Department of Electrical Engineering, IIT Madras
EE6141 : Multi-Carrier Communications

Marks 50* **Simulation Assignment #1 (OFDM Sync.&PAPR)** Due on: **April. 28, 2015**

Note: *Assignment to be submitted by email to* sriblue7@gmail.com *on or before* ***5pm on Tuesday, April 28, 2015****. Mark the name of the pdf file* ee6141-sa1-rollnumber.pdf. *Independent work is expected from each student, and access to your code and additional information may be demanded if required.*

* The total marks will finally be scaled to 20 marks.

For all the problems in this assignment, the following 512-point FFT based OFDM system is to be used.

Sl #	Attribute	Value / Definition
1.	Subcarrier Bandwidth	$f_{\text{sub}} = 10\text{KHz} = 1/T$ (T is useful symbol duration)
2.	FFT size	$N = 512$
3.	OFDM Signal Bandwidth	W=5.12 MHz
4.	Sampling Rate	$1/T_S = W = 5.12$ MSps
5.	Cyclic Prefix duration	$T_{CP} = 12.5$ μsec
6.	Frame duration (S)	Typically, $S = 5$ OFDM symbols; preamble, if present, will be the 1st symbol in the frame.
7.	OFDM Symbol duration	$T_{OFDM} = T + T_{CP} = 112.5$ μsec
8.	Guard Subcarrier (GS) labels	Upper GS: $n \in \{256 \text{ to } 249\}$ DC subcarrier: $n = 0$ Lower GS: $n \in \{-249 \text{ to } -255\}$

1. [2+2+2+3+6=15 marks] PAPR: Assume i.i.d, unit-energy, QPSK symbols are imposed on all the useful subcarriers. The Peak to Average Power Ratio (PAPR) is defined on the base-band samples at the Tx DAC input as follows:

$$\text{PAPR} \cong \frac{\max \ |\tilde{x}(k,m)|^2}{\frac{1}{S(N+N_{CP})}\sum_{k=1}^{S}\sum_{m=1}^{N+N_{CP}} |\tilde{x}(k,m)|^2}$$

Calculate the PAPR in the decibel (dB) scale for the following situations:

(a) $S = 1$; $1/T_S$ =5.12 MSps

(b) $S = 5$; $1/T_S$ =5.12 MSps

(c) $S = 50$; $1/T_S$ =5.12 MSps

(d) $S = 50$; $1/T_S$ = 51.20 MSps†

† In this case, you will have to zero-interleave and interpolate $\tilde{x}(k,m)$ by a factor of 10, in order to generate the higher sampling rate. Another option is to use to higher-point FFT (think!) with appropriate zero-padding.

(e) Read Ch-7 Section 7.1 in Cho's book, understand, and then plot the Complementary CDF (CCDF) curve as in Fig. 7.3 in the book, both for case (c) and case (d). Comment on your results.

2. [2+2+2+9 = 15 marks] SC Frequency Sync: A preamble symbol is to be designed to ensure that the *entire* frequency offset can be estimated by the Schmidl-Cox (SC) algorithm. Assume that the non-zero subcarriers in the preamble use i.i.d QPSK symbols. The maximum frequency offset seen on the received samples is $\Delta f = 8.57\text{KHz}$. The samples at the receiver's ADC output can be modeled by $\tilde{y}(k,m) = e^{j2\pi\Delta f m T_s}\,\tilde{r}(k,m)$, where in turn the noisy measurement $\tilde{r}(k,m) = h_m * \tilde{x}(k,m) + v(m)$ ®. Here, "$*$" represents linear convolution, and $\tilde{x}(k,m)$ is obtained by adding the CP to $x(k,m)$, with $\bar{x}(k) = F\bar{d}(k)$ where F is the NxN full DFT (FFT) matrix with scaling factor $1/\sqrt{N}$ to ensure that statistically the average gain of each $\tilde{x}(k,m)$ is unity. Further, assume in ® that h_m=1 and that $v(m)$ is zero-mean, circular Gaussian with variance σ_v^2 . Therefore, the (average) received SNR based on $\tilde{y}(k,m)$ is given by SNR = $1/\sigma_v^2$, which can then be varied by varying the noise variance.

(a) Specify the preamble symbol in the frequency domain, and describe it’s time-domain properties using a labeled simulated result.

