orthonormal to all $\mathbf{v}_l$, $l<k$ by subtracting a linear combination of all previous $\mathbf{v}_l$ from $\mathbf{h}_k^*$:

$$\mathbf{v}_k = \frac{\mathbf{P}_k \mathbf{h}_k^*}{\left\| \mathbf{P}_k \mathbf{h}_k^* \right\|}, \text{where} \quad \begin{aligned} \mathbf{P}_1 &= \mathbf{I}_{n_T} \\ \mathbf{P}_{k+1} &= \mathbf{P}_k \left( \mathbf{I}_{n_T} - \mathbf{v}_k \mathbf{v}_k^H \right) \end{aligned}$$

Goal:

$\mathbf{v}_l^H \mathbf{v}_k = 0$, for $l < k$ $\qquad \mathbf{v}_k^H \mathbf{v}_k = 1$

$\mathbf{h}_k^T \mathbf{v}_j = 0$, for $k < j$

# Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)

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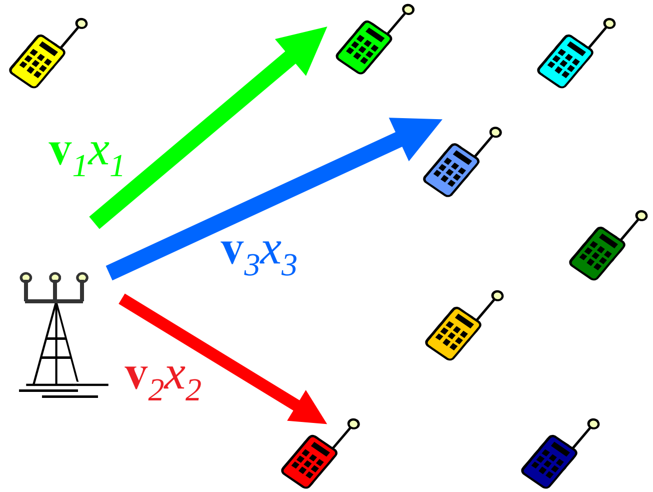
Sequential generation of precoding vectors $\mathbf{v}_k$ with Gram-Schmidt procedure allows

- optimized ordering of users
- scheduling of users ($K > n_T$)

1 Rx antenna per user




# users $K$ > # Tx antennas $n_T$
→ only $n_T$ users can be scheduled

**Heuristic scheduling rule:**

At each step $k$, $k=1,\ldots,n_T$:

- Compute precoding vector $\mathbf{v}_l$ for all users $l$ which are not yet served.
- For each user $l$, determine the effective channel $\mathbf{h}_l^T\mathbf{v}_l$ and the resulting sum capacity increment which would be achieved if user $l$ was scheduled at step $k$.
- Allocate the respective resource unit to the user who achieves the highest sum capacity increment.

**Gram-Schmidt Procedure:**

$$\mathbf{v}_k = \frac{\mathbf{P}_k \mathbf{h}_k^*}{\left\|\mathbf{P}_k \mathbf{h}_k^*\right\|}, \text{where} \quad \mathbf{P}_1 = \mathbf{I}_{n_T}, \quad \mathbf{P}_{k+1} = \mathbf{P}_k\left(\mathbf{I}_{n_T} - \mathbf{v}_k \mathbf{v}_k^H\right)$$

Goal:

$\mathbf{v}_l^H \mathbf{v}_k = 0$, for $l < k$ $\qquad$ $\mathbf{v}_k^H \mathbf{v}_k = 1$

$\mathbf{h}_k^T \mathbf{v}_j = 0$, for $k < j$

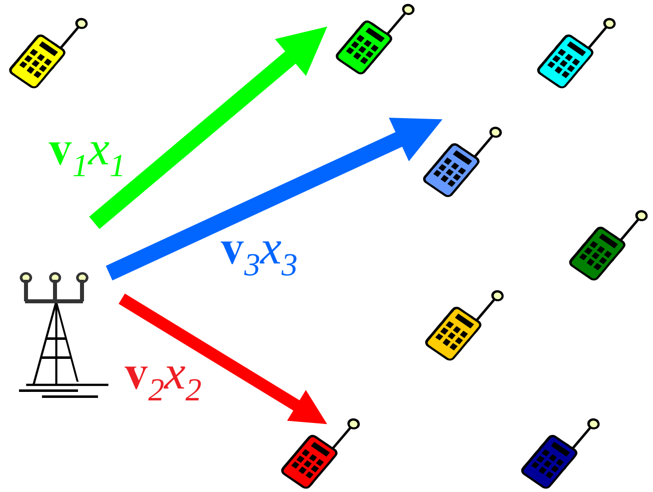
# Sum Capacity of Sequential Encoding with Dirty Paper Coding (DPC) and Zero-Forcing (ZF)



Sequential generation of precoding vectors $\mathbf{v}_k$ with Gram-Schmidt procedure allows

- optimized ordering of users
- scheduling of users ($K > n_T$)

1 Rx antenna per user




**Heuristic scheduling rule:**

At each step $k$, $k=1,\ldots,n_T$:

- Compute precoding vector $\mathbf{v}_l$ for all users $l$ which are not yet served.
- For each user $l$, determine the effective channel $\mathbf{h}_l^T\mathbf{v}_l$ and the resulting sum capacity increment which would be achieved if user $l$ was scheduled at step $k$.
- Allocate the respective resource unit to the user who achieves the highest sum capacity increment.

**Sum capacity:**

$$C = \sum_{j=1}^{n_T} \log_2\left(1 + \frac{P_x^{(\Pi(j))}}{P_n^{(\Pi(j))}}\left|\mathbf{h}_{\Pi(j)}^T \mathbf{v}_{\Pi(j)}\right|\right)$$

$\Pi(j)$: Ordering: The $j^{\text{th}}$ spatial resource is allocated to user $\Pi(j)$.

$P_x^{(\Pi(j))}$: Power which is allocated to user $\Pi(j)$ (waterfilling).

$P_n^{(\Pi(j))}$: Noise power per Rx antenna at user $\Pi(j)$.

# System Model

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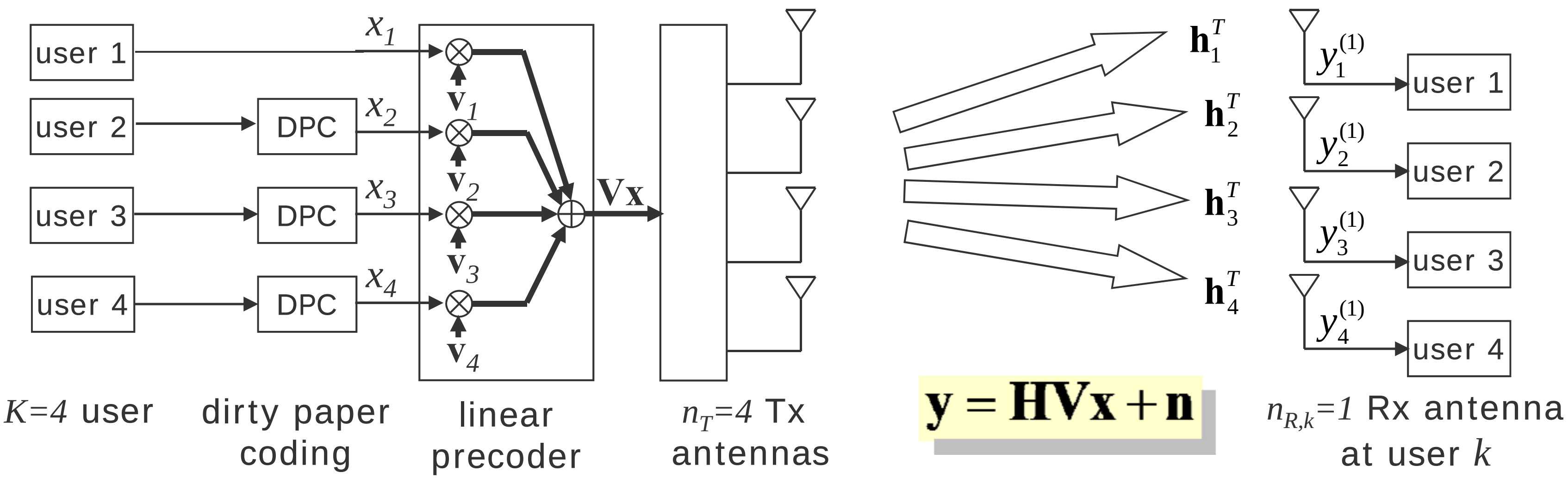


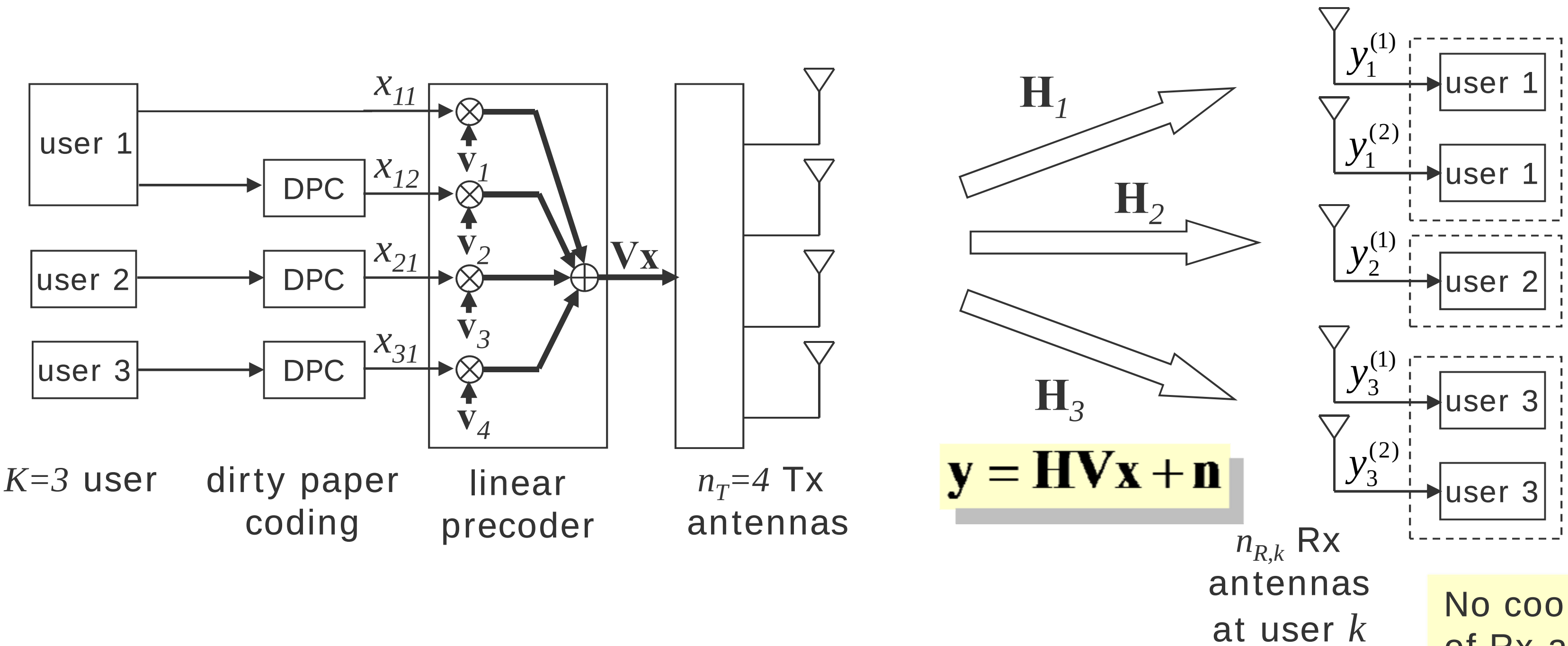

No cooperation of Rx antennas.

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_K \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \qquad = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1K} \\ v_{21} & v_{22} & \cdots & v_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_T 1} & v_{n_T 2} & \cdots & v_{n_T K} \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \\ \vdots \\ \mathbf{h}_K^T \end{bmatrix} = \begin{bmatrix} h_1^{(11)} & \cdots & h_1^{(n_T 1)} \\ h_2^{(11)} & \cdots & h_2^{(n_T 1)} \\ \vdots & \ddots & \vdots \\ h_K^{(11)} & \cdots & h_K^{(n_T 1)} \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = \begin{bmatrix} y_1^{(1)} \\ y_2^{(1)} \\ \vdots \\ y_K^{(1)} \end{bmatrix}$$

# System Model



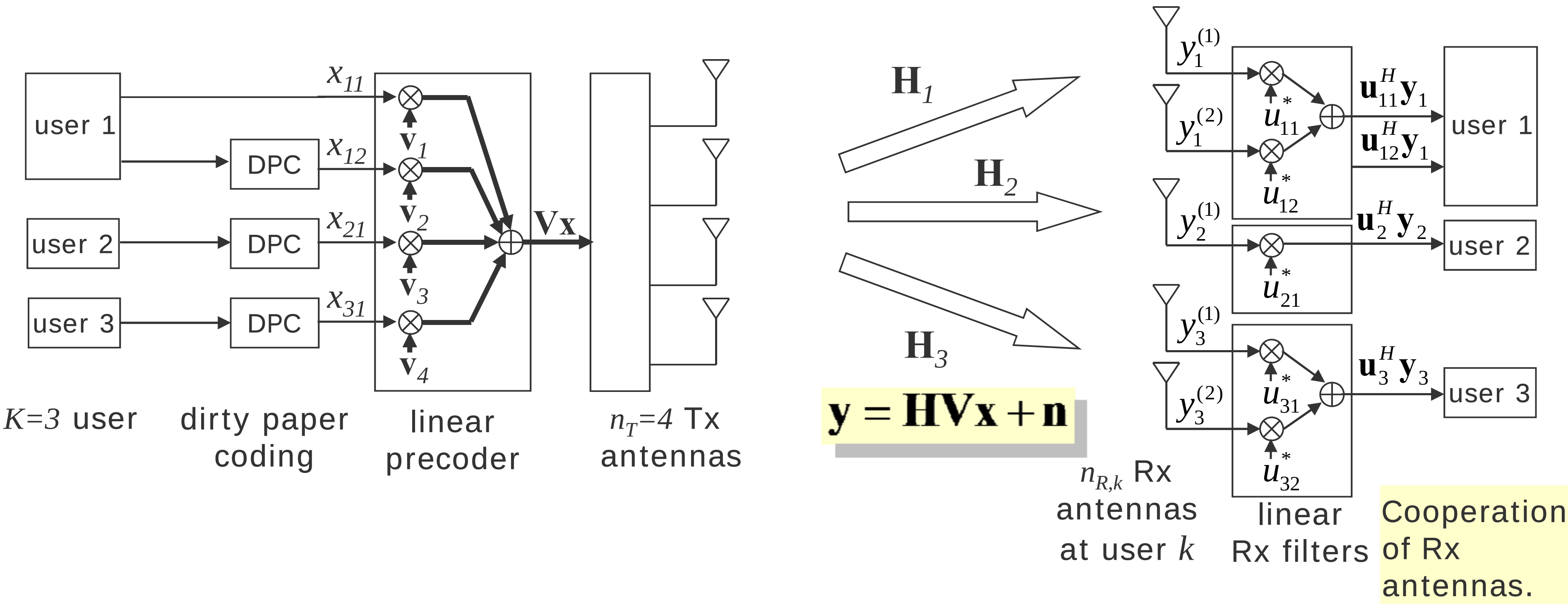

$$\mathbf{y} = \mathbf{HVx} + \mathbf{n}$$

No cooperation of Rx antennas.

$$\mathbf{x} = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{31} \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_K \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1K} \\ v_{21} & v_{22} & \cdots & v_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_T 1} & v_{n_T 2} & \cdots & v_{n_T K} \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11}^T \\ \vdots \\ \mathbf{h}_{1,n_{R,1}}^T \\ \vdots \\ \mathbf{h}_{K1}^T \\ \vdots \\ \mathbf{h}_{Kn_{R,K}}^T \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11}^T \\ \mathbf{h}_{12}^T \\ \mathbf{h}_{21}^T \\ \mathbf{h}_{31}^T \\ \mathbf{h}_{32}^T \end{bmatrix} = \begin{bmatrix} h_1^{(11)} & \cdots & h_1^{(n_T 1)} \\ h_1^{(12)} & \cdots & h_1^{(n_T 2)} \\ h_2^{(11)} & \cdots & h_2^{(n_T 1)} \\ h_3^{(11)} & \cdots & h_3^{(n_T 1)} \\ h_3^{(12)} & \cdots & h_3^{(n_T 2)} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = \begin{bmatrix} y_1^{(1)} \\ y_1^{(2)} \\ y_2^{(1)} \\ y_3^{(1)} \\ y_3^{(2)} \end{bmatrix}$$

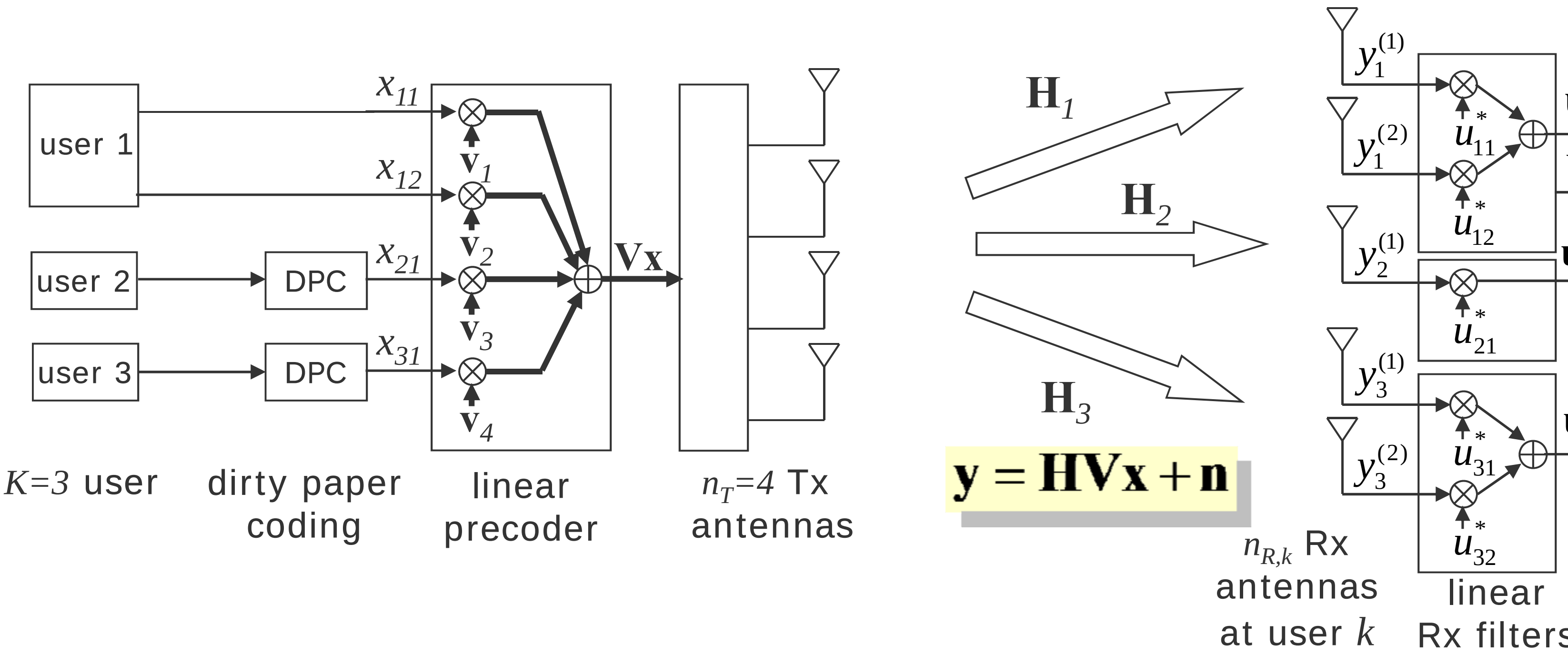
# System Model




$$\mathbf{x} = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{31} \end{bmatrix}, \qquad \mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_K \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1K} \\ v_{21} & v_{22} & \cdots & v_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_T 1} & v_{n_T 2} & \cdots & v_{n_T K} \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11}^T \\ \vdots \\ \mathbf{h}_{1,n_{R,1}}^T \\ \vdots \\ \mathbf{h}_{K1}^T \\ \vdots \\ \mathbf{h}_{Kn_{R,K}}^T \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11}^T \\ \mathbf{h}_{12}^T \\ \mathbf{h}_{21}^T \\ \mathbf{h}_{31}^T \\ \mathbf{h}_{32}^T \end{bmatrix} = \begin{bmatrix} h_1^{(11)} & \cdots & h_1^{(n_T 1)} \\ h_1^{(12)} & \cdots & h_1^{(n_T 2)} \\ h_2^{(11)} & \cdots & h_2^{(n_T 1)} \\ h_3^{(11)} & \cdots & h_3^{(n_T 1)} \\ h_3^{(12)} & \cdots & h_3^{(n_T 2)} \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = \begin{bmatrix} y_1^{(1)} \\ y_1^{(2)} \\ y_2^{(1)} \\ y_3^{(1)} \\ y_3^{(2)} \end{bmatrix}$$

# System Model




Cooperation of Rx antennas.

$$\mathbf{x} = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{31} \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_K \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1K} \\ v_{21} & v_{22} & \cdots & v_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_T 1} & v_{n_T 2} & \cdots & v_{n_T K} \end{bmatrix},$$

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# Sequential Encoding with Dirty Paper Coding (DPC) and Block Zero-Forcing (Block ZF)

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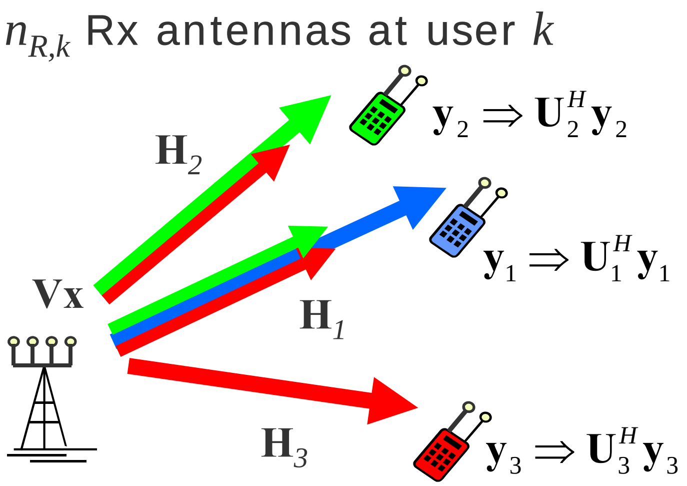
- $n_{R,k} \leq n_T$ Rx antennas at user $k$ allow to allocate $n_{R,k}$ spatial streams to user $k$.
- Interference between spatial streams of a particular user $k$ can be resolved by Rx processing.
- Only interference between spatial streams of different users has to be avoided by Tx processing.

