Department of Electrical Engineering, Indian Institute of Technology, Madras

EE 6002: Multicarrier Communications

May 9, 2013 **Simulation based Tutorial on OFDM Channel Estimation** 10 marks

Answer sheets must be submitted to the course instructor by Monday 10am, May 13 at ESB-350. Again,it goes with saying that honest, independent effort is expected from all.

Consider our familiar 10MHz OFDM system with FFT size of N=1024. Assuming that the used subcarriers are indexed from 512 to -511, the guard sub-carriers are indexed by (512 to 472) and (-472 to -511). The sampling rate is taken to be f_S=10MHz, and the cyclic prefix length is N_{CP}=N/8. The multipath channel has 5 uncorrelated paths with Rayleigh statistics (i.e., each path gain is complex, following a circular Gaussian distribution), and the corresponding power-delay profile (PDP) is given by the following table:

Delay (in μsec)	0	0.7	1.3	2.5	4.6
Gain (in dB) (before normalization)	0	-3	-5	-6	-10

Normalize the (linear) values of the channel gains in this PDP so that they add up to unity. The OFDM symbol carrying embedded pilots has a comb of pilot sub-carriers with a spacing of 8 (n,n-9,n-18,…) starting from subcarrier # 470. Pilot symbols are QPSK modulated with unit symbol energy. The average SNR seen by the receiver can then be given by SNR = $1/\sigma_v^2$ where σ_v^2/N is the variance of the measurement noise $V[n]$ on the n^{th} sub-carrier of the frequency domain, IBI and ICI free measurement model. The scalar measurement model is given by $Y[n] = X[n]H[n] + V[n]$ and the vector measurement model on the N_P equi-spaced pilot subcarriers is given by $\mathbf{Y}[n] = \mathbf{X}[n]\mathbf{H}[n] + \mathbf{V}[n]$ where $\mathbf{X}$[n] is N_P x N_P, and has all the pilot symbols placed on the diagonal.

Simulate 1000 independent channel realizations (Monte-Carlo trials) using the above PDP. For each realization of CIR $\mathbf{h}[k]$, let the corresponding CFR be $\mathbf{H}$[k]. The aim is to estimate this CFR using some of the well known channel estimation algorithms, and compare their performance. Given the Nx1 error vector $\mathbf{e}[k] = \mathbf{H}[k] - \hat{\mathbf{H}}[k]$, the Mean Square Error (MSE) using the 1000 Monte-Carlo runs is approximated as follows: $\text{MSE} = E(\|\mathbf{e}(k)\|_2^2) \approx \frac{1}{1000}\sum_{k=1}^{1000}\|\mathbf{e}(k)\|_2^2$ where $\|\mathbf{a}\|_2^2 = \mathbf{a}^H\mathbf{a}$. The MSE performance curve is obtained by plotting 10 $\log_{10}$(MSE) in the y-axis versus 10 $\log_{10}$(SNR) on the x-axis, where both are in dB scale. Let the SNR vary in 3 dB steps from 0dB to 30dB. *All the below MSE curves must be plotted on the **same** graph, for the same set of channel realizations.*

(a) What is the MSE curve of the Zero-Forcing CFR estimate $\hat{\mathbf{H}}_{ZF}$ measured only on the pilot locations?

(b) If the ZF estimate is linearly interpolated to the remaining subcarriers, what is the MSE performance? *Bonus*: Instead of linear interpolation, you are welcome to try any other frequency-domain interpolation technique. Describe the technique and provide the MSE curve.

(c) Instead of the schemes in (b), use the FFT-based interpolator discussed in the class. Use any other method including windowing to enhance the MSE performance of this $\hat{\mathbf{H}}_{FFT}$, and plot the same.

(d) Now, develop the modified Least Square (MLS) estimator, where the estimator assumes only that the CIR is confined to be < N_{CP}. Plot the MSE of this $\hat{\mathbf{H}}_{MLS}$.

(e) Do you need to regularize the pseudo-inverse computation for the MLS? If so, do an appropriate diagonal loading, and again compute the MSE performance.

(f) If the receiver knows the knowledge of the delay profile (i.e., multipath delay locations), can the performance of the corresponding MLS+ estimator $\hat{\mathbf{H}}_{MLS+}$ be improved? Describe your method clearly, and provide the performance curve.

(g) Following the derivation of the modified Linear MMSE channel estimator done in class, set up the same using the knowledge of the PDP and the noise variance (at every SNR). Plot the performance of the corresponding $\hat{\mathbf{H}}_{m-MMSE}$.

(h) After plotting the MSE performance all the above channel estimators on the same graph, what conclusions can you draw?