Department of Electrical Engineering, Indian Institute of Technology, Madras

EE 6002: Multicarrier Communications

March 19, 2013 **Mid-Sem (Take Home)** 50 marks

Answer sheets must be submitted by Tuesday 9am, March 26. Independent effort is expected from all.

1. Pilot Design and Overheads [10 marks] : Consider a 10MHz OFDM system with FFT size of N=1024. Assuming that the subcarriers are indexed from 512 to -511, the guard sub-carriers are indexed by (512 to 472) and (-472 to -511). The sampling rate is taken to be f_S=10MHz. Answer now the following:

(a) What is the subcarrier band-width Δf in KHz?

(b) If the Cyclic Prefix (CP) length N_{cp} is $1/8^{th}$ of the useful symbol duration, what is the duration (in μsec) of the full OFDM symbol?

(c) If QPSK modulation is employed on all the useful subcarriers, what is the "gross" spectral efficiency of this OFDM system (in bits/sec/Hz)?

(d) This system is deployed to cover a circular area of 3km radius, where the delay spread is 5μsec or less. Also, the maximum Doppler frequency is expected to be 200Hz. Generally, it is advisable to have at least 1 or 2 pilot subcarriers (preferably 2 pilots) within a "coherence band", and pilots in time as often as every 10% of the "coherence time" (which is approximately 36^0 phase change). Given this, how will you distribute pilot subcarriers in a 2-D manner (i.e., over frequency and time)?

(e) If 2.5ppm clocks are used on both the Tx and Rx nodes, how often should the preamble symbol be repeated in time so that between preambles, there is no more than half-a-sample of slip? (*Hint*: First find the slip-rate in number of sample slips per second.)

(f) Putting (d) and (e) together, make a neat sketch of the OFDM blocks, say drawn over two preamble intervals. The preamble can be assumed to also mark the beginning of a new frame. What is the frame rate in frames/sec?

(g) What is the "nett" spectral efficiency, after accounting for preamble and pilots?

(h) Now, the cell-radius is increased so that the maximum delay spread can at most equal the CP length defined in part (b). Recalculate your answer to part (d), and hence, redo your answer to part (g).

2. Preamble Design for Frequency Offset Estimation [10 marks] : Let us continue with the same 10MHz OFDM system with FFT of N=1024 as in Pbm.1 ☹ ! This system has a maximum carrier frequency f_C=6GHz, and the oscillators (clocks) can have at most 5ppm error on both the Tx and on the Rx side. Assuming that the subcarriers are indexed similar to Pbm.1, answer the following:

(a) What is the maximum possible frequency offset f_O possible at the receiver? (Assume that there is no Doppler, and that f_O is caused only due to the oscillator clock drift.)

(b) If the preamble symbol uses only every 4^{th} subcarrier (i.e., n,n+4,n+8,….), what is the maximum frequency offset that can be corrected in the time domain, using the Schmidl-Cox method of correlation?

(c) Explain how the remaining (integer) part of the frequency error will be corrected. What are the maximum and minimum values of this integer frequency offset possible?

(d) Now, if it was possible to change the design of the preamble, how will you change it so that there is no integer offset to be estimated (unlike part (c)) ?

(e) Describe using discrete-time notation the Schmidl-Cox (SC) algorithm that you will use for your choice in part (d). What will be the averaging window?

(f) For your choice as in (d), can you handle some more frequency offset which is induced due to Doppler? If so, how much Doppler (in Hz) can you handle using this design?

3. IBI and ICI [10 marks] : Recall that the *N*-point (inverse) DFT to obtain the time-domain samples can be given by $x[k,m] = \frac{1}{N}\sum_{n=-\frac{N}{2}+1}^{N/2} X[k,n]e^{j\frac{2\pi}{N}mn}$ where $m = -N_{CP}, -N_{CP}+1, \cdots, -1, 0, 1, 2, \cdots, N-1$., and to make the index go from 1 to $N+N_{CP}$ we can define $z[k(N+N_{CP})T_S + N_{CP}T_S + mT_S] = x[k,m]$ where the sampling time is T_S.

(a) Now, if we would like to instead use the summation from n=0 to N-1 for the I-DFT above, then how does the index for m, as well as the definition of $z[.]$, change?