$\Rightarrow$Suppress only interference to Rx antennas of other users by zero-forcing (block ZF, cooperative ZF)).



# Sequential Encoding with Dirty Paper Coding (DPC) and Block Zero-Forcing (Block ZF)

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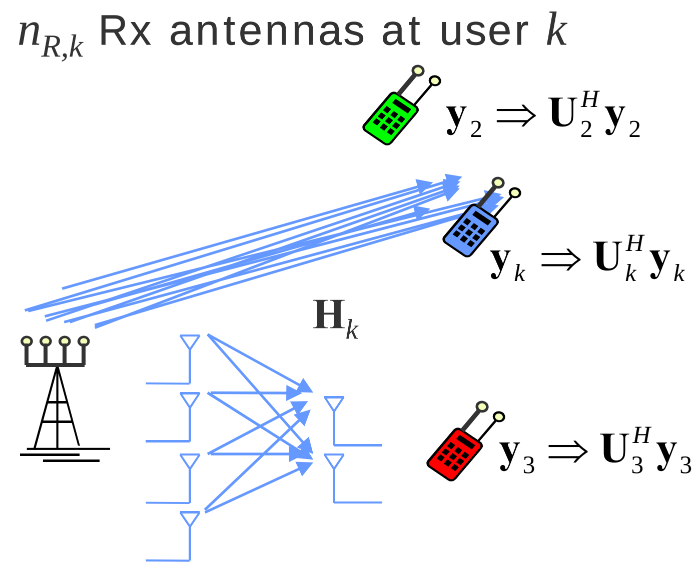


⇒Suppress only interference to Rx antennas of other users by zero-forcing (block ZF, cooperative ZF).

⇒ Diagonalize effective channel (including ZF filter) of desired user by singular value decomposition (SVD):

$$\mathbf{H}_k \mathbf{P}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$$

$$\mathbf{U}_k^H \mathbf{y}_k = \underbrace{\mathbf{U}_k^H \mathbf{U}_k}_{\mathbf{I}_{n_{R,k}}} \mathbf{\Lambda}_k \underbrace{\mathbf{V}_k^H \mathbf{V}_k}_{\mathbf{I}_{n_{R,k}}} \mathbf{x}_k + \mathbf{U}_k^H \mathbf{n}_k = \begin{bmatrix} \lambda_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \lambda_{n_{R,k}} \end{bmatrix} + \mathbf{U}_k^H \mathbf{n}_k$$

# Sequential Encoding with Dirty Paper Coding (DPC) and Block Zero-Forcing (Block ZF)

- $n_{R,k} < n_T$ Rx antennas at user $k$ allow to allocate $n_{R,k}$ spatial streams to user $k$.
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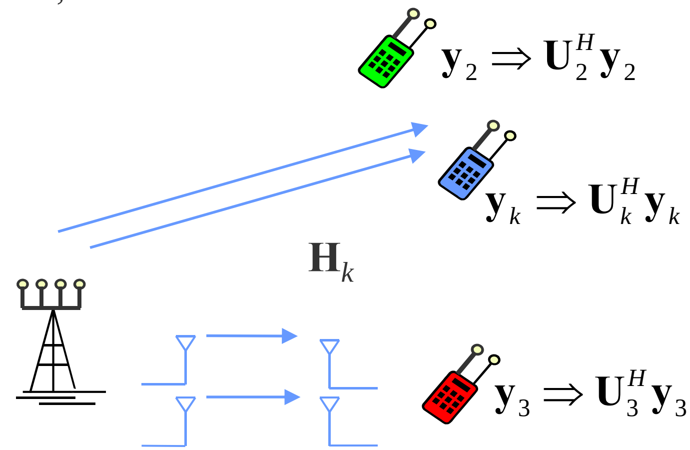


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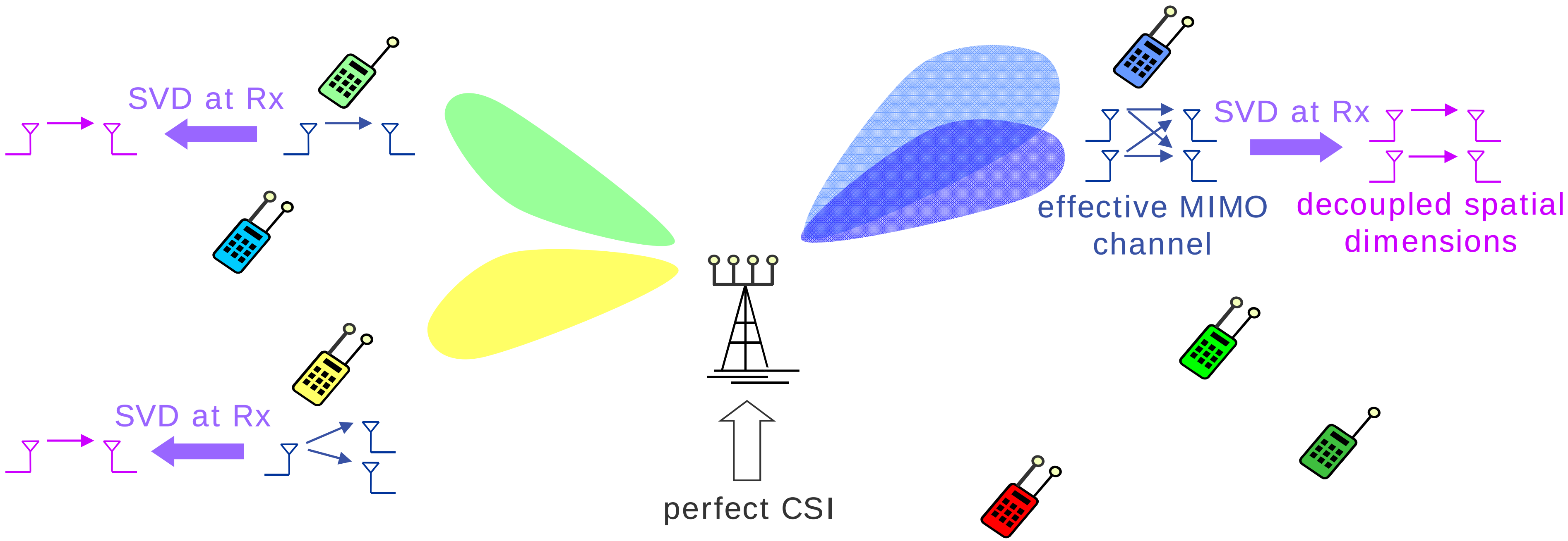
Capacity is increased compared to zero forcing with non-cooperative receive antennas since cooperation among receive antennas of the same user is exploited.

# Multiuser MIMO – Principle of CZF-SESAM



CZF-SESAM: **C**ooperative **Z**ero **F**orzing – **S**equential **E**ncoding **S**equential **A**llocation **M**ethod




**Idea:**

- Use beamforming at Tx and Rx, and successive encoding/interference cancellation based on Dirty Paper Coding at Tx in order to provide interference-free channel dimensions to several users.
- Exploit multiuser-diversity by resource allocation in space, frequency and time.

**Critical Assumption:**

- Perfect channel state information (CSI) at Tx and Rx.

**Optimization criterion:**
Maximization of sum capacity/cell throughput

# Multiuser MIMO – Principle of CZF-SESAM

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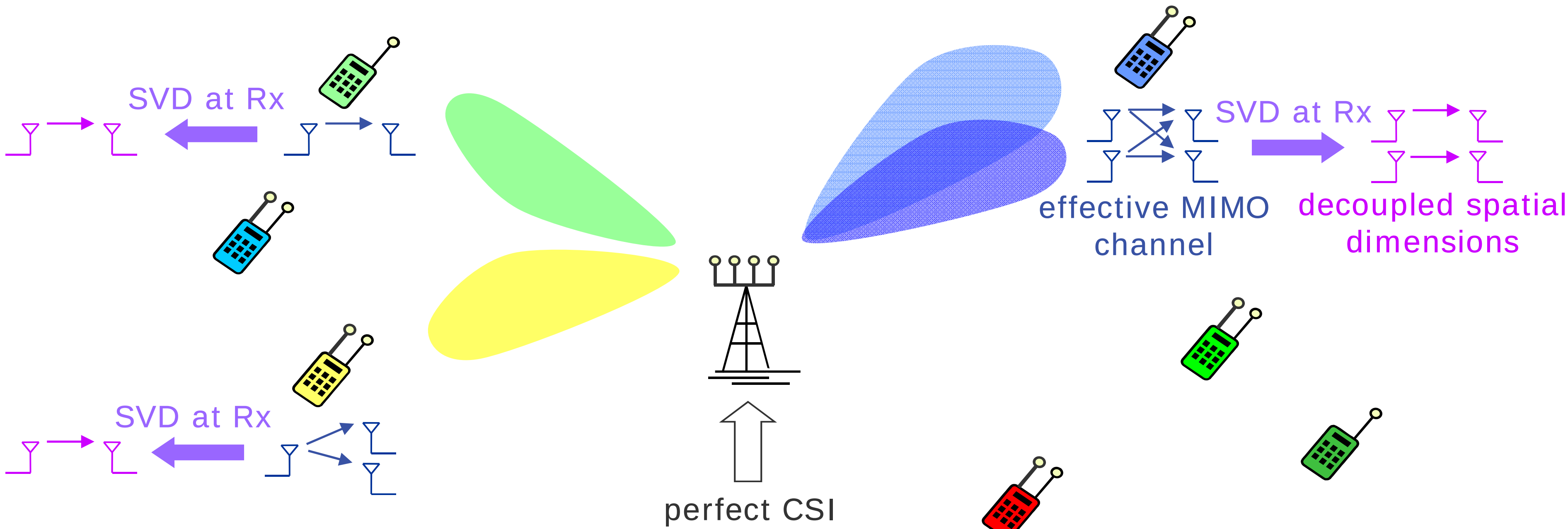
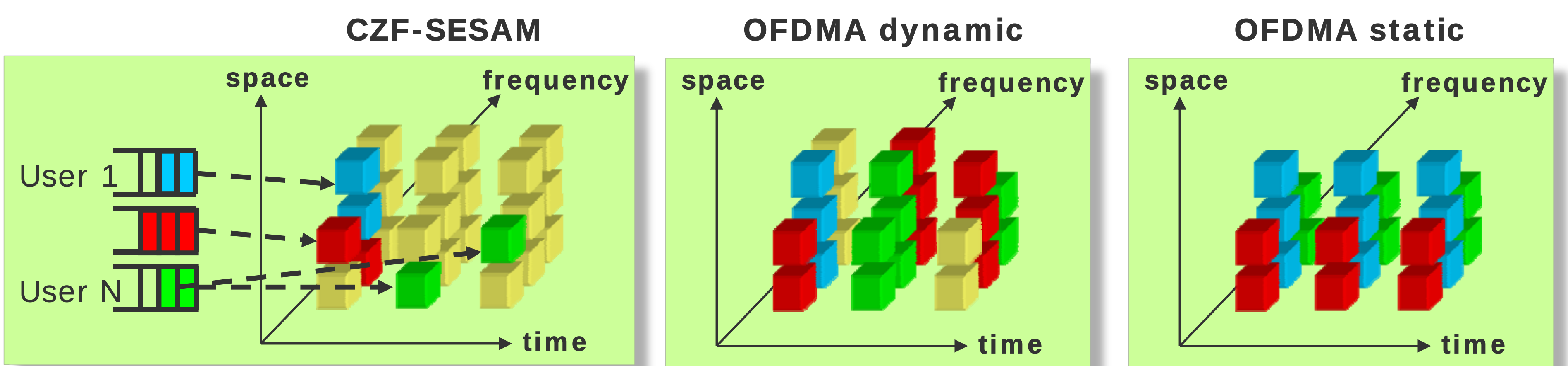
CZF-SESAM: **C**ooperative **Z**ero **F**orzing – **S**equential **E**ncoding **S**equential **A**llocation **M**ethod



# Performance

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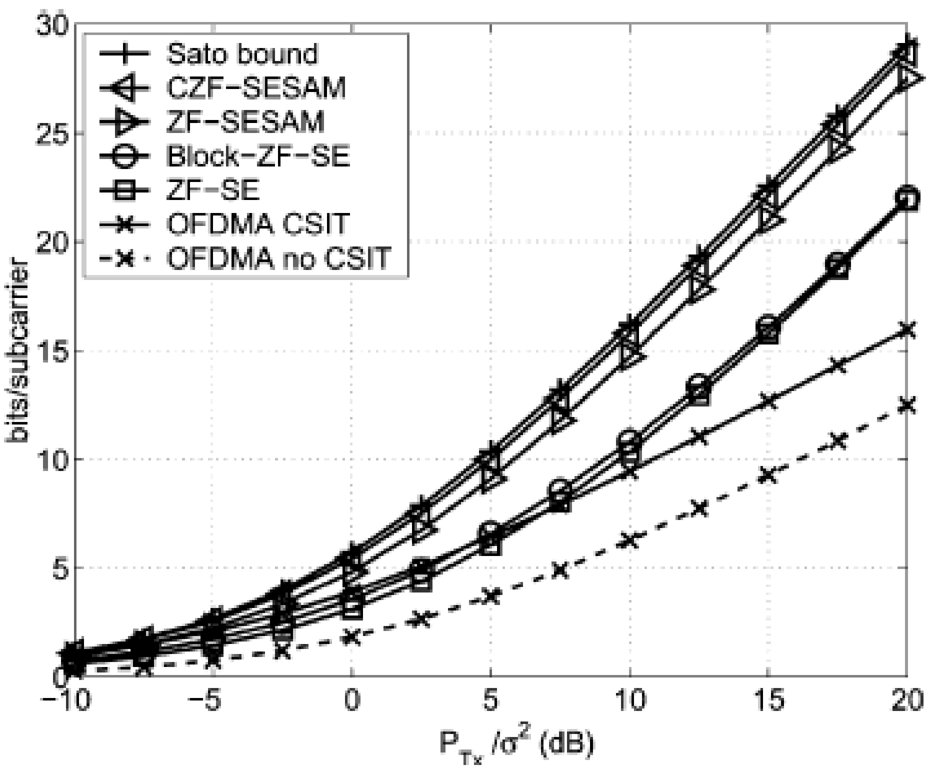
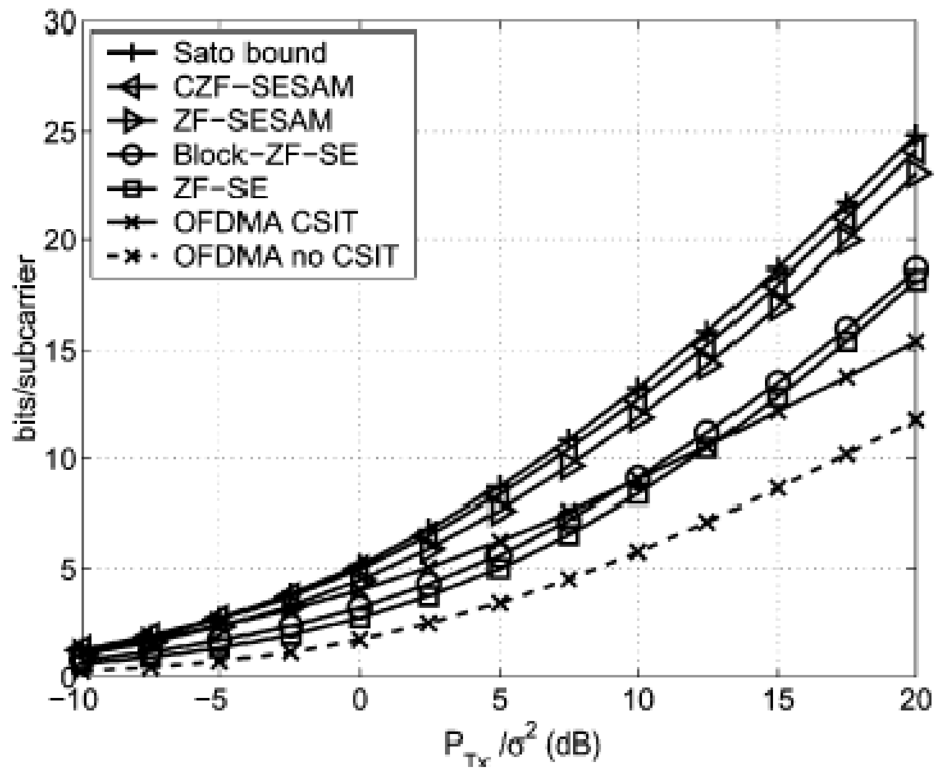
4 Tx antennas,  2 Rx antennas per user, 10 users, Rayleigh fading

**uncorrelated channel**




**correlated channel**



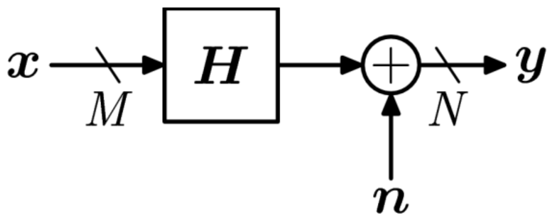
# Outline (2)

- Non-linear MU-MIMO algorithms based on Dirty Paper Coding (DPC) and zero-forcing (ZF)
  - Sequential encoding with DPC and ZF for single receive antennas
  - Sequential encoding with DPC and block zero-forcing (block ZF)
  - SESAM: A capacity approaching algorithm
  - Comparison of achievable rates

- Theoretical limits
  - Capacity of the SU-MIMO channel
  - Capacity region of the MIMO multiple-access channel (MAC)
  - Sum capacity of the MIMO broadcast channel (Sato bound)
  - DPC and dual MAC region of the MIMO broadcast channel
  - Capacity region of the MIMO broadcast channel

# Capacity of the SU MIMO Channel

*[Foschini & Gans '98, Telatar '99]*




Multivariate proper complex Gaussian distributed signal and noise:

$$\boldsymbol{x} \sim \mathcal{N}_{\mathbb{C}}\left(\boldsymbol{0}_M, \boldsymbol{K}_{\boldsymbol{x}}\right),$$

$$\boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}\left(\boldsymbol{0}_N, \boldsymbol{K}_{\boldsymbol{n}}\right) \quad \text{with } \boldsymbol{K}_{\boldsymbol{n}} > 0$$

Mutual information:

$$\begin{aligned} I(\boldsymbol{x};\boldsymbol{y}) &= h(\boldsymbol{y}) - h(\boldsymbol{y}|\boldsymbol{x}) = h(\boldsymbol{y}) - h(\boldsymbol{n}) \\ &= \log\left|\pi e\, \boldsymbol{K}_{\boldsymbol{y}}\right| - \log\left|\pi e\, \boldsymbol{K}_{\boldsymbol{n}}\right| \\ &= \log \frac{\left|\boldsymbol{K}_{\boldsymbol{n}} + \boldsymbol{H}\boldsymbol{K}_{\boldsymbol{x}}\boldsymbol{H}^{\mathsf{H}}\right|}{\left|\boldsymbol{K}_{\boldsymbol{n}}\right|} \end{aligned}$$

Capacity if transmit power is limited to *P*:

$$C_{\mathsf{SU}} = \max_{\substack{\boldsymbol{K}_{\boldsymbol{x}} \in \mathbb{C}^{M\times M}: \\ \boldsymbol{K}_{\boldsymbol{x}} \geq 0,\ \mathrm{tr}\,\boldsymbol{K}_{\boldsymbol{x}} \leq P}} I(\boldsymbol{x};\boldsymbol{y})$$

# Capacity of the SU MIMO Channel

*[Foschini & Gans '98, Telatar '99]*

Capacity if transmit power is limited to *P*:

$$C_{\mathsf{SU}} = \max_{\substack{\boldsymbol{K_x} \in \mathbb{C}^{M \times M}: \\ \boldsymbol{K_x} \succeq 0,\ \mathrm{tr}\, \boldsymbol{K_x} \leq P}} \log \frac{\left|\boldsymbol{K_n} + \boldsymbol{H}\boldsymbol{K_x}\boldsymbol{H}^{\mathsf{H}}\right|}{|\boldsymbol{K_n}|}$$

$$\begin{aligned} \frac{\left|\boldsymbol{K_n} + \boldsymbol{H}\boldsymbol{K_x}\boldsymbol{H}^{\mathsf{H}}\right|}{|\boldsymbol{K_n}|} &= \left|\boldsymbol{K_n}\boldsymbol{K_n}^{-1} + \boldsymbol{H}\boldsymbol{K_x}\boldsymbol{H}^{\mathsf{H}}\boldsymbol{K_n}^{-1}\right| \\ &= \left|\boldsymbol{I}_M + \boldsymbol{K_x}\boldsymbol{H}^{\mathsf{H}}\boldsymbol{K_n}^{-1}\boldsymbol{H}\right| \end{aligned}$$

