Department of Electrical Engineering, IIT Madras

EE6141 : Multi-Carrier Communications

Marks 50* **Simulation Assignment #2** **(OFDM Channel Estimation)** Due on: **May 20, 2015**

Note: *Assignment to be emailed to* vigneshkumar.ceg@gmail.com *on or before* ***6pm on Wednesday, May 20, 2015.*** *Mark the name of the pdf file* ee6141-**sa2**-rollnumber.pdf. *Independent work is expected from each student, and access to your code and additional information may be demanded if required.*

* The total marks will finally be scaled to 15 marks.

For all the problems in this assignment, the following 512-point FFT based OFDM system is to be used.

Sl #	Attribute	Value / Definition
1.	Subcarrier Bandwidth	f_{sub} = 10KHz = $1/T$ (T is useful symbol duration)
2.	FFT size	N = 512
3.	OFDM Signal Bandwidth	W=5.12 MHz
4.	Sampling Rate	$1/T_S = W$ = 5.12 MSps
5.	Cyclic Prefix duration	T_{CP} = 12.5 μsec
6.	Pilot locations in each OFDM symbol	Every ninth subcarrier, i.e., subcarrier locations (n, n+8, n+16, …) starting from n = -239
7.	OFDM Symbol duration	$T_{OFDM} = T + T_{CP}$ = 112.5 μsec
8.	Guard Subcarrier (GS) labels *Note*: They are different from Assg.#1	Upper GS: $n \in \{256 \text{ to } 241\}$ DC subcarrier: n = 0 Lower GS: $n \in \{-241 \text{ to } -255\}$

The above OFDM signal is transmitted over either of the two multi-path models $h_m = \sum_{i=0}^{L-1} a_i \, \delta(m - \tau_i)$ as given below, defined by their power delay profiles (PDPs).

*Channel Model #***1**

Path Gain σ_i^2(in dB)	0	-3	-8	-15
Path Delay τ_i (in μsecs)	0	0.5	1.7	2.2

*Channel Model #***2**

Path Gain σ_i^2(in dB)	-2	0	-1	-6	-9	-14
Path Delay τ_i (in μsecs)	0	1.8	3.5	5.7	8.1	12.3

Hint: To normalize average channel gain to unity, in each of these models, rescale the (linear value of) the path variance σ_i^2 to ensure that over the L paths, $\sum_{i=0}^{L-1} \sigma_i^2 = 1$. Each zero-mean path gain a_i, where $E[|a_i|^2] = \sigma_i^2$, is a complex Gaussian random variable with each dimension having a variance of $\sigma_i^2/2$. The impulse-response snap-shot $h[k,m]$ corresponding to a given PDP is obtained by calling a circular Gaussian rv L times, and scaling the gain based on the power profile.

The frequency response snap-shot $H[k,n]$ is obtained by zero-padding plus FFT (of typically large size to visualize shape easily). For each of the above PDPs, take a 512 point FFT of the instantaneous $h[k,m]$ (by appropriate zero-padding) to get $H[k,n]$. The measurement model in frequency domain is given by

$$Y[k,n] = H[k,n]X[k,n] + V[k,n], n = 1 \text{ to } N,$$

where the noise component $V[k,n]$ in every sub-carrier is i.i.d, zero-mean, circular Gaussian with variance σ_v^2. Therefore, the (average) received SNR based is given by SNR = $1/\sigma_v^2$, which can then be varied by varying the noise variance. Let the number of pilot subcarriers be N_P and the number of used subcarriers by N_U. Then, in the above measurement equation, $X[k,n]$ are unknown QPSK symbols on the data locations, while on the pilot subcarrier locations, we assume $X[k,n]= X_P[k,n]$ where $X_P[k,n]$ are known symbols drawn from QPSK alphabet.

Simulate k =1 to 1000 independent channel realizations (Monte-Carlo trials), over 1000 OFDM symbols, using each of the above PDPs. For each realization of CIR $h[k,m]$, let the corresponding CFR be $H[k,n]$. The aim is to estimate this CFR using the OFDM channel estimation algorithms given below, and compare their performance. For doing this, we first find the $N \times 1$ estimation error vector $\boldsymbol{e}(k) = \boldsymbol{H}[k] - \widehat{\boldsymbol{H}}[k]$, and compute the Mean Square Error (MSE) using the 1000 Monte-Carlo runs, which is approximated as follows:

$$MSE = E(\|\boldsymbol{e}(k)\|_2^2) \approx \frac{1}{N \times 1000} \sum_{k=1}^{1000} \|\boldsymbol{e}(k)\|_2^2 \text{ where } \|\boldsymbol{a}\|_2^2 = \boldsymbol{a}^H \boldsymbol{a}\,.$$

The MSE performance curve is obtained by plotting $10\, log_{10}(MSE)$ in the y-axis versus $10 log_{10}(SNR)$ on the x-axis, where both are in dB scale. Let the SNR vary in 3 dB steps from 0dB to 30dB. All the below MSE curves (for a given channel PDP) must be plotted on the same graph, for the same set of 1000 channel realizations.

Mark Allotment (Total 50 marks): 3+3+5+5+5+8+6+7+8

(a) What is the MSE curve of the Zero-Forcing CFR estimate $\widehat{\boldsymbol{H}}_{ZF}$ measured *only* on the N_P pilot locations?

(b) If the ZF estimate is linearly interpolated to the remaining (N_U - N_P) subcarriers, what is the MSE performance measured now on *all* the N_U used sub-carrier locations?

(c) *Advanced*: Instead of linear interpolation, you are welcome to try any other frequency- domain interpolation technique. Describe the technique and provide the (hopefully, improved) MSE curve measured.

(d) Instead of the schemes in (b), use the FFT-based interpolator discussed in the class. Use any other method including windowing to enhance the MSE performance of this $\widehat{\boldsymbol{H}}_{FFT}$, and plot the same. Explain your method(s) clearly.

(e) Now, develop the Modified Least Square (MLS) estimator, where the estimator assumes only that the CIR is confined to be $< N_{CP}$. Plot the MSE of this $\widehat{\boldsymbol{H}}_{MLS}$.

(f) *Advanced*: Do you need to regularize (loading on the diagonal) the pseudo-inverse computation for the MLS? If so, do an appropriate diagonal loading, and again compute the MSE performance. Explain your steps clearly.

(g) If the receiver knows the knowledge of the delay profile (i.e., multipath delay locations), can the performance of the corresponding MLS+ estimator $\widehat{\boldsymbol{H}}_{MLS+}$ be improved? Describe your method clearly, and provide the performance curve.

(h) Following the derivation of the modified Linear MMSE channel estimator done in class, set up the same using the knowledge of the PDP and the noise variance (at every SNR). Plot the MSE performance of this $\widehat{\boldsymbol{H}}_{m-LMMSE}$.

(i) *Advanced*: Those of you who are familiar with the way to determine the Cramer-Rao Lower Bound (CRLB) for the MSE, do so by first computing the Maximum Likelihood (ML) Estimate. Compare the CRLB with the MSE performance all the above channel estimators on the same graph. Does the relative performance of the various algorithms change between channel model #1 and #2? If so, why? What conclusions can you draw?