(b) We define the ergodic Mean Square Error in the frequency offset estimate by MSE $\triangleq \frac{1}{J}\sum_{j=1}^{J}\left(\Delta f_j - \Delta\hat{f}_j\right)^2$, where $\Delta\hat{f}_j$ is the estimate from the “j^{th}” trial of the SC algorithm with independent signal and noise samples in each trial. Use J =10 for your MSE simulations. Vary the SNR between 0dB and 30dB in steps of 3dB, and measure the MSE in each case. Plot the MSE in the dB scale on the y-axis, and SNR (also in the dB scale) on the x-axis.

(c) Can you improve the performance of the frequency offset estimator by doing more averaging over the preamble symbol? If possible, explain your approach clearly. Plot the resultant MSE for your approach on the same plot as in part (b). Comment on your result.

<u>Note</u>: The below section is worth 9 marks.
(d) Repeat (a) thro (c) for the case where Δf = 26.13KHz . When you compare the best possible MSE in this case (following your answer to part (c)), to that for the lower offset, what do you observe? Carefully explain your approach in part (c) for this case.

3. [3+3=6 marks] SC Timing Sync: For the preamble designed for Δf = 26.13KHz in part (d) of Q.**2** above:
(a) Given that it is a single-tap channel, what is the maximum amount of averaging that you can do within the SC algorithm? Give the expression, and how many correlation terms are involved in the same? (A single correlation term would imply the product of two measurements spaced “appropriately” apart in time).
(b) Provide the plot of the SC correlation output from which the FFT window (timing instant) is derived. Plot this for SNR=6dB, over 2 consecutive frames of size S = 5 OFDM symbols where the first symbol is the preamble in each frame.

4. [4 marks] Frequency-selective Fading: The above OFDM signal is transmitted over either of the two multi-path models $h_m = \sum_{i=0}^{L-1} a_i\, \delta(m-\tau_i)$ as given below, defined by their power delay profiles (PDPs).

*Channel Model #***1**

Path Gain σ_i^2(in dB)	0	-3	-8	-15
Path Delay τ_i (in μsecs)	0	0.5	1.7	2.2

*Channel Model #***2**

Path Gain σ_i^2(in dB)	-2	0	-1	-6	-9	-14
Path Delay τ_i (in μsecs)	0	1.8	3.5	5.7	8.1	12.3

Hint: To normalize average channel gain to unity, in each of these models, rescale the (linear value of) the path variance σ_i^2 to ensure that over the L paths, $\sum_{i=0}^{L-1}\sigma_i^2 = 1$.
Each zero-mean path gain a_i, where $E[|a_i|^2] = \sigma_i^2$, is a complex Gaussian random variable with each dimension having a variance of $\sigma_i^2/2$. The impulse-response snap-shot h[n] corresponding to a given PDP is obtained by

calling a circular Gaussian rv L times, and scaling the gain based on the power profile. The frequency response snap-shot $H[k,n]$ is obtained by zero-padding plus FFT (of typically large size to visualize shape easily). For each of the above PDPs, take a 2048 point FFT of the instantaneous h_m (by appropriate zero-padding) to get $H[k,n]$. Plot in dB scale the squared gain, i.e., $10\log_{10}(|H(k,n)|^2)$, to interpret the coherence band-width of models #1 and #2 and comment. Repeat for each model over 5 different (independent) channel realisations, and plot in the same figure.

5. [5 marks] SC-based Timing Sync for Selective Channel: Now, replace the h_m in equation ® Q.**2** with the channel model #2 in Q.**4**.
(a) Provide the plot of the SC correlation output from which the FFT window (timing instant) is derived. Plot this for SNR=10dB, over 2 consecutive frames of size S = 5 OFDM symbols where the first symbol is the preamble in each frame.
(b) Repeat (a) for channel model #1 in Q.4.

6. [5 marks] CP-Corr based Timing Sync for Selective Channel: For h_m in equation ® Q.**2** with the channel model #1 in Q.**4**, describe the CP-correlation based approach to derive timing sync (FFT window). Plot this for SNR=10dB, over 10 consecutive OFDM symbols, since there is no preamble. Describe the CP-corr window you have used, and the number of correlation terms involved. Will this choice be suitable for both models #1 and #2?

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