EigenValue Decomposition (EVD): $\boldsymbol{H}^{\mathsf{H}}\boldsymbol{K_n}^{-1}\boldsymbol{H} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^{\mathsf{H}}$

Orthonormal modal matrix: $\boldsymbol{Q} \in \mathbb{C}^{M \times D}$

Diagonal matrix of eigenvalues: $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \ldots, \lambda_D) \in \mathbb{R}_{0,+}^{D \times D}$

$\boldsymbol{\Lambda} > 0, \quad D = \mathrm{rank}\, \boldsymbol{H}$

# Capacity of the SU MIMO Channel



*[Foschini & Gans '98, Telatar '99]*

Capacity if transmit power is limited to *P*:

$$C_{\mathrm{SU}} = \max_{\substack{\boldsymbol{K_x} \in \mathbb{C}^{M\times M}: \\ \boldsymbol{K_x} \succeq 0,\ \mathrm{tr}\,\boldsymbol{K_x} \leq P}} \log \frac{\left|\boldsymbol{K_n} + \boldsymbol{H}\boldsymbol{K_x}\boldsymbol{H}^{\mathrm{H}}\right|}{\left|\boldsymbol{K_n}\right|}$$

$$\begin{aligned}\frac{\left|\boldsymbol{K_n} + \boldsymbol{H}\boldsymbol{K_x}\boldsymbol{H}^{\mathrm{H}}\right|}{\left|\boldsymbol{K_n}\right|} &= \left|\boldsymbol{I}_M + \boldsymbol{K_x}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{K_n}^{-1}\boldsymbol{H}\right| \\ &= \left|\boldsymbol{I}_M + \boldsymbol{K_x}\boldsymbol{Q}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Q}^{\mathrm{H}}\right| \\ &= \left|\boldsymbol{I}_D + \boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{Q}^{\mathrm{H}}\boldsymbol{K_x}\boldsymbol{Q}\boldsymbol{\Lambda}^{\frac{1}{2}}\right| \\ &= \left|\boldsymbol{I}_D + \boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{K_\xi}\boldsymbol{\Lambda}^{\frac{1}{2}}\right| \\ &\leq \prod_{i=1}^{D}\left(1 + \lambda_i k_{\xi_i}\right)\end{aligned}$$

Definition of transformed signal:

$$\boldsymbol{\xi} = \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{x} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_D, \boldsymbol{K_\xi})$$

Equality holds for diagonal matrix:

$$\boldsymbol{K_\xi} = \mathrm{diag}(k_{\xi_1}, \ldots, k_{\xi_D})$$

(Hadamard inequality)

# Capacity of the SU MIMO Channel

*[Foschini & Gans ’98, Telatar ’99]*

Capacity if transmit power is limited to *P*:

$$C_{\mathsf{SU}} = \max_{\substack{\boldsymbol{K}_{\boldsymbol{x}} \in \mathbb{C}^{M \times M}: \\ \boldsymbol{K}_{\boldsymbol{x}} \succeq 0,\ \operatorname{tr} \boldsymbol{K}_{\boldsymbol{x}} \leq P}} \log \frac{\left|\boldsymbol{K}_{\boldsymbol{n}} + \boldsymbol{H}\boldsymbol{K}_{\boldsymbol{x}}\boldsymbol{H}^{\mathsf{H}}\right|}{\left|\boldsymbol{K}_{\boldsymbol{n}}\right|}$$

$$\frac{\left|\boldsymbol{K}_{\boldsymbol{n}} + \boldsymbol{H}\boldsymbol{K}_{\boldsymbol{x}}\boldsymbol{H}^{\mathsf{H}}\right|}{\left|\boldsymbol{K}_{\boldsymbol{n}}\right|} \leq \prod_{i=1}^{D} \left(1 + \lambda_i k_{\xi_i}\right)$$

$$C_{\mathsf{SU}} = \max_{\substack{\{k_{\xi_i} \in \mathbb{R}_{0,+}\}_{i=1}^{D}: \\ \sum_{i=1}^{D} k_{\xi_i} \leq P}} \log \prod_{i=1}^{D} \left(1 + \lambda_i k_{\xi_i}\right)$$

# Capacity of the SU MIMO Channel



*[Foschini & Gans '98, Telatar '99]*

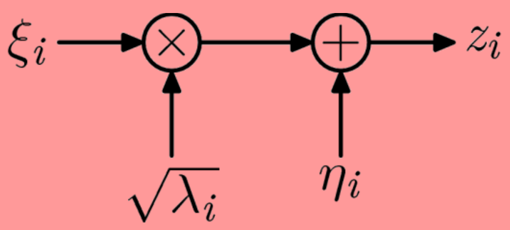
Capacity if transmit power is limited to *P*:

$$C_{\mathsf{SU}} = \max_{\substack{\boldsymbol{K_x} \in \mathbb{C}^{M \times M}: \\ \boldsymbol{K_x} \succeq 0,\ \mathrm{tr}\,\boldsymbol{K_x} \leq P}} \log \frac{\left|\boldsymbol{K_n} + \boldsymbol{H}\boldsymbol{K_x}\boldsymbol{H}^{\mathsf{H}}\right|}{\left|\boldsymbol{K_n}\right|}$$

With optimal variances $k_{\xi_i}$ according to waterfilling:

$$C_{\mathsf{SU}} = \sum_{i=1}^{D} \log\left(1 + \lambda_i k^*_{\xi_i}\right), \quad k^*_{\xi_i} = \max\left(0, \mu - \frac{1}{\lambda_i}\right), \quad \sum_{i=1}^{D} k^*_{\xi_i} = P$$

Same capacity than the following *D* decoupled SISO channels:

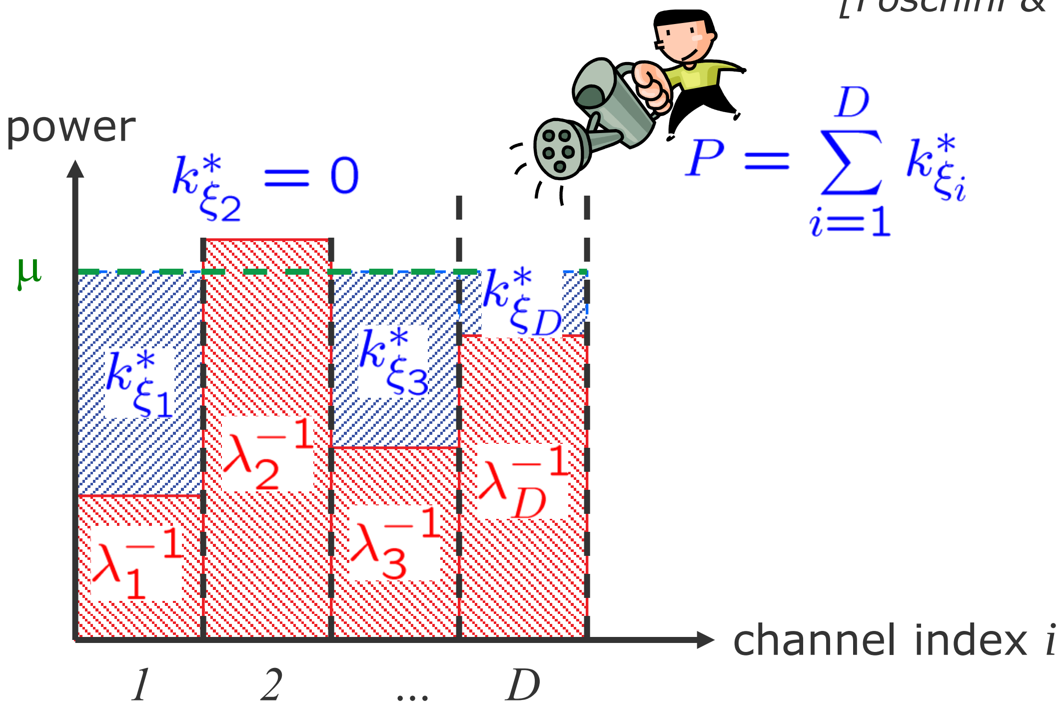



$$\xi_i \sim \mathcal{N}_{\mathbb{C}}(0, k^*_{\xi_i})$$
$$\eta_i \sim \mathcal{N}_{\mathbb{C}}(0, 1)$$

# Capacity of the SU MIMO Channel



*[Foschini & Gans '98, Telatar '99]*

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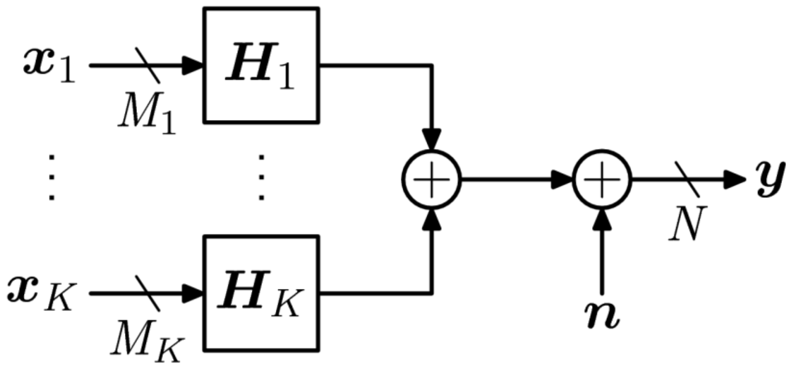
# Capacity Region of the MIMO MAC



[Cheng & Verdu '93, Yu et al. '04]

MIMO Multiple-Access Channel (MAC):




For all $k \in \{1,\ldots,K\}$:

$$\boldsymbol{x}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_M, \boldsymbol{K}_{\boldsymbol{x}_k})$$

$$\boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_N, \boldsymbol{I}_N)$$

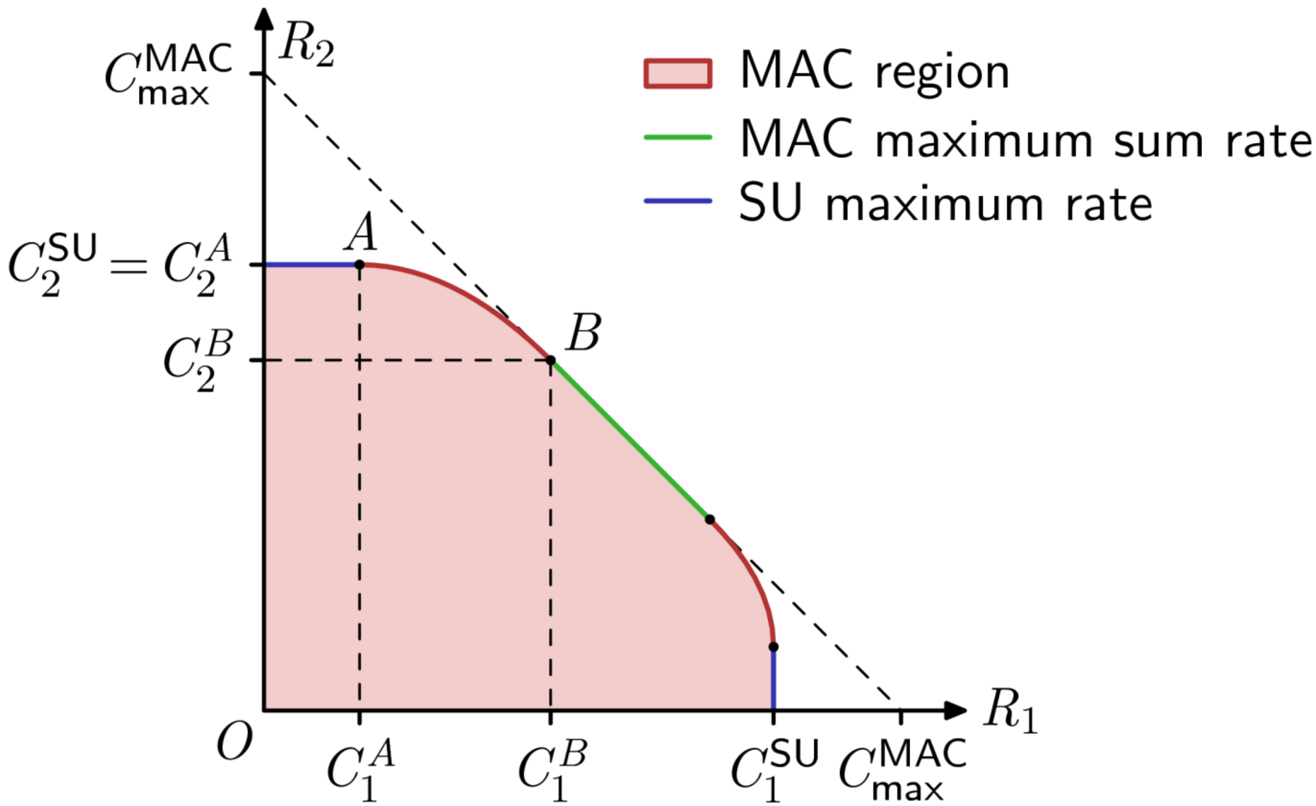
Capacity region if transmit power of user $k$ is limited to $P_k$:

$$\mathcal{C}_{\text{MAC}} = \bigcup_{\substack{\boldsymbol{K}_{\boldsymbol{x}_k} \in \mathbb{C}^{M_k \times M_k}: \\ \boldsymbol{K}_{\boldsymbol{x}_k} \succeq 0,\ \operatorname{tr} \boldsymbol{K}_{\boldsymbol{x}_k} \le P_k \\ \forall k \in \{1,\ldots,K\}}} \left\{ \begin{array}{l} (R_1, \ldots, R_K) \in \mathbb{R}^K_{0,+} : \\ \sum_{k \in \mathbb{K}} R_i \le \log \left| \boldsymbol{I}_N + \sum_{k \in \mathbb{K}} \boldsymbol{H}_k \boldsymbol{K}_{\boldsymbol{x}_k} \boldsymbol{H}_k^{\mathrm{H}} \right| \\ \forall \mathbb{K} \subseteq \{1, \ldots, K\} \end{array} \right\}$$

Note that for Gaussian channels, no convex hull is needed!

# Capacity Region of the MIMO MAC with 2 Users



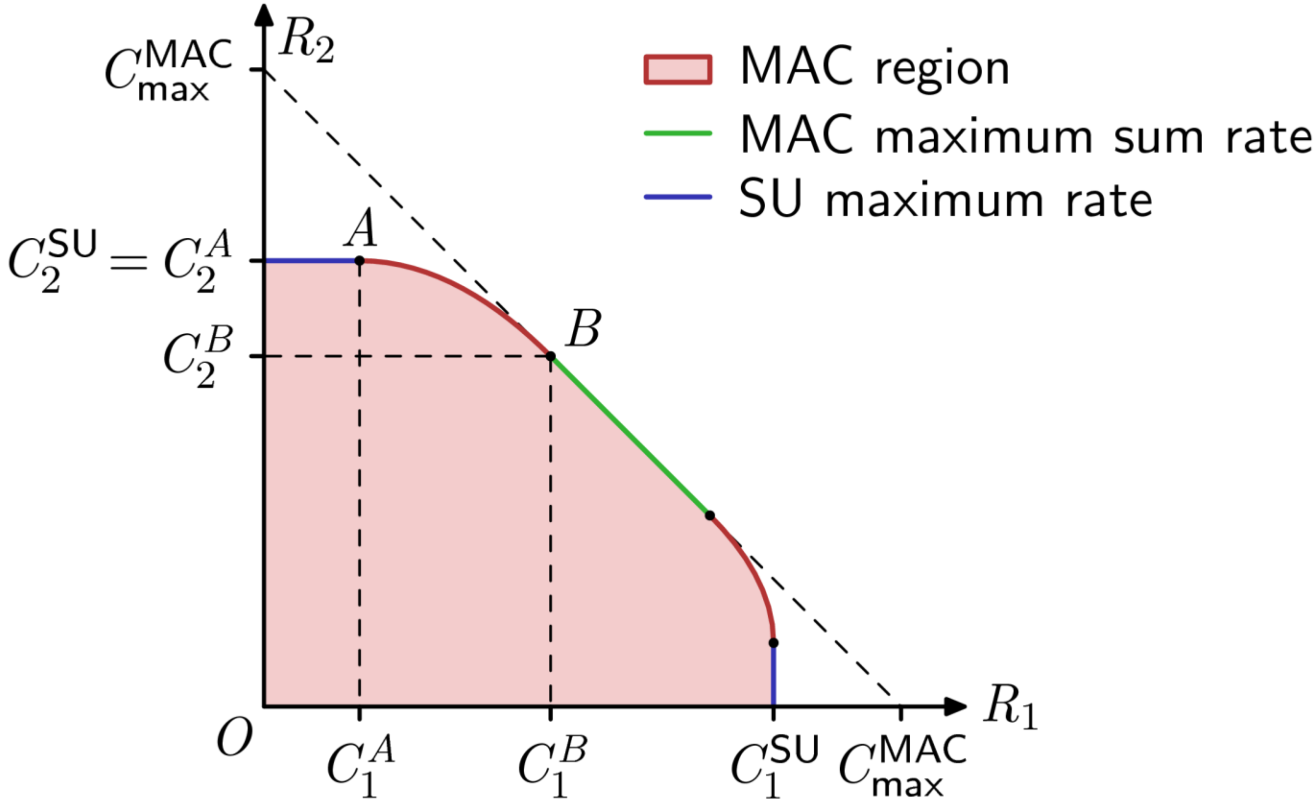



Application of successive decoding to achieve certain rate combinations (e.g., points *A* and *B*):

- Decode user 1 considering signal of user 2 as noise.
- Subtract interference from user 1 and decode user 2 following traditional SU MIMO

# Capacity Region of the MIMO MAC with 2 Users

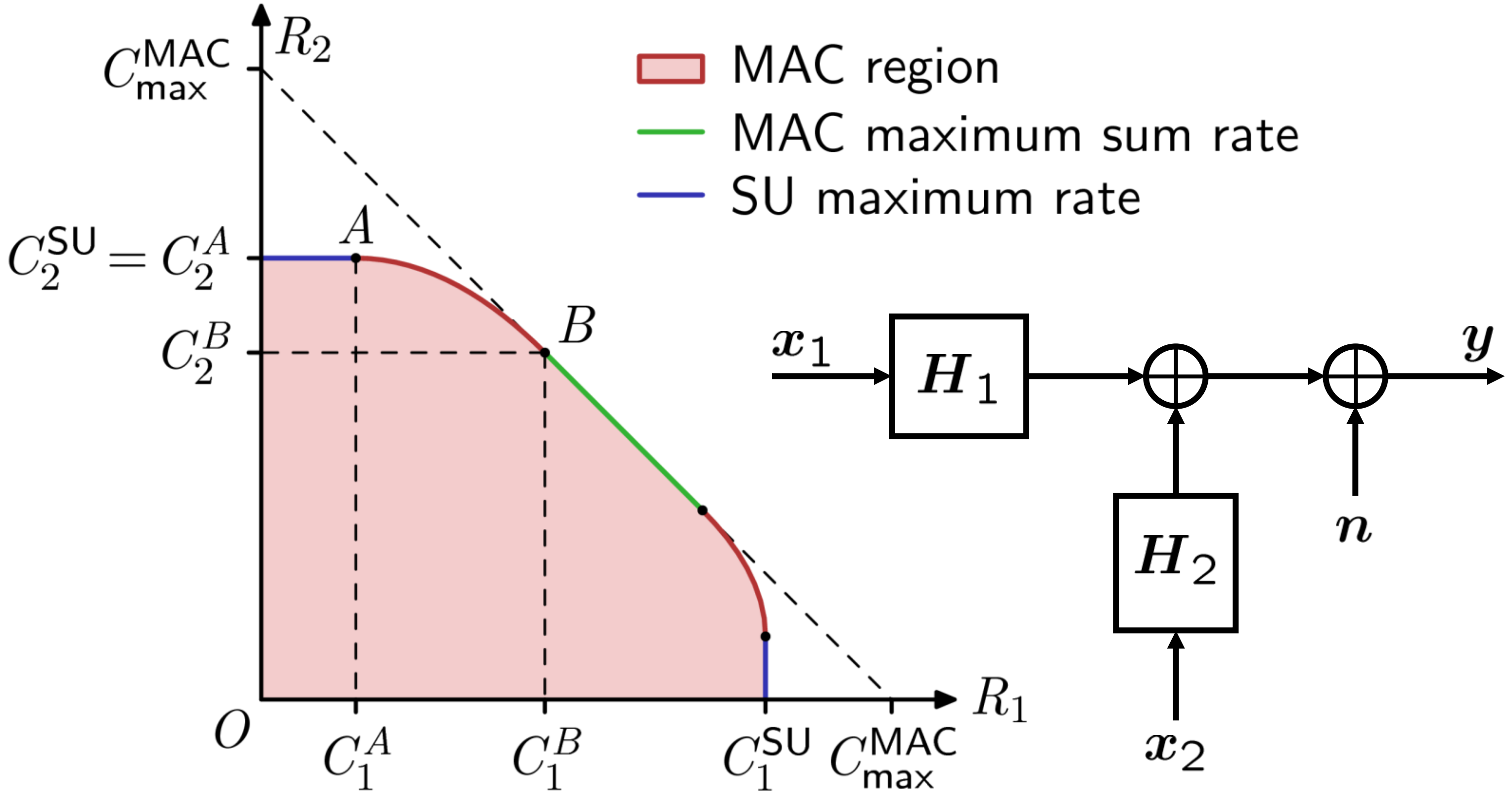





SU MIMO optimal covariance matrices ($k \in \{1,2\}$):

$$\boldsymbol{K}_{\boldsymbol{x}_k}^{\text{SU}} = \underset{\substack{\boldsymbol{K}_{\boldsymbol{x}_k} \in \mathbb{C}^{M_k \times M_k}: \\ \boldsymbol{K}_{\boldsymbol{x}_k} \geq 0,\ \operatorname{tr} \boldsymbol{K}_{\boldsymbol{x}_k} \leq P_k}}{\operatorname{argmax}} \log \left| \boldsymbol{I}_N + \boldsymbol{H}_k \boldsymbol{K}_{\boldsymbol{x}_k} \boldsymbol{H}_k^{\text{H}} \right|$$