(b) Take an example where N=8, and N_{CP}=1. Let the channel $h[mT_S] = h[m] = \delta[m]$. For the following choices of timing offset Δ , determine the expressions for the signal term, and the IBI, ICI, and the IBI-induced ICI terms with reference to the DC subcarrier (i.e., as measure on the subcarrier n=0). Also, what is the Signal to Interference ratio (SIR) expression in each case, where all the 8 sub-carriers (including the DC subcarrier) are using QPSK modulation with symbol energy E_S. Assume that all the QPSK symbols are i.i.d. Choices of Δ are: (i) Δ =0; (ii) Δ =1; (iii) Δ =2; and (iv) Δ =4.

4. Frequency Domain Cross Correlation [10 marks] : A 128-point FFT is used to define a OFDM system with bandwidth 5.12MHz. Since the worst-case frequency offset possible in this system was expected to be at most 50KHz, a preamble symbol is defined with only alternate sub-carriers carrying a PN-code (from a set of Q possible codes of appropriate length). The channel delay spread is limited to 1μsec, and a CP length in sample durations, of $N_{CP}=N/4$ is used. Of the 128 tones, a total of sixteen subcarriers are used for spectral shaping at the two band-edges, and also for ensuring zero DC.

(a) Indicate the non-zero sub-carrier indices used by any data-carrying OFDM symbol.

(b) Now, what will be the non-zero subcarrier indices for the preamble symbol?

(c) Describe both of these answers, by an appropriately labeled sketch of the frequency domain allocation.

(d) Other than the SC algorithm or the CP-Correlation or Cross-correlation methods discussed in class, can you suggest any other timing recovery algorithm for this problem? A proper mathematical description is required. ***Hint***: You may do well to look up ML or MMSE estimators from books/papers for this problem.

(e) Following the SC algorithm for timing and carrier recovery, the code-search (to understand the base-station ID) is to be performed in the frequency domain. Given that the channel is not known and neither is channel estimation done as yet, can you suggest a cross-correlation technique that will reveal the closest matching code? ***Hint***: Non-coherent cross-correlation could be considered.

5. Simulation based problem [10 marks] **:** Consider again the same 5.12MHz OFDM system with N=128 and N_{CP}=32, and the spectral shaping parameters also as given in Pbm. 4. The received filtered, noisy samples, which are sampled at $T_S=1/f_S=1/5.12$MHz are given by $z(mT_S) = z[m] = \{ f_0\delta[m] + f_1\delta[m-4] + f_2\delta[m-9] + f_3\delta[m-21]\} * x[m] + v[m]$ where the "*" represents linear convolution and $x[m]$ are the transmitted OFDM signal samples. The measurement noise $v[m]$ is AWGN with variance σ_v^2 and is mutually uncorrelated with $x[m]$. The complex channel coefficients (before energy normalization) are given by f_0= 1.0+j0.8, f_1= -0.5-j0.4, f_2= -0.8+j0.3, and f_3= 0.4-j0.2, and data $I(n)$ used to obtain $x[m]$ are QPSK symbols with variance $\sigma_I^2 = 1$. The received signal to noise ratio is then given by SNR=$1/\sigma_v^2$. Given that every 4th OFDM symbol in a frame is a preamble symbol, and only every 4th subcarrier is used in the preamble symbol, answer the following:

(a) Four different preambles are to be defined, so that in the frequency domain, their non-zero subcarriers do not overlap. Design the 4 preambles so that each of them are zero-DC. Next, plot for SNR=20dB, individually, their corresponding $N+N_{CP}$=160 samples. Are all of them “equally” SC compatible? Explain.

(b) Take any 1 of these 4 preambles. Given SNR=10dB, design the best possible Schmidl-Cox (SC) based algorithm for determining the preamble symbol and the FFT window using only 1 frame of samples. Explain the modification you did to the conventional SC, and substantiate you design using simulations. Compare your result with:

(b1) Conventional SC

(b2) CP Correlation method with an appropriate averaging window

(c) Given a cellular deployment, assume that the 4 different preambles you have made in part (a) are used by the synchronized Base-Stations (BS) in a reuse-1/4 manner across 4 adjacent cells. Now, if a user is at a cell-edge, and sees more than one BS, will the required property that SC needs be seen in the time-domain samples? Answer this using your simulations for various scenarios. Assume independent data (and channels) for the different signals. (To generate independent channels, choose different channel taps but with nearly the same delay spread, for the different BS signals. Ensure that all the channels have unit gain, by proper normalization.) Make the desired signal and the interfering signal(s) have the same received power by proper normalization. Let SNR=20dB.