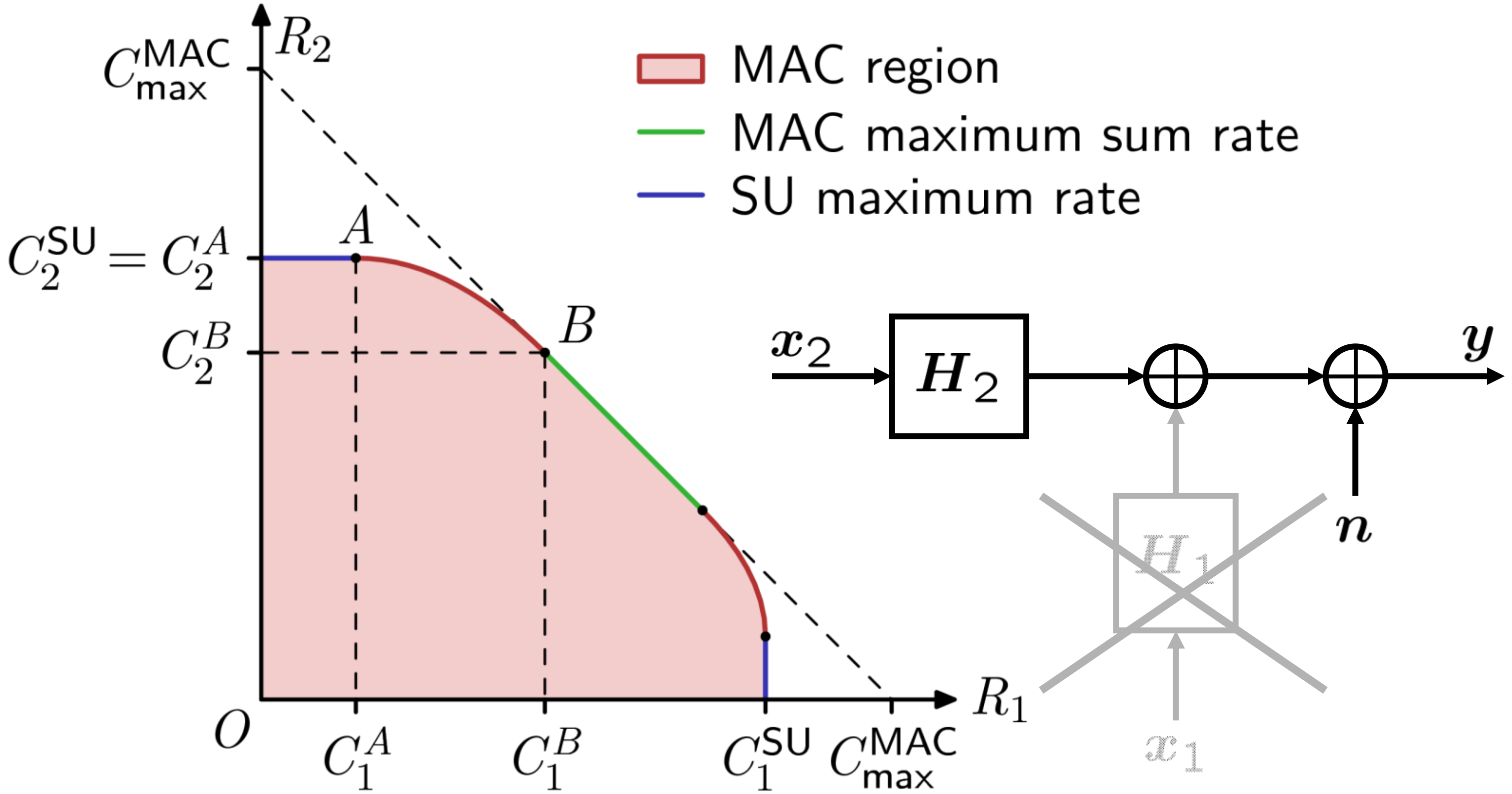
# Capacity Region of the MIMO MAC with 2 Users




Point $A$: $$C_1^A = \max_{\substack{\boldsymbol{K}_{\boldsymbol{x}_1} \in \mathbb{C}^{M_1 \times M_1}: \\ \boldsymbol{K}_{\boldsymbol{x}_1} \succeq 0,\ \mathrm{tr}\, \boldsymbol{K}_{\boldsymbol{x}_1} \leq P_1}} \log \frac{\left| \boldsymbol{I}_N + \boldsymbol{H}_1 \boldsymbol{K}_{\boldsymbol{x}_1} \boldsymbol{H}_1^{\mathsf{H}} + \boldsymbol{H}_2 \boldsymbol{K}_{\boldsymbol{x}_2}^{\mathsf{SU}} \boldsymbol{H}_2^{\mathsf{H}} \right|}{\left| \boldsymbol{I}_N + \boldsymbol{H}_2 \boldsymbol{K}_{\boldsymbol{x}_2}^{\mathsf{SU}} \boldsymbol{H}_2^{\mathsf{H}} \right|}$$

# Capacity Region of the MIMO MAC with 2 Users



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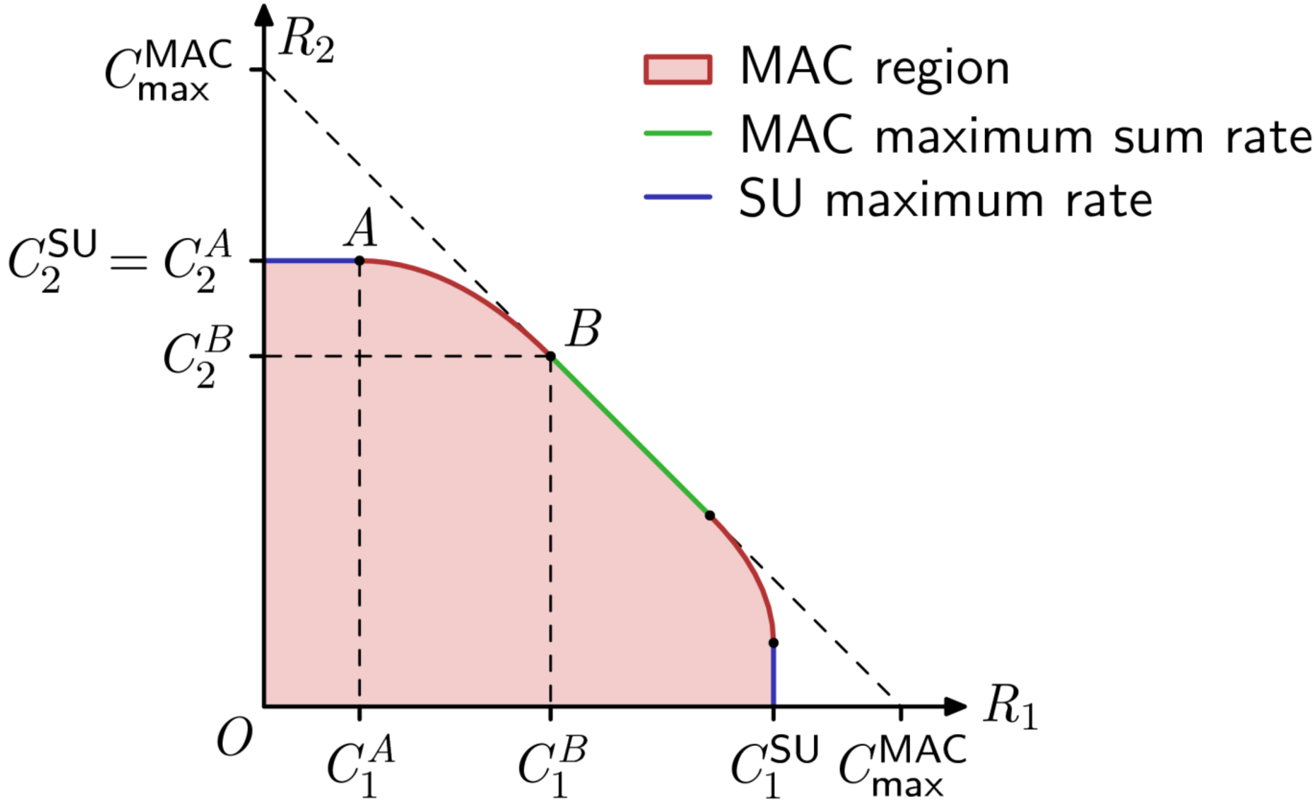
$$C_2^A = C_2^{\text{SU}} = \log \left| \boldsymbol{I}_N + \boldsymbol{H}_2 \boldsymbol{K}_{\boldsymbol{x}_2}^{\text{SU}} \boldsymbol{H}_2^{\text{H}} \right|$$

# Capacity Region of the MIMO MAC with 2 Users



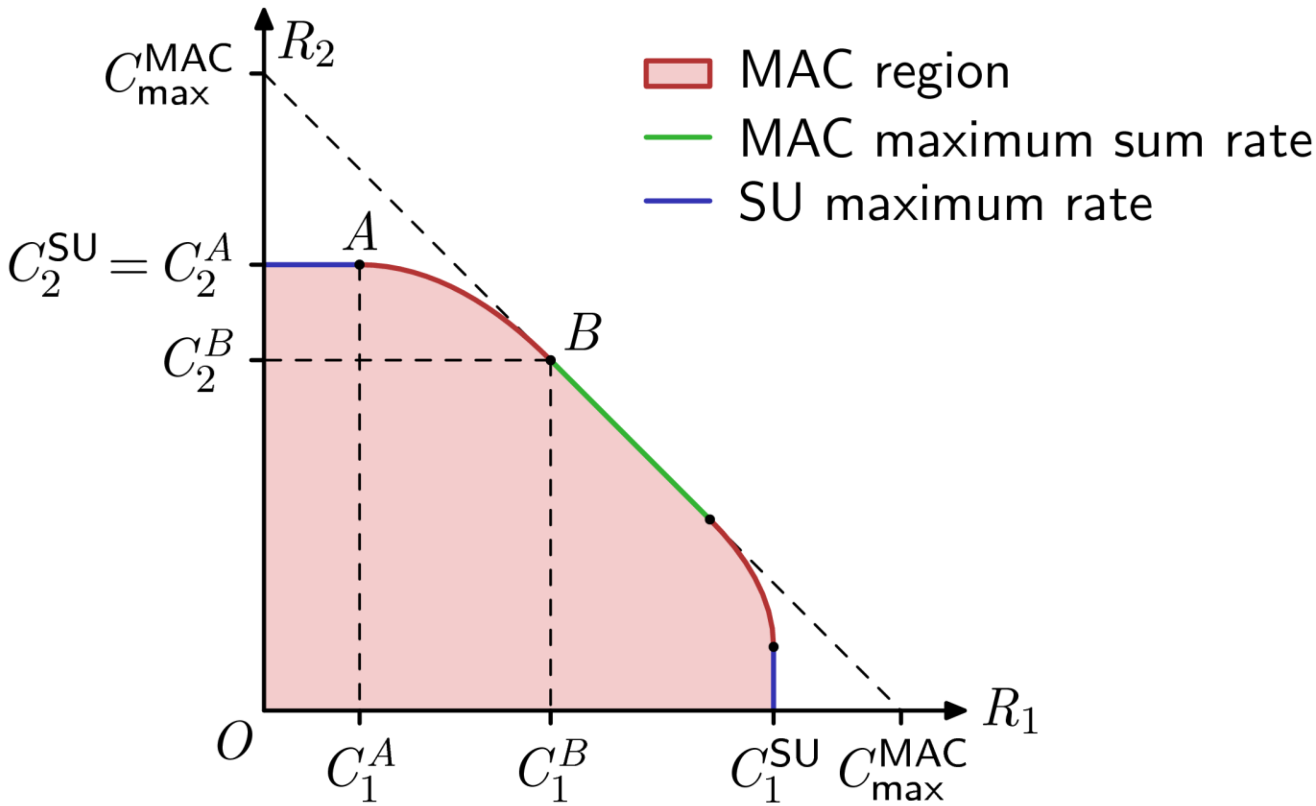



MIMO MAC maximum sumrate covariance matrices:

$$\left(\boldsymbol{K}_{\boldsymbol{x}_1}^{\mathsf{MAC}}, \boldsymbol{K}_{\boldsymbol{x}_2}^{\mathsf{MAC}}\right) = \underset{\substack{(\boldsymbol{K}_{\boldsymbol{x}_1}, \boldsymbol{K}_{\boldsymbol{x}_2}): \\ \boldsymbol{K}_{\boldsymbol{x}_k} \in \mathbb{C}^{M_k \times M_k},\ \boldsymbol{K}_{\boldsymbol{x}_k} \succeq 0, \\ \operatorname{tr} \boldsymbol{K}_{\boldsymbol{x}_k} \leq P_k\ \forall k \in \{1,2\}}}{\operatorname{argmax}} \log \left| \boldsymbol{I}_N + \sum_{k \in \{1,2\}} \boldsymbol{H}_k \boldsymbol{K}_{\boldsymbol{x}_k} \boldsymbol{H}_k^{\mathsf{H}} \right|$$

# Capacity Region of the MIMO MAC with 2 Users



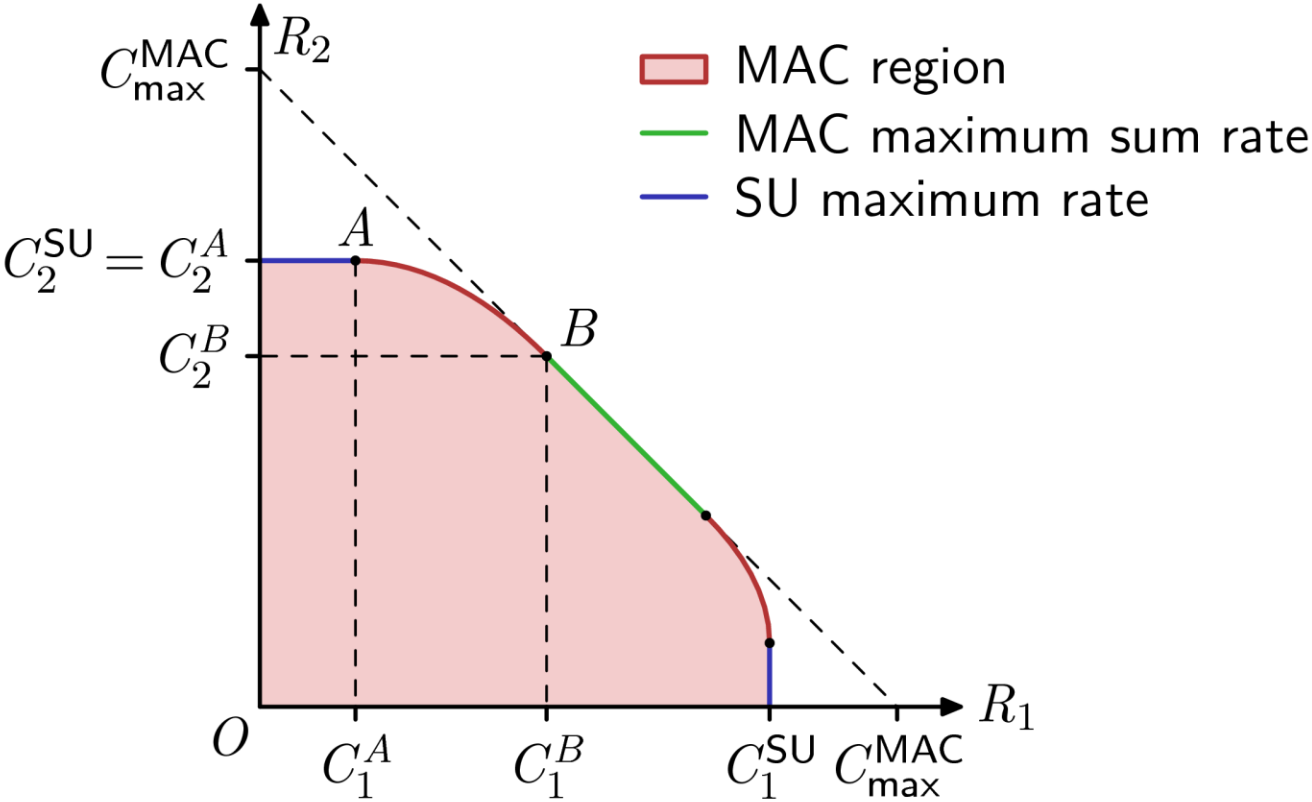



Point $B$:

$$C_1^B = \log \frac{\left| \boldsymbol{I}_N + \boldsymbol{H}_1 \boldsymbol{K}_{\boldsymbol{x}_1}^{\mathrm{MAC}} \boldsymbol{H}_1^{\mathrm{H}} + \boldsymbol{H}_2 \boldsymbol{K}_{\boldsymbol{x}_2}^{\mathrm{MAC}} \boldsymbol{H}_2^{\mathrm{H}} \right|}{\left| \boldsymbol{I}_N + \boldsymbol{H}_2 \boldsymbol{K}_{\boldsymbol{x}_2}^{\mathrm{MAC}} \boldsymbol{H}_2^{\mathrm{H}} \right|}$$

$$C_2^B = C_{\max}^{\mathrm{MAC}} - C_1^B = \log \left| \boldsymbol{I}_N + \boldsymbol{H}_2 \boldsymbol{K}_{\boldsymbol{x}_2}^{\mathrm{MAC}} \boldsymbol{H}_2^{\mathrm{H}} \right|$$

# Capacity Region of the MIMO MAC with 2 Users

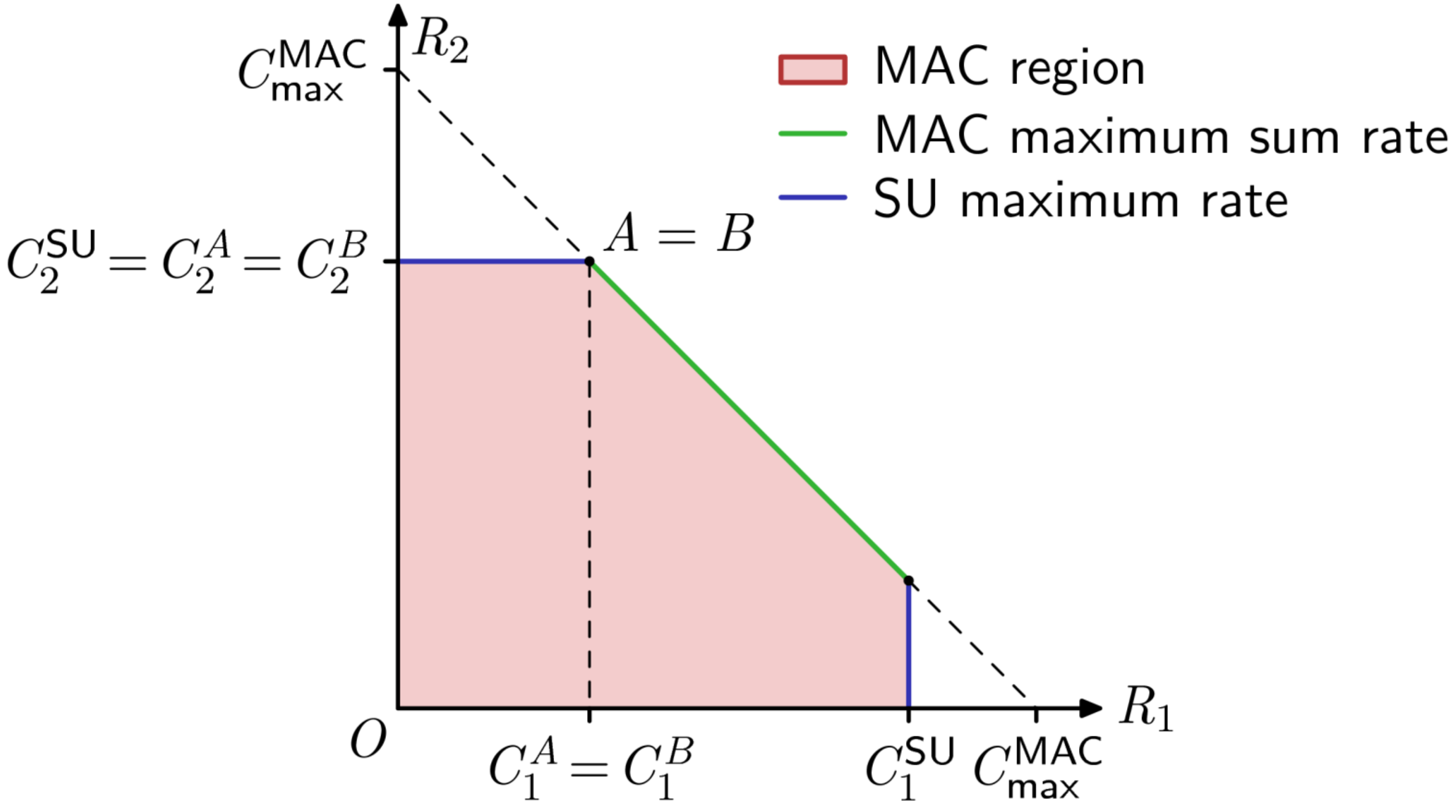



Special case: $M_k$=1 transmit antenna for all $k \in \{1,2\}$

- Scalar covariance matrices: $\boldsymbol{K}_{\boldsymbol{x}_k}^{\text{MAC}} = \boldsymbol{K}_{\boldsymbol{x}_k}^{\text{SU}} = P_k$
- Points $A$ and $B$ fall together.
- Capacity region degenerates to a pentagon.

# Capacity Region of the MIMO MAC with 2 Users






Special case: $M_k$=1 transmit antenna for all $k \in \{1,2\}$

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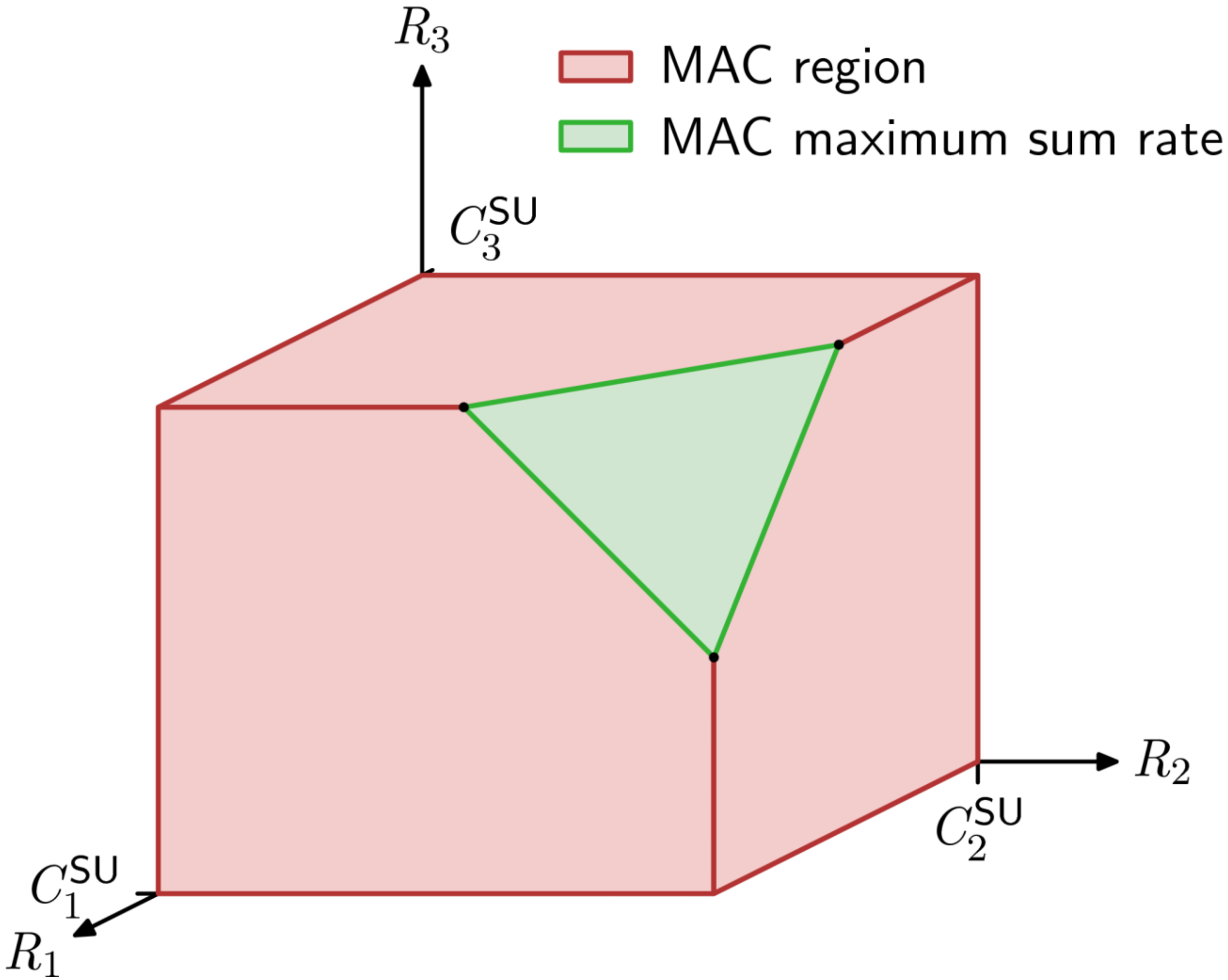
- Points *A* and *B* fall together.
- Capacity region degenerates to a pentagon.

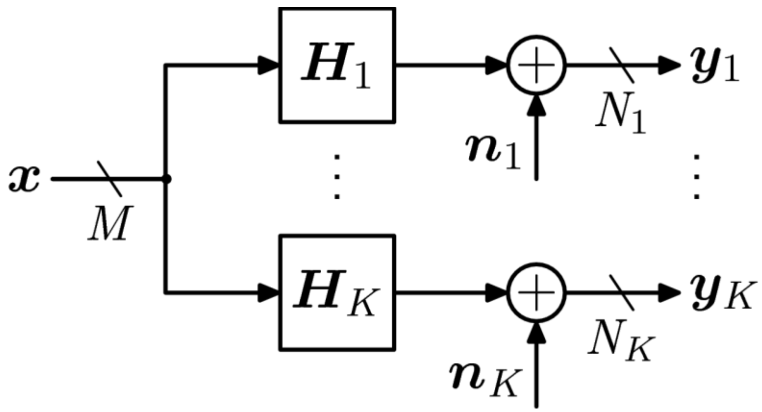
# Capacity Region of the MIMO MAC with 3 Users

Special case: $M_k$=1 transmit antenna for all $k \in \{1,2,3\}$



# Sum Capacity of the MIMO BC Channel






$$\boldsymbol{x} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_M, \boldsymbol{K}_{\boldsymbol{x}})$$

$$\boldsymbol{x} = \boldsymbol{x}_1 + \cdots + \boldsymbol{x}_K$$

For all $k \in \{1,\ldots,K\}$:

$$\boldsymbol{n}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_N, \boldsymbol{I}_{N_k})$$

The capacity of the cooperative SU MIMO system

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_1 \\ \vdots \\ \boldsymbol{H}_K \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{n}_1 \\ \vdots \\ \boldsymbol{n}_K \end{bmatrix} \Leftrightarrow \boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \in \mathbb{C}^N \qquad N = \sum_{k=1}^{K} N_k$$

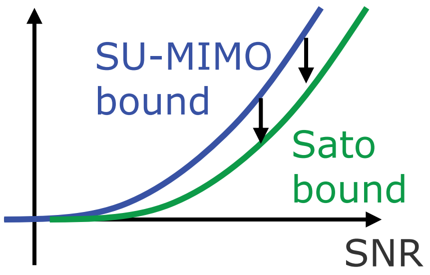
is an upper bound on the sum capacity of noncooperative MU MIMO BC channel.

# Sum Capacity of the MIMO BC Channel



- Correlations between noise vectors at different receivers affect the capacity of the cooperative SU MIMO system **but not** the MIMO BC capacity region.

$$\mathcal{S} = \left\{ \boldsymbol{K_n} \in \mathbb{C}^{N\times N} : \boldsymbol{K_n} = \begin{bmatrix} \boldsymbol{I}_{N_1} & \cdots & \bullet \\ \vdots & \ddots & \vdots \\ \bullet & \cdots & \boldsymbol{I}_{N_K} \end{bmatrix} \right\}$$




- Idea: Find the worst case noise to get the tightest bound!

Sato bound of the MIMO BC channel :

$$C_{\mathsf{Sato}} = \inf_{\boldsymbol{K_n}\in\mathcal{S}} \max_{\substack{\boldsymbol{K_x}\in\mathbb{C}^{M\times M}: \\ \boldsymbol{K_x}\geq 0,\ \mathrm{tr}\,\boldsymbol{K_x}\leq P}} \log \frac{\left|\boldsymbol{K_n} + \boldsymbol{H}\boldsymbol{K_x}\boldsymbol{H}^{\mathsf{H}}\right|}{|\boldsymbol{K_n}|}$$

# Sum Capacity of the MIMO BC Channel

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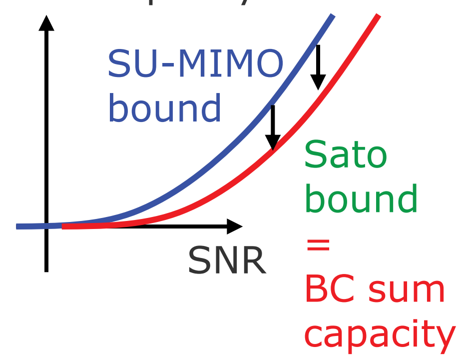


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Dirty Paper Coding (DPC) can be used to achieve the sum capacity of the MIMO BC channel!

# DPC Region of the MIMO BC Channel

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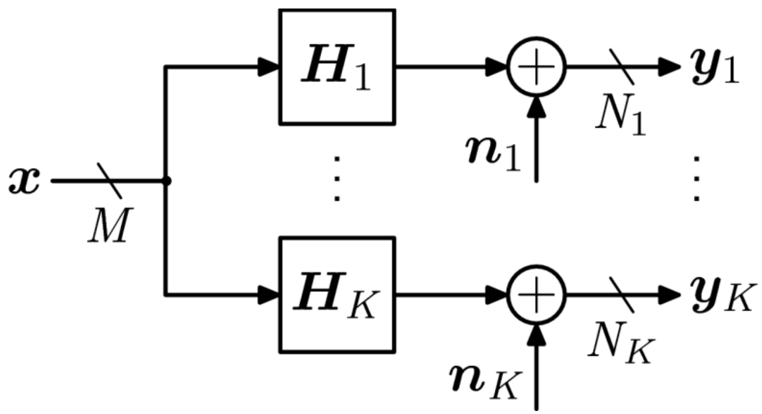



$$\boldsymbol{x} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_M, \boldsymbol{K}_{\boldsymbol{x}})$$

$$\boldsymbol{x} = \boldsymbol{x}_1 + \cdots + \boldsymbol{x}_K$$

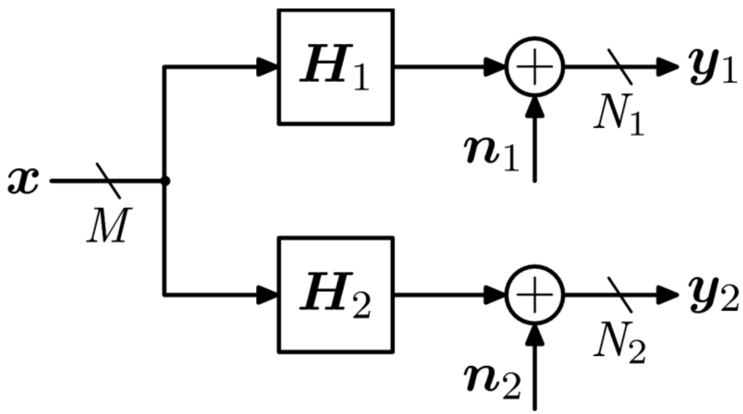
For all $k \in \{1,\dots,K\}$:

$$\boldsymbol{n}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_N, \boldsymbol{I}_{N_k})$$

DPC region if transmit power is limited to *P*:

$$\mathcal{C}_{\mathrm{DPC}} = \operatorname*{convex\ hull}_{\substack{(\pi,\boldsymbol{K}_{\boldsymbol{x}_1},\dots,\boldsymbol{K}_{\boldsymbol{x}_K}): \\ \pi:\{1,\dots,K\}\to\{1,\dots,K\}, \\ \boldsymbol{K}_{\boldsymbol{x}_k}\in\mathbb{C}^{M\times M},\ \boldsymbol{K}_{\boldsymbol{x}_k}\succeq 0, \\ \mathrm{tr}\sum_{k=1}^{K}\boldsymbol{K}_{\boldsymbol{x}_k}\le P \\ \forall k\in\{1,\dots,K\}}} \bigcup \left\{ \begin{array}{l} (R_1,\dots,R_K)\in\mathbb{R}_{0,+}^K : \\ R_{\pi(k)} = \log \dfrac{\left|\boldsymbol{I}_N + \boldsymbol{H}_{\pi(k)}\left(\sum_{\ell=k}^{K}\boldsymbol{K}_{\boldsymbol{x}_\ell}\right)\boldsymbol{H}_{\pi(k)}^{\mathrm{H}}\right|}{\left|\boldsymbol{I}_N + \boldsymbol{H}_{\pi(k)}\left(\sum_{\ell=k+1}^{K}\boldsymbol{K}_{\boldsymbol{x}_\ell}\right)\boldsymbol{H}_{\pi(k)}^{\mathrm{H}}\right|} \\ \forall k\in\{1,\dots,K\} \end{array} \right\}$$

# DPC Region of the MIMO BC Channel




$$\boldsymbol{x} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \boldsymbol{K}_{\boldsymbol{x}})$$
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For all $k \in \{1,2\}$:
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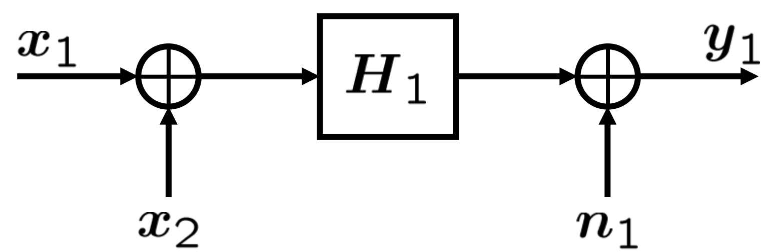
Example: DPC for 2 users (here, with ordering $\pi(k)=k$)

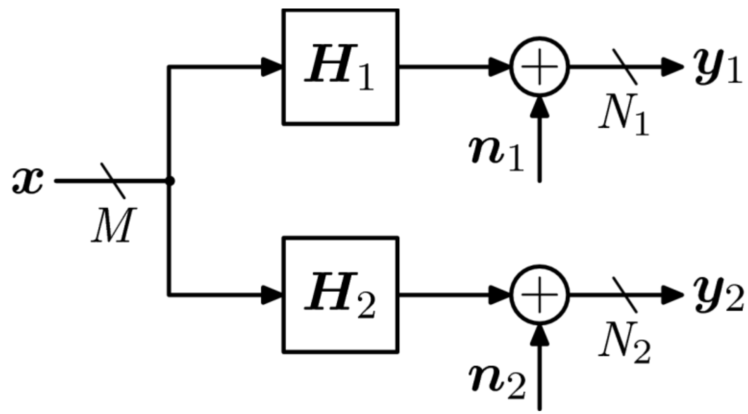
1. Encoder user 1 without considering the interference caused by user 2.

$$R_1 = \log \frac{\left|\boldsymbol{I}_N + \boldsymbol{H}_1 \left(\boldsymbol{K}_{\boldsymbol{x}_1} + \boldsymbol{K}_{\boldsymbol{x}_2}\right) \boldsymbol{H}_1^{\mathsf{H}}\right|}{\left|\boldsymbol{I}_N + \boldsymbol{H}_1 \boldsymbol{K}_{\boldsymbol{x}_2} \boldsymbol{H}_1^{\mathsf{H}}\right|}$$



# DPC Region of the MIMO BC Channel



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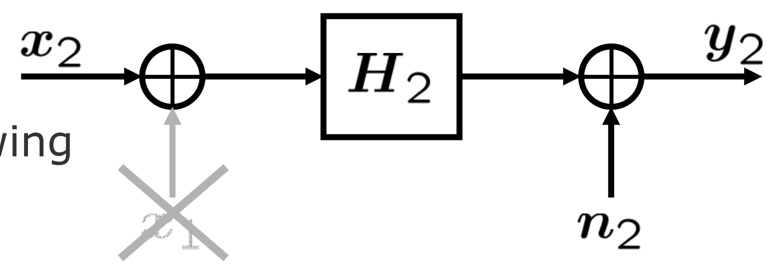
2. After DPC which removes completely the interference from user 1, encode user 2 following traditional SU MIMO.

$$R_2 = \log \left|\boldsymbol{I}_N + \boldsymbol{H}_2 \boldsymbol{K}_{\boldsymbol{x}_2} \boldsymbol{H}_2^{\mathrm{H}}\right|$$



# DPC Region of the MIMO BC Channel

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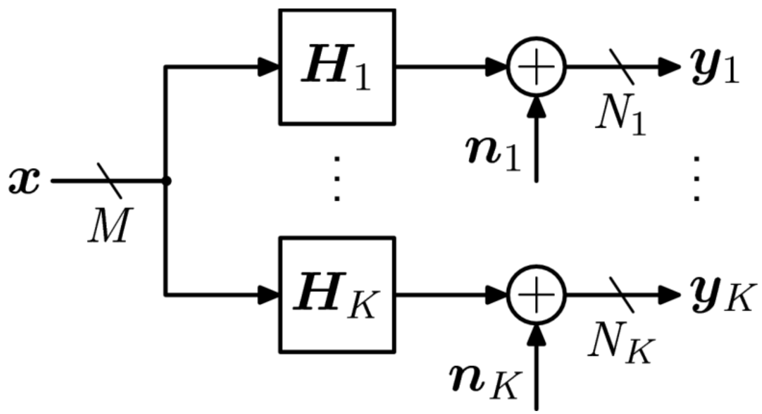


$$\boldsymbol{x} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_M, \boldsymbol{K}_{\boldsymbol{x}})$$

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For all $k \in \{1,\ldots,K\}$:

$$\boldsymbol{n}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_N, \boldsymbol{I}_{N_k})$$

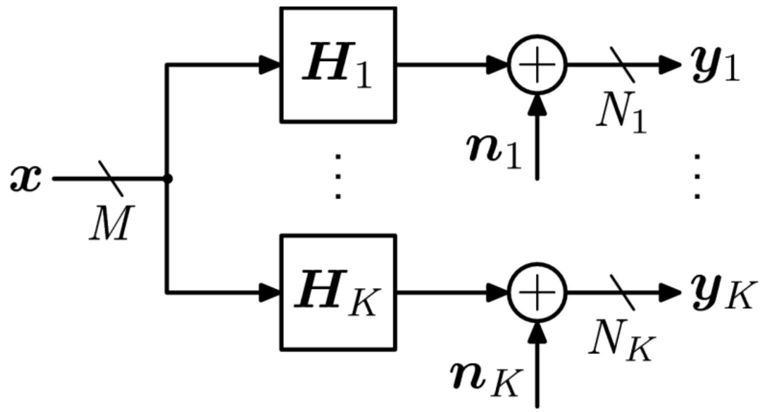
*[Caire & Shamai '00, Yu & Cioffi '01]*

- The DPC region is an achievable region of the MIMO BC channel.
- However, the rate equations are neither concave nor convex with respect to the covariance matrices.

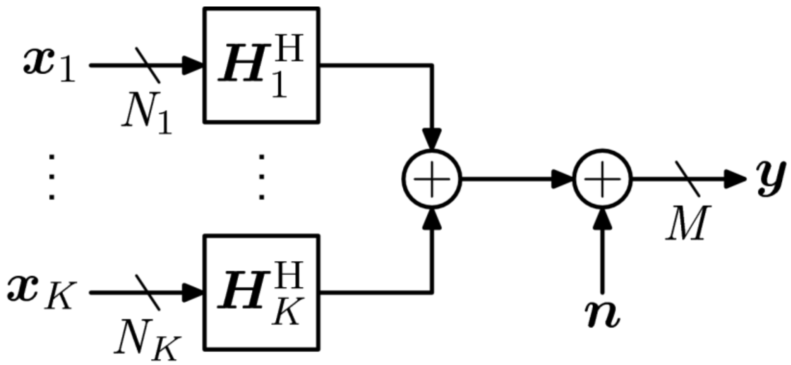
Finding optimal covariance matrices is computationally cumbersome!

[Vishwanath et al. '02]

MIMO BC Channel




Dual MAC (DMAC)



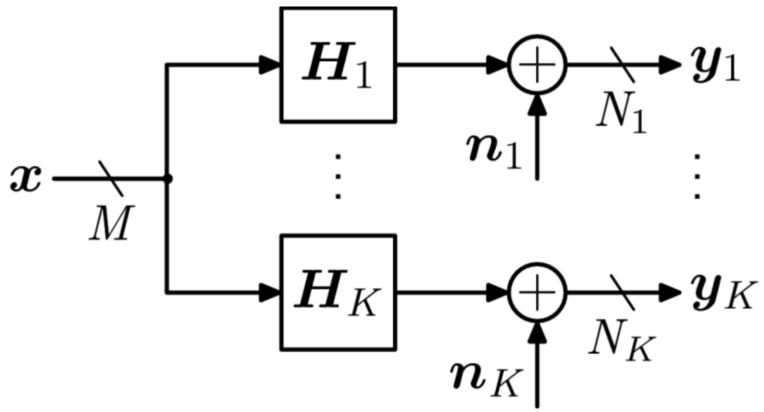

With the additional sum power constraint $P_1+\ldots+P_K \leq P$ of the DMAC:

$$\mathcal{C}_{\mathrm{DMAC}} = \bigcup_{\substack{P_k : \sum_{k=1}^{K} P_k = P \\ \forall k \in \{1,\ldots,K\}}} \mathcal{C}_{\mathrm{MAC}} \Big|_{\substack{\boldsymbol{H}_k \leftarrow \boldsymbol{H}_k^{\mathrm{H}},\ N \leftarrow M,\ M_k \leftarrow N_k \\ \forall k \in \{1,\ldots,K\}}}$$

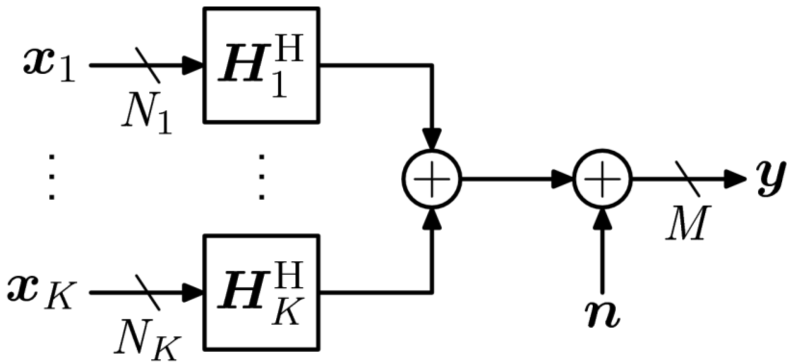
# Relationship Between DPC and Dual MAC Region

[Vishwanath et al. '02]

MIMO BC Channel




Dual MAC (DMAC)

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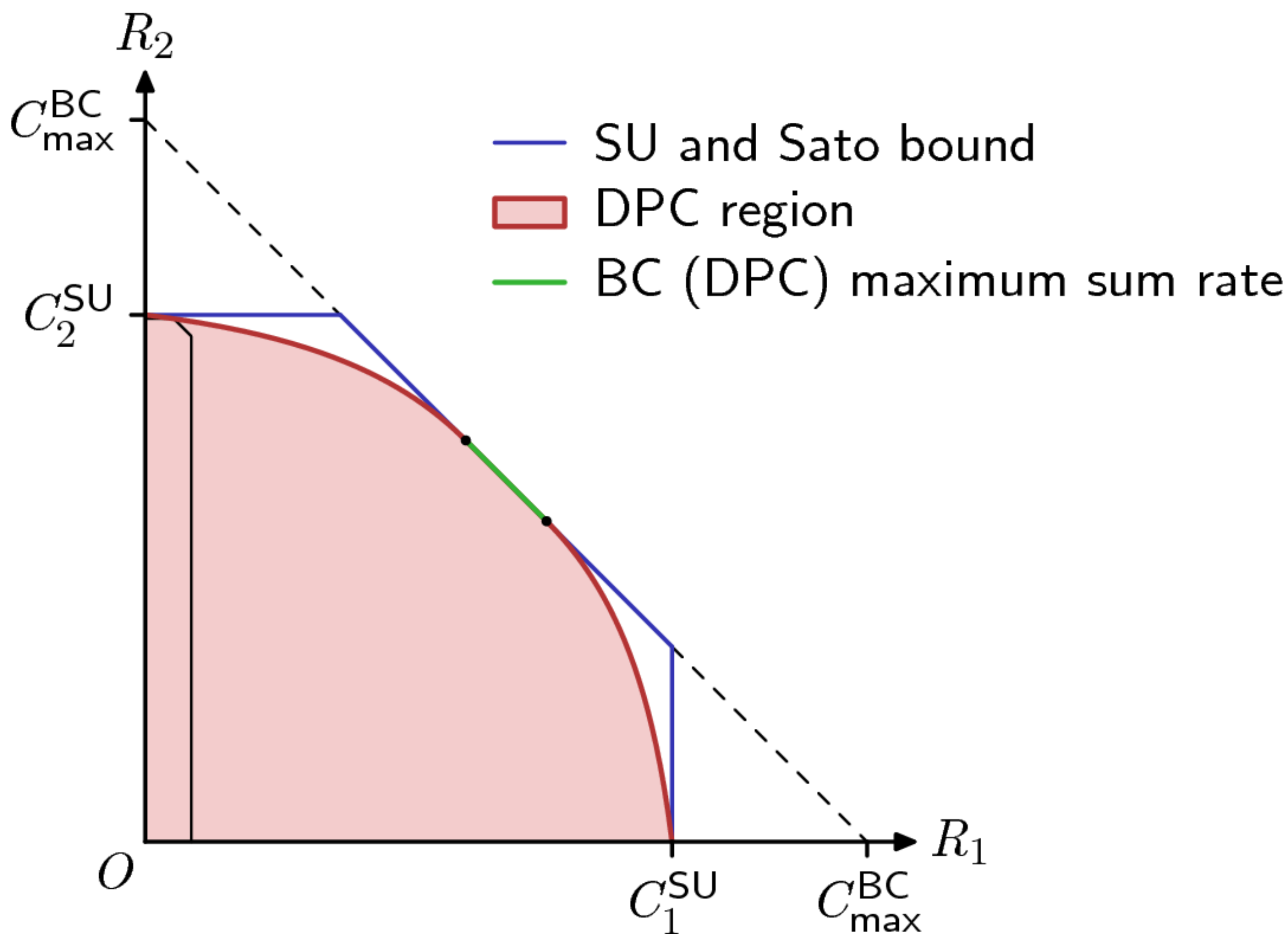
Thus, standard convex optimization techniques can be used to compute optimal covariance matrices (i.e., precoders)!

# Relationship Between DPC and Dual MAC Region



Special case: $N_k$=1 receive antennas for all $k \in \{1,2\}$



# Relationship Between DPC and Dual MAC Region

[Weingarten et al. '04]

The DPC region is the capacity region of the MIMO BC channel!

Consquences:

- DPC is the optimal encoding strategy.
- Union of DMAC regions is also equal to the MIMO BC region.
- Convex optimization techniques can be used to compute optimal precoders.
- For example: iterative waterfilling method!

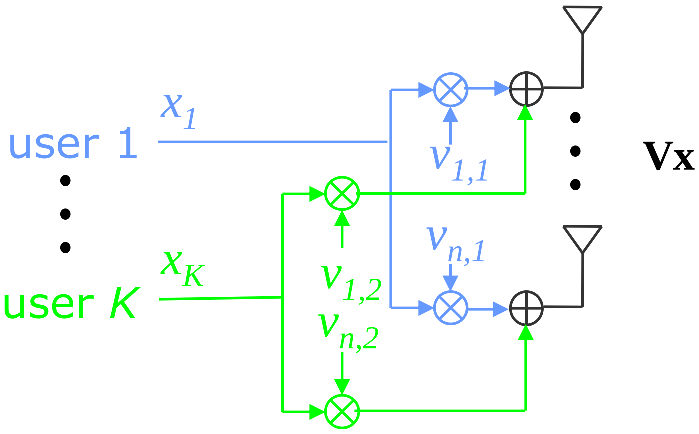
Problem: High computational complexity due to iterative nature!

# Outline (3)

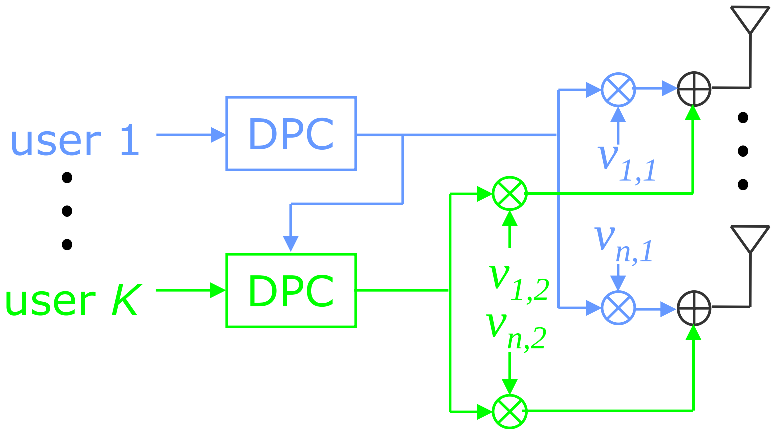
- Linear MU-MIMO schemes for 3GPP Long Term Evolution (LTE) and 3GPP LTE Advanced
  - Linear versus nonlinear precoding
  - MU- versus SU-MIMO
  - Summary of MIMO techniques in 3GPP-LTE
  - Precoder codebook based 3GPP-LTE MU-MIMO
  - Channel codebook based ZF precoding
  - Performance comparisons
  - MU-MIMO Status in 3GPP-LTE-Advanced

# Linear versus Non-Linear Precoding

## Linear Precoding




## Non-Linear Precoding




$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_K \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_K \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1K} \\ v_{21} & v_{22} & \cdots & v_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n_T 1} & v_{n_T 2} & \cdots & v_{n_T K} \end{bmatrix},$$

# Nonlinear versus Linear Precoding

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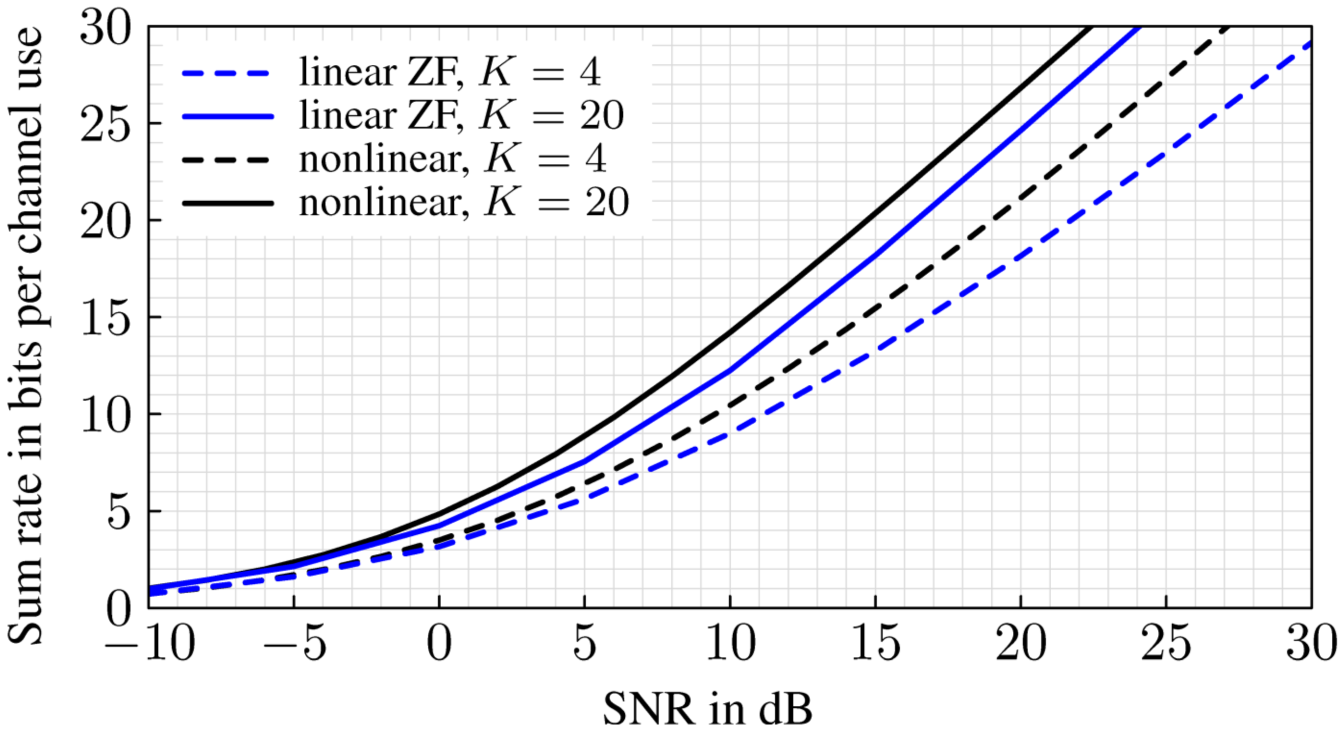


**System parameters**

- 4 Tx antennas
- 1 Rx antenna
- K users
- semicorrelated channel
- perfect CSIT
- optimal transmitter and receiver

**Pros and cons of nonlinear precoding**

☺
- beneficial for very high data rates as required for IMT-Advanced
- trade-off between performance and complexity

☹
- high computational complexity
- needs high hardware requirements (e.g., strictly linear amplifiers)

# MU- versus SU-MIMO Transmission

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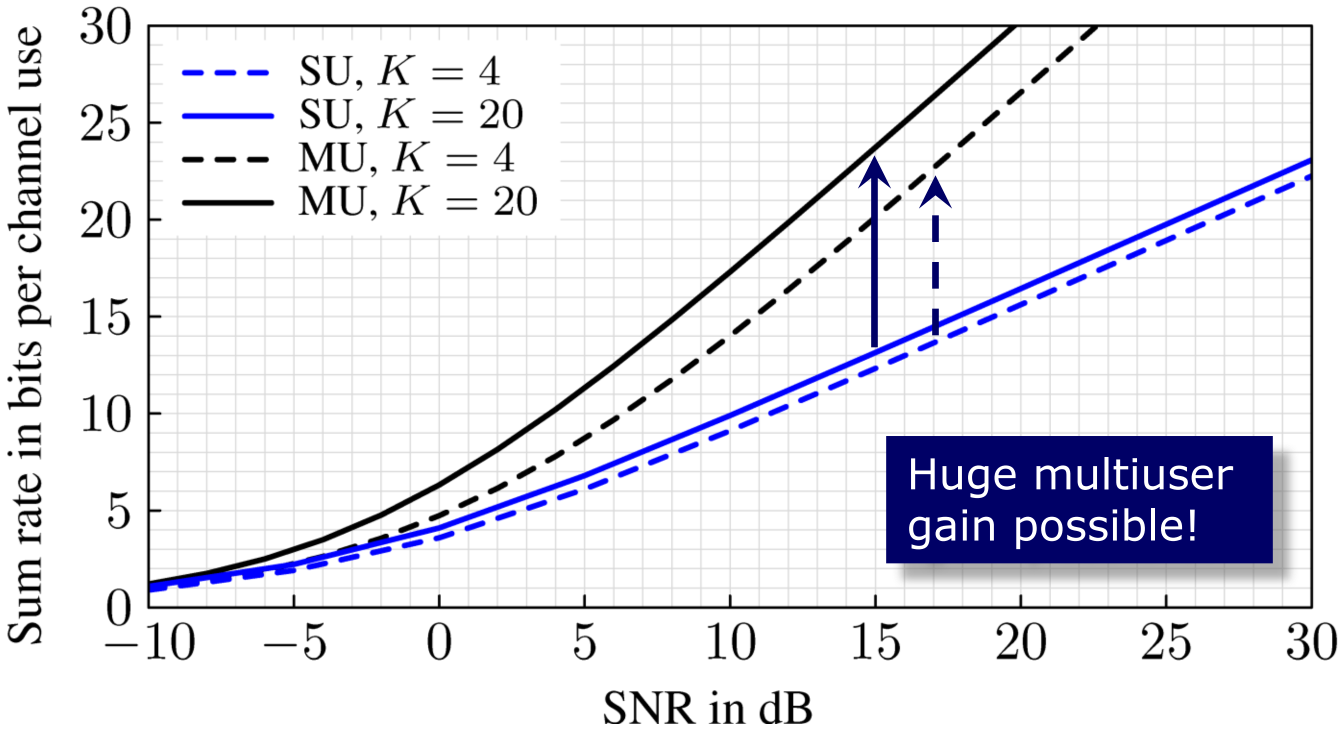



**System parameters**

- 4 Tx antennas
- 2 Rx antennas
- K users
- uncorrelated channel
- perfect **C**hannel **S**tate **I**nformation at the **T**ransmitter (**CSIT**)
- optimal transmitter and receiver

**Pros and cons of MU-MIMO transmission**

☺

- exploiting MU diversity
- increasing system capacity if UEs have less antennas than Node B
- improving performance in case of low-rank channels

☹

- high computational complexity
- needs high hardware requirements (e.g., strictly linear amplifiers)

# SU-MIMO versus MU-MIMO



| | SU-MIMO | MU-MIMO |
| --- | --- | --- |
| ☺ | • high user throughput<br>• high peak data rates | • high system capacity<br>• full exploitation of multiuser diversity |
| ☹ | • multiple transmit antennas are not fully exploited<br>• mulituser diversity is not fully exploited | • degradation of peak data rates due to MU interference (ZF is not working perfectly due to imperfect CSIT) |

- **SU-MIMO**
  - not supported in Release 8
  - however, transmit diversity is not excluded: closed loop antenna selection for data channel is supported as an option for **F**requency **D**ivision **D**uplex (**FDD**) and half-duplex FDD

- **MU-MIMO**
  - **S**patial **D**ivision **M**ultiple **A**ccess (**SDMA**) is included
  - one data stream per user

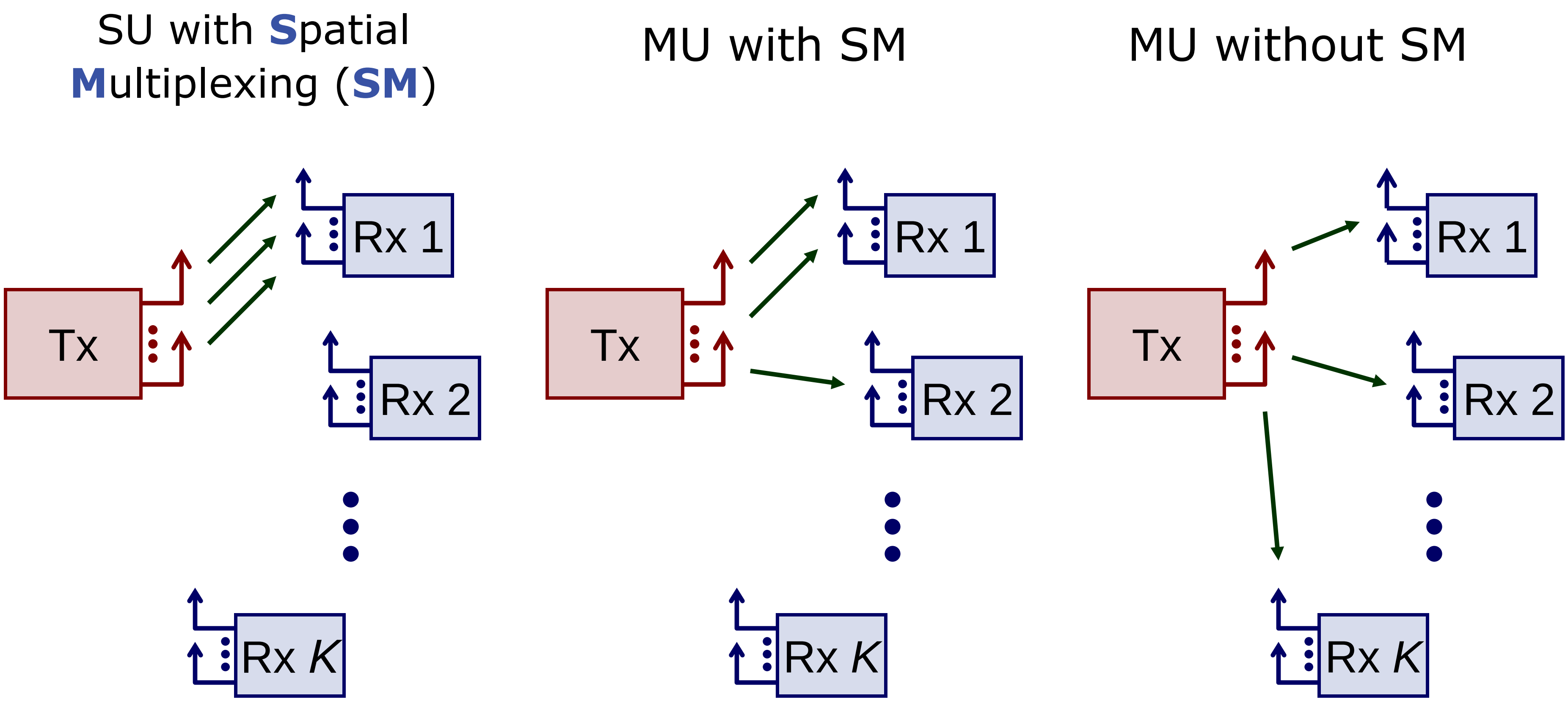
# 3GPP-LTE: Downlink Transmission

- **SU-MIMO**
  - precoder codebook based precoding
  - direction-of-arrival based beamforming
  - transmit diversity techniques
    - **S**pace-**F**requency **B**lock **C**odes (**SFBC**)
    - **F**requency **S**witching **T**ransmit **D**iversity (**FSTD**)
  - combinations of precoding and transmit diversity like **C**yclic **D**elay **D**iversity (**CDD**)

- **MU-MIMO**
  - sophisticated methods are not included
    - unitary precoder codebook based precoding
    - channel codebook based ZF beamforming
  - precoding based on combinations of SU-MIMO rank 1 codebook entries
  - CQI computation assuming no MU interference
  - one data stream per user

# MIMO Strategies in a Multiuser System






Exploiting Multiuser Diversity!

# MU-MIMO System with Linear Precoding

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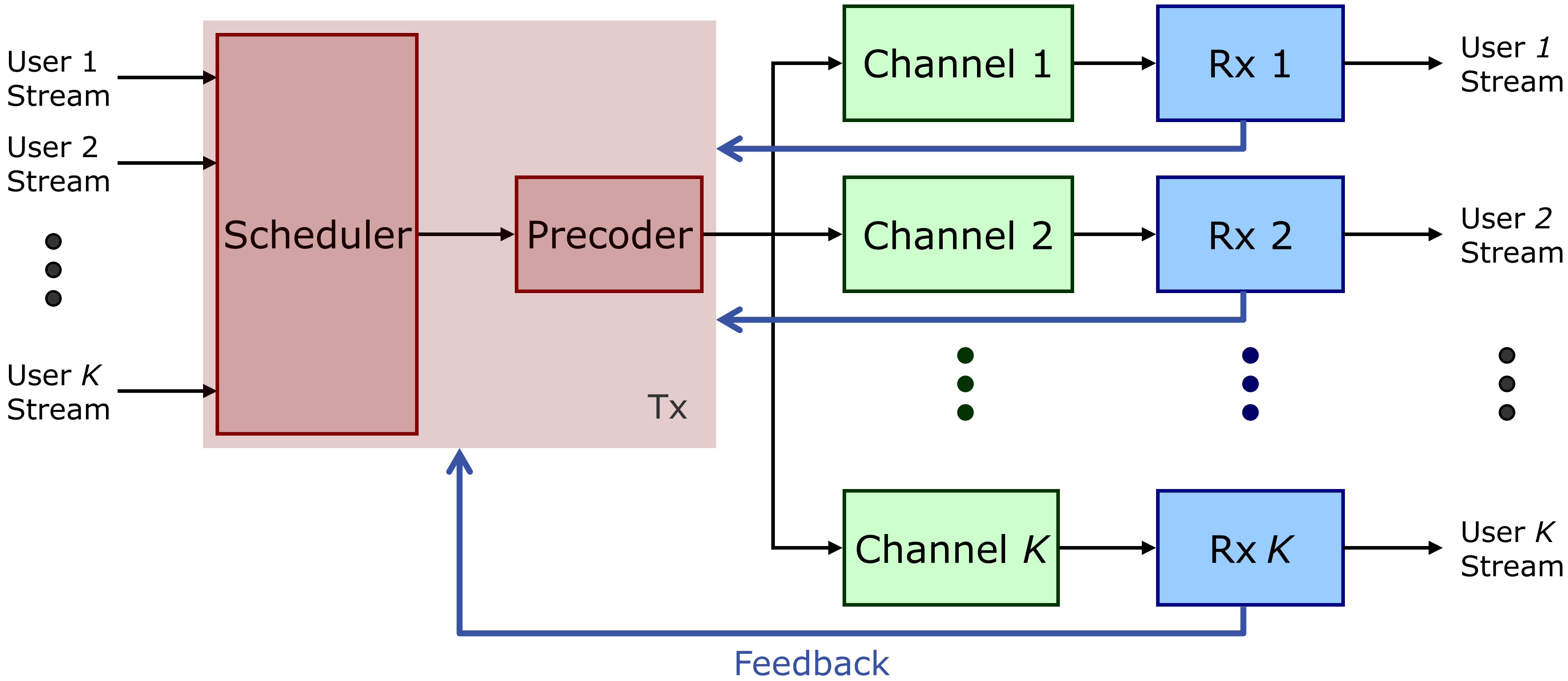


- scheduler and linear precoder needs CSIT
- feedback channel with finite rate

**What is the best limited feedback information?**

# Multiuser MIMO System with Linear Precoding

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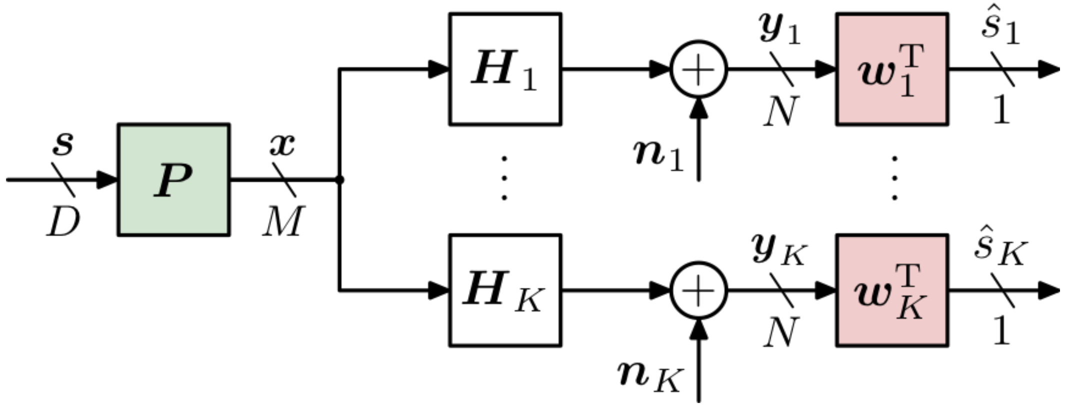



Channel model:

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x} + \boldsymbol{n}_k \in \mathbb{C}^N$$

Linear precoding:

$$\boldsymbol{x} = \sum_{k \in \mathbb{K}} \boldsymbol{p}_k s_k = \boldsymbol{P}\boldsymbol{s}, \quad P_{\mathrm{Tx}} = \mathrm{E}\left\{\boldsymbol{x}^{\mathrm{H}}\boldsymbol{x}\right\}$$ (transmit power)

Linear MMSE receivers for estimating the *k*th user's symbol:

$$\hat{s}_k = \boldsymbol{w}_k^{\mathrm{T}} \boldsymbol{y}_k \in \mathbb{C}, \quad \boldsymbol{w}_k = \left(\boldsymbol{H}_k^* \boldsymbol{P}^* \boldsymbol{P}^{\mathrm{T}} \boldsymbol{H}_k^{\mathrm{T}} + \frac{D}{P_{\mathrm{Tx}}} \boldsymbol{I}_N\right)^{-1} \boldsymbol{H}_k^* \boldsymbol{p}_k^*$$

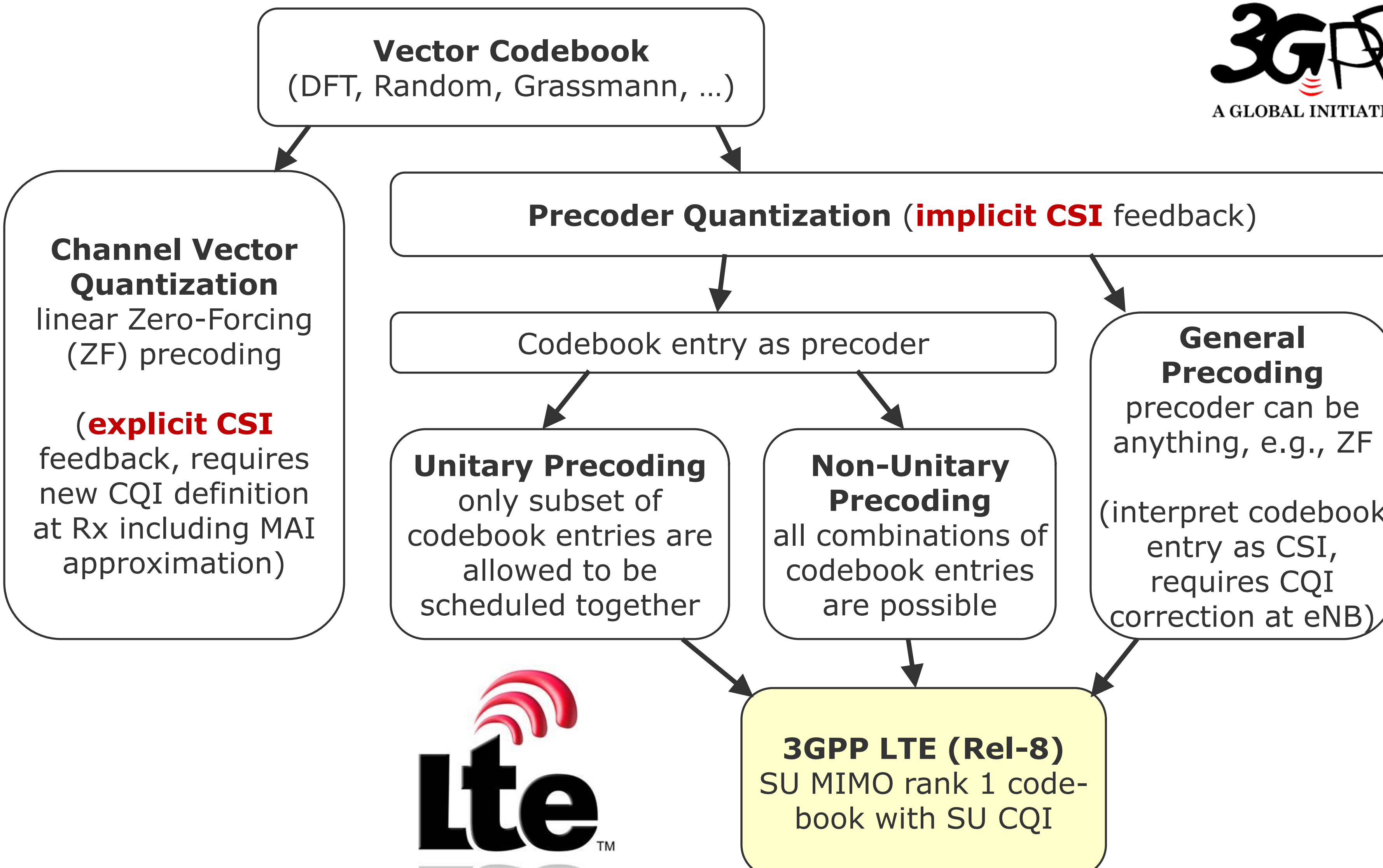
SINR:

$$\gamma_k = \frac{\left|\boldsymbol{w}_k^{\mathrm{T}} \boldsymbol{H}_k \boldsymbol{p}_k\right|^2}{\|\boldsymbol{w}_k\|_2^2 \frac{D}{P_{\mathrm{Tx}}} + \sum_{\substack{i \in \mathbb{K} \\ i \neq k}} \left|\boldsymbol{w}_k^{\mathrm{T}} \boldsymbol{H}_k \boldsymbol{p}_i\right|^2}$$

Sum Rate:

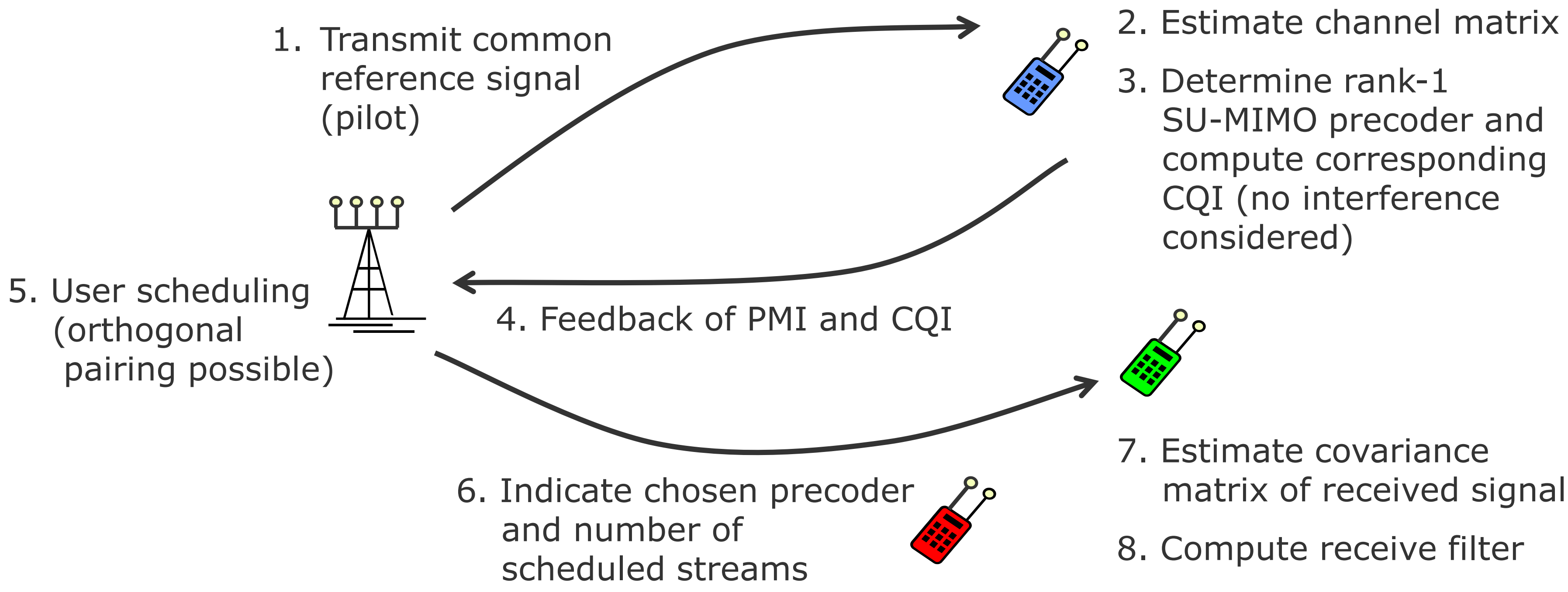
$$R_{\mathrm{sum}} = \sum_{k \in \mathbb{K}} \log_2\left(1 + \gamma_k\right)$$

# 3GPP-LTE: Summary of Discussed MU-MIMO Techniques



# 3GPP-LTE MU-MIMO Scheme

- One data stream (codeword) per user
- Reuse of SU-MIMO codebooks (2 Tx: DFT, 4 Tx: Householder)




- PMI: Precoder Matrix Index
- CQI: Channel Quality Indicator

## 3GPP-LTE: Rank-1 Single-User MIMO Precoder Codebooks



### 2 Tx antennas
### DFT-based codebook

| index | precoder |
|---|---|
| 0 | $\begin{bmatrix}1\\0\end{bmatrix}$ |
| 1 | $\begin{bmatrix}0\\1\end{bmatrix}$ |
| 2 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ |
| 3 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ |
| 4 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix}$ |
| 5 | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix}$ |

- Constant modulus property ($|w_{n,ij}| = 1$)
- Nested property (lower rank precoding matrix is submatrix of higher rank precoding matrix)

### 4 Tx antennas
### Householder codebook

$$W_n = I - 2u_n u_n^H / u_n^H u_n \Rightarrow W_n^{(1)}$$

| index | $u_n$ |
|---|---|
| 0 | $u_0 = [1 \quad -1 \quad -1 \quad -1]^T$ |
| 1 | $u_1 = [1 \quad -j \quad 1 \quad j]^T$ |
| 2 | $u_2 = [1 \quad 1 \quad -1 \quad 1]^T$ |
| 3 | $u_3 = [1 \quad j \quad 1 \quad -j]^T$ |
| 4 | $u_4 = [1 \quad (-1-j)/\sqrt{2} \quad -j \quad (1-j)/\sqrt{2}]^T$ |
| 5 | $u_5 = [1 \quad (1-j)/\sqrt{2} \quad j \quad (-1-j)/\sqrt{2}]^T$ |
| 6 | $u_6 = [1 \quad (1+j)/\sqrt{2} \quad -j \quad (-1+j)/\sqrt{2}]^T$ |
| 7 | $u_7 = [1 \quad (-1+j)/\sqrt{2} \quad j \quad (1+j)/\sqrt{2}]^T$ |
| 8 | $u_8 = [1 \quad -1 \quad 1 \quad 1]^T$ |
| 9 | $u_9 = [1 \quad -j \quad -1 \quad -j]^T$ |
| 10 | $u_{10} = [1 \quad 1 \quad 1 \quad -1]^T$ |
| 11 | $u_{11} = [1 \quad j \quad -1 \quad j]^T$ |
| 12 | $u_{12} = [1 \quad -1 \quad -1 \quad 1]^T$ |
| 13 | $u_{13} = [1 \quad -1 \quad 1 \quad -1]^T$ |
| 14 | $u_{14} = [1 \quad 1 \quad -1 \quad -1]^T$ |
| 15 | $u_{15} = [1 \quad 1 \quad 1 \quad 1]^T$ |

## CQI table used at UE



### 4-bit CQI table

| CQI index | modulation | coding rate x 1024 | efficiency |
|---|---|---|---|
| 0 | out of range | | |
| 1 | QPSK | 78 | 0.1523 |
| 2 | QPSK | 120 | 0.2344 |
| 3 | QPSK | 193 | 0.3770 |
| 4 | QPSK | 308 | 0.6016 |
| 5 | QPSK | 449 | 0.8770 |
| 6 | QPSK | 602 | 1.1758 |
| 7 | 16QAM | 378 | 1.4766 |
| 8 | 16QAM | 490 | 1.9141 |
| 9 | 16QAM | 616 | 2.4063 |
| 10 | 64QAM | 466 | 2.7305 |
| 11 | 64QAM | 567 | 3.3223 |
| 12 | 64QAM | 666 | 3.9023 |
| 13 | 64QAM | 772 | 4.5234 |
| 14 | 64QAM | 873 | 5.1152 |
| 15 | 64QAM | 948 | 5.5547 |

TS 36.213 ver.8.3.0 (2008.05), Table 7-2-3-1

# ZF Precoding with Channel Codebook



**Z**ero **F**orcing **C**hannel **V**ector **Q**uantization (**ZF-CVQ**)

- UE estimates and quantizes its channel based on the channel codebook (Channel Vector Quantization, CVQ):

$$\mathcal{C} = \{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{2^B}\}$$ (assuming B feedback bits)

- UEs report their quantized channels together with CQIs (e.g. SINRs).

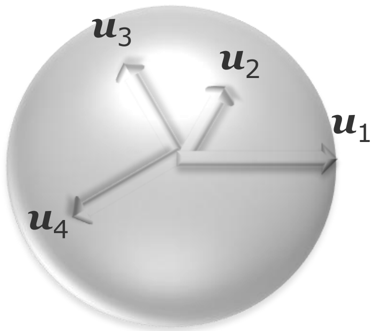
- Node B computes ZF precoder and schedules users based on the CQIs (e.g. maximum throughput scheduling).

**Performance depends strongly on the codebook size $2^B$**

# Vector Codebooks



- **Random codebook**:
  vectors $\boldsymbol{u}_i$ isotropically distributed on a complex unit sphere




$$C = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4\}$$

- **Grassmannian codebook**:
  minimum distance between any pair of vectors is maximum
  ⇒ Grassmanian line packing problem

- **Fourier (DFT) codebook**:
  example for 4 Tx antennas and a codebook size of $2^B$=8
  (3 bit index, 8x8 DFT matrix section)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & e^{-j\pi/4} & e^{-j\pi/2} & e^{-j3\pi/4} & e^{-j\pi} & e^{-j5\pi/4} & e^{-j3\pi/2} & e^{-j7\pi/4} \\ 1 & e^{-j\pi/2} & e^{-j\pi} & e^{-j3\pi/2} & e^{-j2\pi} & e^{-j5\pi/2} & e^{-j3\pi} & e^{-j7\pi/2} \\ 1 & e^{-j3\pi/4} & e^{-j3\pi/2} & e^{-j9\pi/4} & e^{-j3\pi} & e^{-j15\pi/4} & e^{-j9\pi/2} & e^{-j21\pi/4} \end{bmatrix}$$

Advantages:
cheap implementation and good performance in correlated scenarios!

# Principles of Channel Vector Quantization (CVQ)

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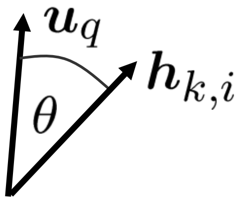
- Assumption: more than one receive antenna at UEs ($N$>1)
- Vector codebook: $\mathcal{C} = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{2^B}\}$
- Quantization of channel matrix $\boldsymbol{H}_k$:
  - each row $\boldsymbol{h}_{k,i}$ , $i$=1,...,$N$, of the channel matrix $\boldsymbol{H}_k$ is quantized individually
  - Problem: huge feedback information in case of many receive antennas

$$\hat{\boldsymbol{h}}_{k,i} = \boldsymbol{u}_\ell, \quad \ell = \arg \max_{q \in \{1,\ldots,2^B\}} \frac{\left|\boldsymbol{u}_q^{\mathrm{H}} \boldsymbol{h}_{k,i}\right|}{\left\|\boldsymbol{h}_{k,i}\right\|_2}$$




- Quantization of composite channel vector:
  - combination of receive filter and physical channel matrix, i.e., the composite channel vector $\boldsymbol{g}_k=\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{w}_k$, is quantized
  - Advantage: only one vector per user needs to be quantized, independent of $N$
  - Problem: receive filter $\boldsymbol{w}_k$ depends on finally chosen precoder which is not known

$$\hat{\boldsymbol{g}}_k = \boldsymbol{u}_\ell, \quad \ell = \arg \max_{q \in \{1,\ldots,2^B\}} \frac{\left|\boldsymbol{u}_q^{\mathrm{H}} \boldsymbol{g}_k\right|}{\left\|\boldsymbol{g}_k\right\|_2}$$

# Quantization with Minimum Euclidean Distance (N>1)

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**Idea:** Quantization of channel-receiver chain $\boldsymbol{g}_k = \boldsymbol{H}_k^{\mathrm{T}} \boldsymbol{w}_k$ (assuming one data stream per user)

**Problem:** Receiver not known at quantization step because it depends on precoder!

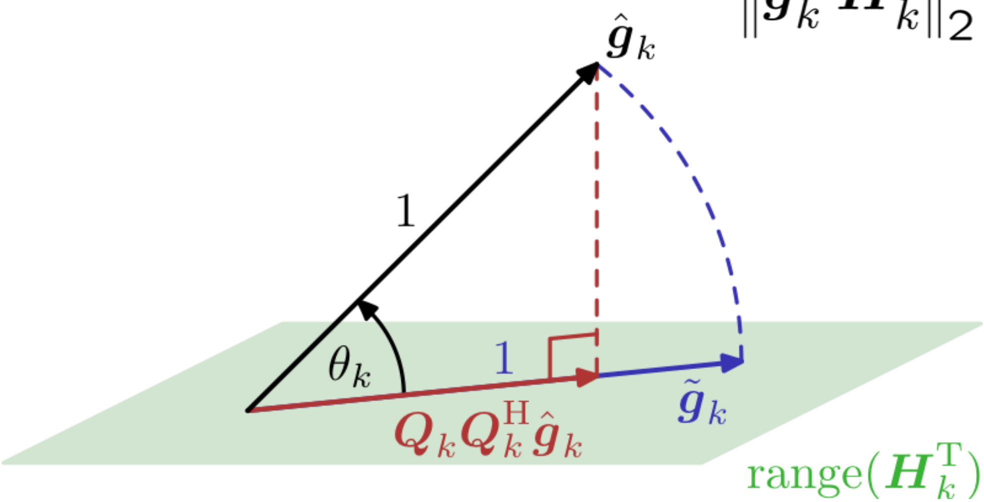
Quantization of linear combination of the rows of $\boldsymbol{H}_k$ with minimum error:

$$\hat{\boldsymbol{g}}_k = \boldsymbol{u}_\ell, \quad \ell = \operatorname*{argmax}_{q \in \{1,\ldots,2^B\}} \|\boldsymbol{Q}_k \boldsymbol{u}_q\|_2$$

with the QR factorization of the channel matrix $\boldsymbol{H}_k = \boldsymbol{Q}_k \boldsymbol{R}_k$.

Closest corresponding composite channel vector:

$$\boldsymbol{g}_k = \boldsymbol{H}_k^{\mathrm{T}} \boldsymbol{w}_k = \frac{\tilde{\boldsymbol{g}}_k}{\left\|\tilde{\boldsymbol{g}}_k^{\mathrm{T}} \boldsymbol{H}_k^{\dagger}\right\|_2}, \qquad \tilde{\boldsymbol{g}}_k = \frac{\boldsymbol{Q}_k \boldsymbol{Q}_k^{\mathrm{H}} \hat{\boldsymbol{h}}_k}{\left\|\boldsymbol{Q}_k \boldsymbol{Q}_k^{\mathrm{H}} \hat{\boldsymbol{h}}_k\right\|_2}$$




Angle between quantized and normalized composite channel vector:

$$\cos \theta_k = |\tilde{\boldsymbol{g}}_k^{\mathrm{H}} \hat{\boldsymbol{h}}_k|$$

Set of scheduled users:

$$\mathbb{K} = \{\pi_1, \pi_2, \ldots, \pi_D\} \qquad \text{e.g.} \qquad \mathbb{K} = \{1, 3, 7\}$$

Channel matrix:

$$\hat{\boldsymbol{G}}_{\mathbb{K}} = \begin{bmatrix} \hat{\boldsymbol{g}}_{\pi_1}^{\mathrm{T}} \\ \hat{\boldsymbol{g}}_{\pi_2}^{\mathrm{T}} \\ \vdots \\ \hat{\boldsymbol{g}}_{\pi_D}^{\mathrm{T}} \end{bmatrix} \qquad \text{e.g.} \qquad \hat{\boldsymbol{G}}_{\mathbb{K}} = \begin{bmatrix} \hat{\boldsymbol{g}}_{1}^{\mathrm{T}} \\ \hat{\boldsymbol{g}}_{3}^{\mathrm{T}} \\ \hat{\boldsymbol{g}}_{7}^{\mathrm{T}} \end{bmatrix}$$

ZF precoder:

$$\boldsymbol{P}_{\mathbb{K}} = \boldsymbol{P}'_{\mathbb{K}} \boldsymbol{\Lambda}_{\mathbb{K}}^{1/2}, \quad \boldsymbol{P}'_{\mathbb{K}} = \hat{\boldsymbol{G}}_{\mathbb{K}}^{\mathrm{H}} \left( \hat{\boldsymbol{G}}_{\mathbb{K}} \hat{\boldsymbol{G}}_{\mathbb{K}}^{\mathrm{H}} \right)^{-1}$$

Equal power allocation:

$$\boldsymbol{\Lambda}_{\mathbb{K}} = \mathrm{diag} \left( \frac{P_{\mathrm{Tx}}}{D \left\| \boldsymbol{p}'_{\mathbb{K},k} \right\|_2^2} \right)_{k=1}^{D}$$

# Definition of CQI

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Cos-angle between channel vector and quantized channel vector:

$$\cos\theta_k = \frac{|\boldsymbol{g}_k^{\mathrm{H}}\hat{\boldsymbol{g}}_k|}{\|\boldsymbol{g}_k\|_2}$$

CQI of user $k$:

$$\mathrm{CQI}_k = \frac{\frac{P_{\mathrm{Tx}}}{M}\|\boldsymbol{g}_k\|_2^2\cos^2\theta_k}{1+\frac{P_{\mathrm{Tx}}}{M}\|\boldsymbol{g}_k\|_2^2\sin^2\theta_k}$$

Throughput approximation:

$$\mathrm{E}\left[\mathrm{SINR}_k\right] \geq \frac{M\lambda_k}{P_{\mathrm{Tx}}}\mathrm{CQI}_k =: \mathrm{SI\hat{N}R}_k \quad \lambda_k = \boldsymbol{e}_k^{\mathrm{T}}\boldsymbol{\Lambda}_{\mathbb{K}}\boldsymbol{e}_k$$

$$\hat{R}_k = \log\left(1+\mathrm{SI\hat{N}R}_k\right)$$

# ZF-CVQ: Comparison of Different Codebooks

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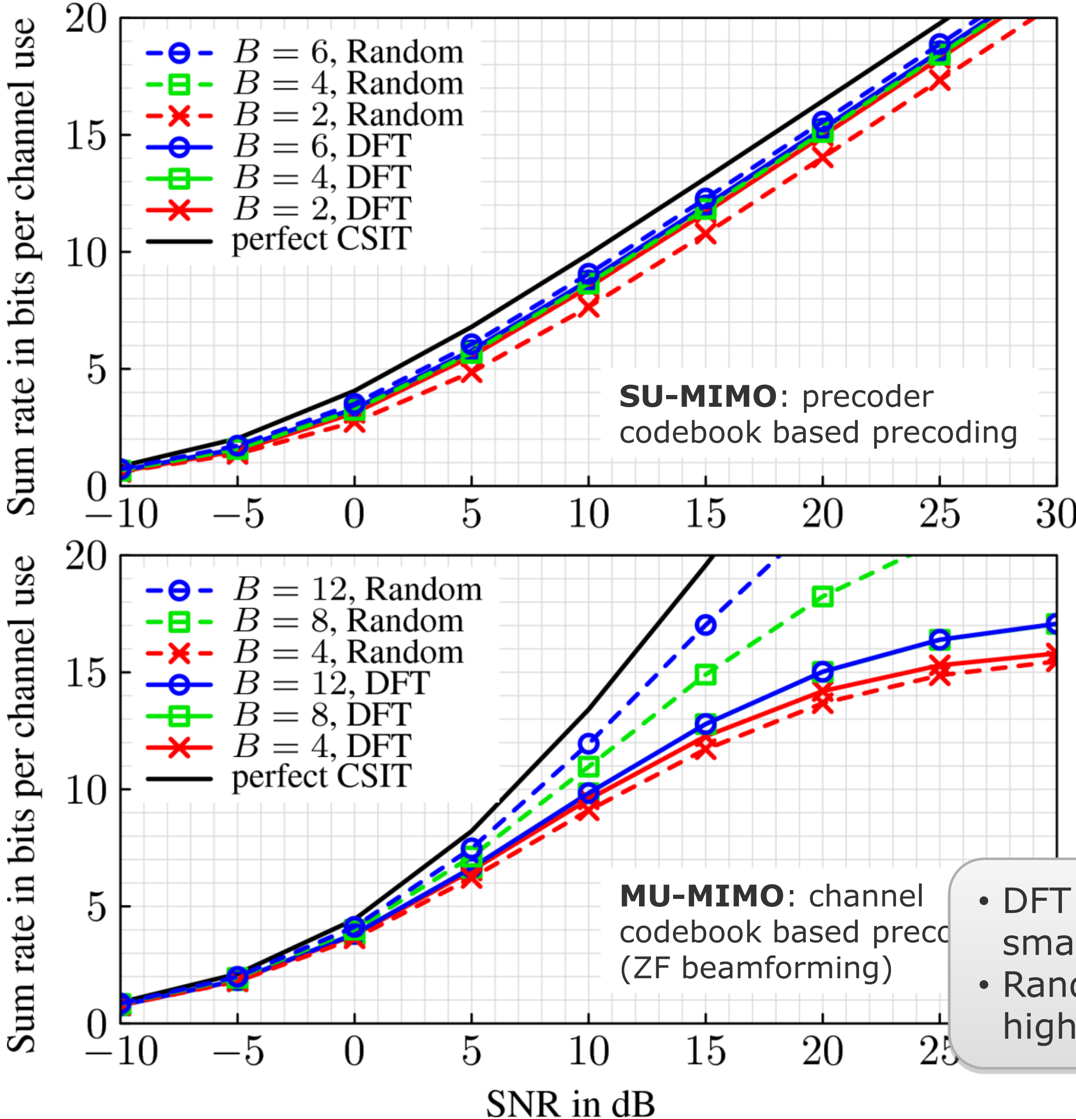


**System parameters**

4 Tx antennas

2 Rx antennas

20 users

uncorrelated channel (best case for random codebook)

feedback: *DB* bits and one real number

Rx: MMSE

MU-MIMO: one data stream per user

- DFT codebook is only good for small number of feedback bits
- Random codebook is preferable for higher number of feedback bits

# ZF-CVQ: MU- versus SU-MIMO Transmission

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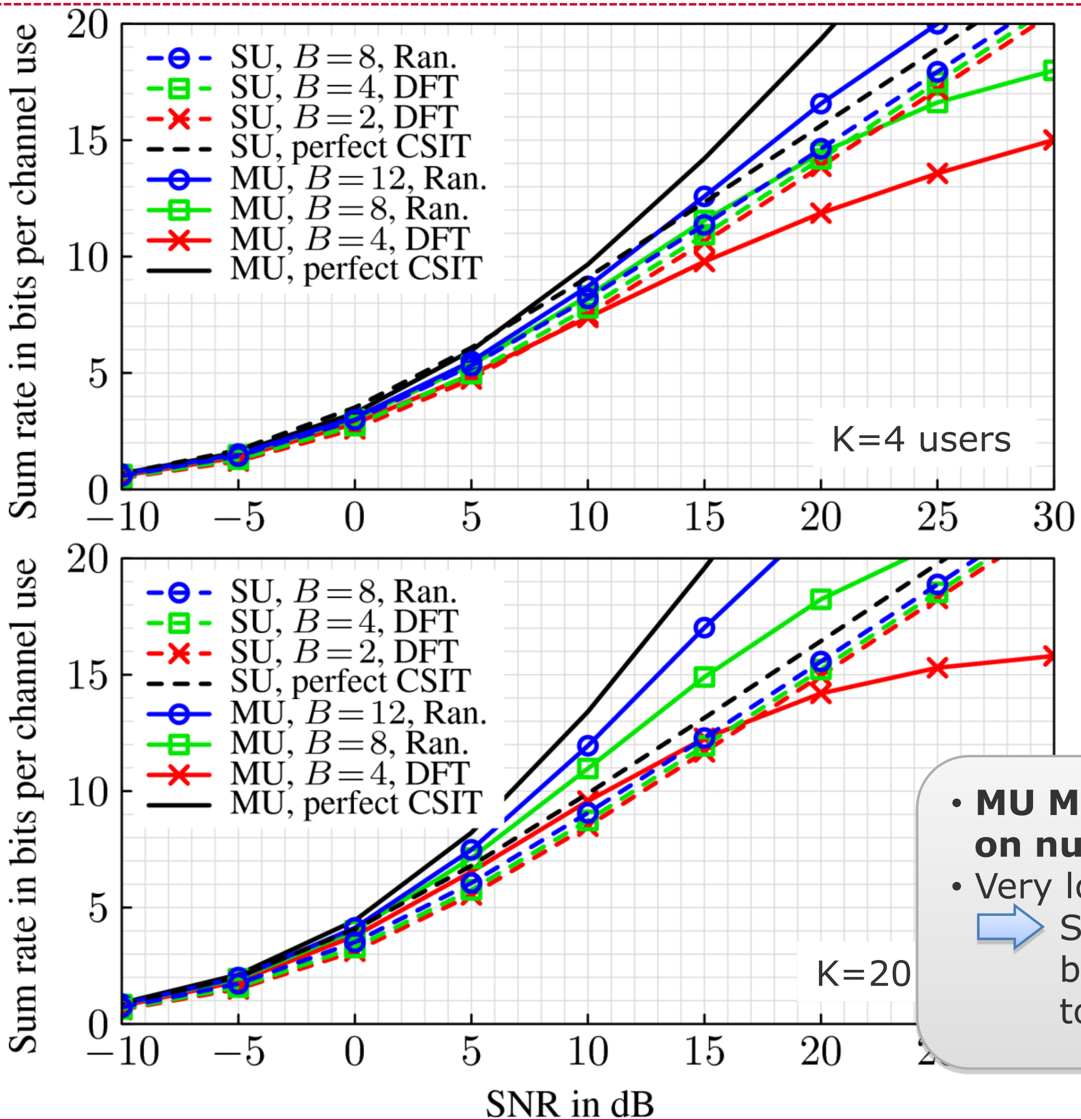



**System parameters**

4 Tx antennas

2 Rx antennas

uncorrelated channel

feedback:
*DB* bits and one real number

Rx: MMSE

MU-MIMO:
- one data stream per user
- max. 4 data streams

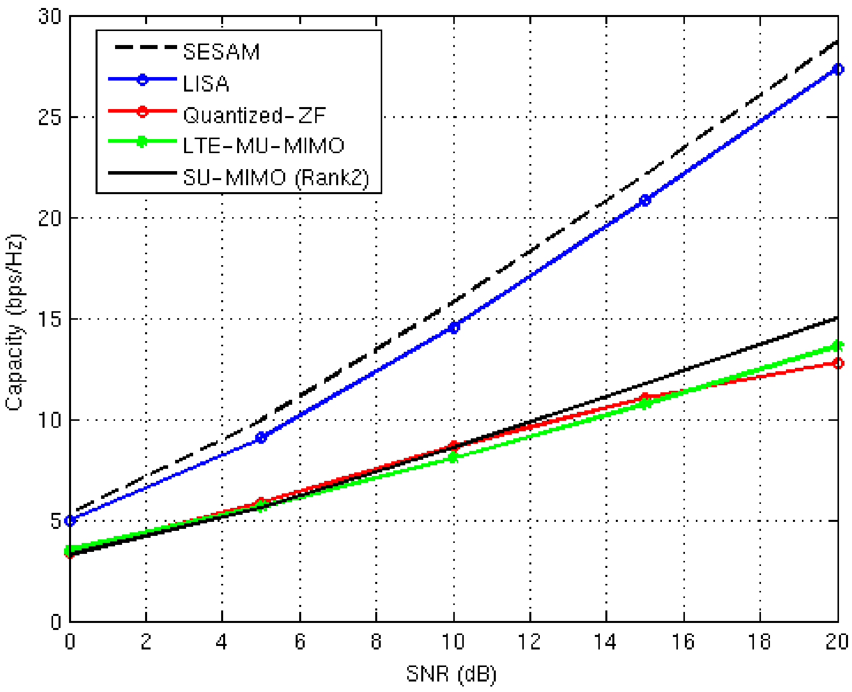
- **MU MIMO gain strongly depends on number of feedback bits!**
- Very low feedback rate
  ⇨ SU MIMO should be preferred because MU interference is too large!

# Comparison to 3GPP-LTE MU-MIMO Scheme

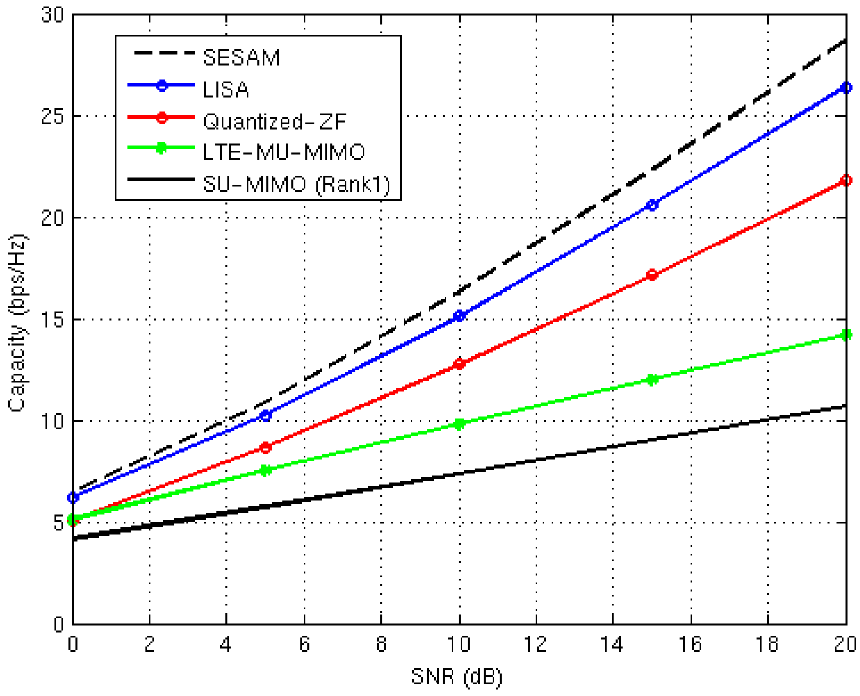


- SESAM, LISA with full CSI
- LTE SU/MU-MIMO, 4-bit Housholder CB
- MU-MIMO with ZF-CVQ, 4-bit DFT CB
- ZF: Max. 4 users
- LTE MU-MIMO: Max. 2 users

**Uncorrelated**



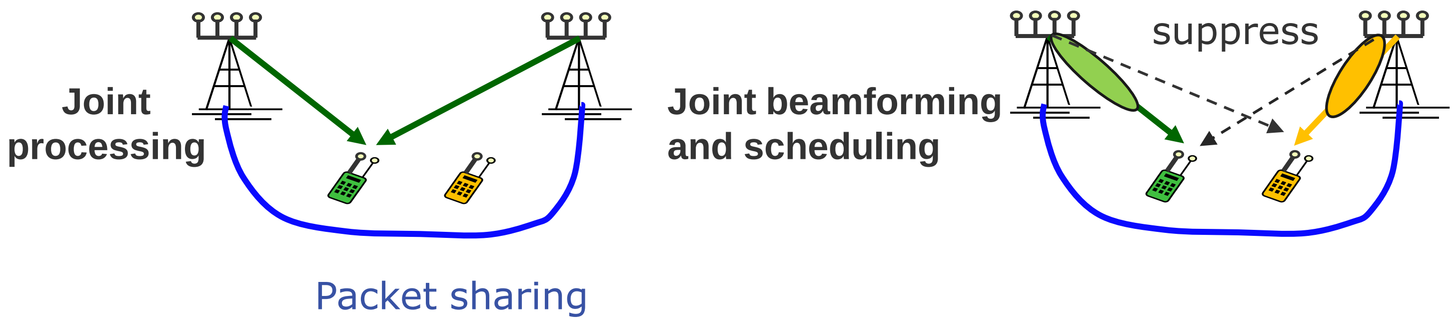

**Highly correlated**



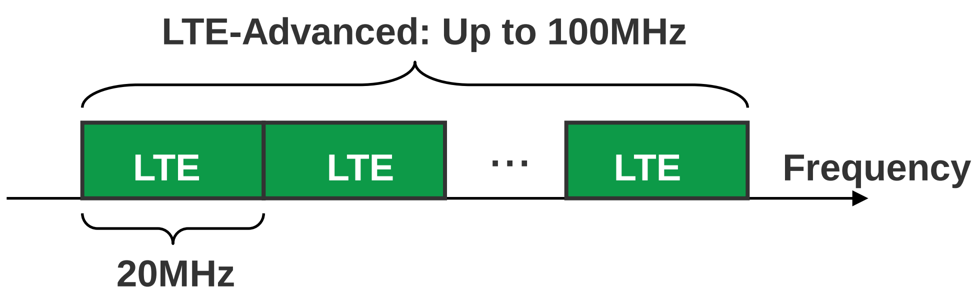

2x4 MIMO

Note: In LTE-MU-MIMO, uses having orthogonal precoding vector are paired.

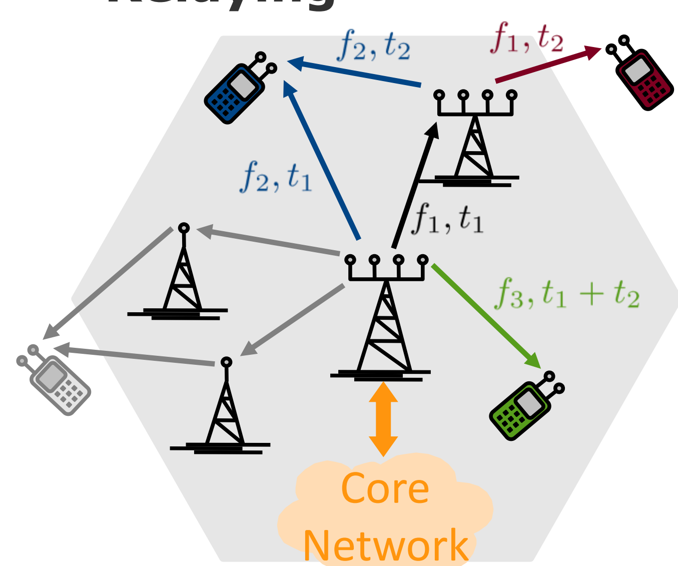
- **Coordinated MultiPoint (CoMP) Transmission/Reception**




- **Bandwidth Aggregation**




- **Relaying**




- **Uplink and Downlink MIMO**

## Status in 3GPP-LTE-Advanced: Uplink Transmission

- **SU-MIMO**
  - DFT-spread OFDMA (or SC-FDMA) as in 3GPP-LTE Release 8 but non-contiguous data transmission with single DFT per component carrier allowed
  - MIMO up to 4x4
  - up to 4 streams with maximal 2 code words

- **MU-MIMO**
  - more than one data stream per user possible

## Status in 3GPP-LTE-Advanced: Downlink Transmission

- **SU-MIMO**
  - OFDMA
  - MIMO up to 8x8
  - up to 8 streams

- **MU-MIMO**
  - sophisticated methods have been discussed
    - unitary precoder codebook based precoding (implicit CSI)
    - channel codebook based ZF beamforming (explicit CSI)
    - even non-linear precoding like THP is under discussion but very unlikely to be included because of excessive feedback overhead and complicated CQI estimation
  - more than one data stream per user possible: requires update on methods discussed before!