Orthogonal Frequency Division Multiplexing (OFDM): A Primer

Dr. Miguel Rodrigues

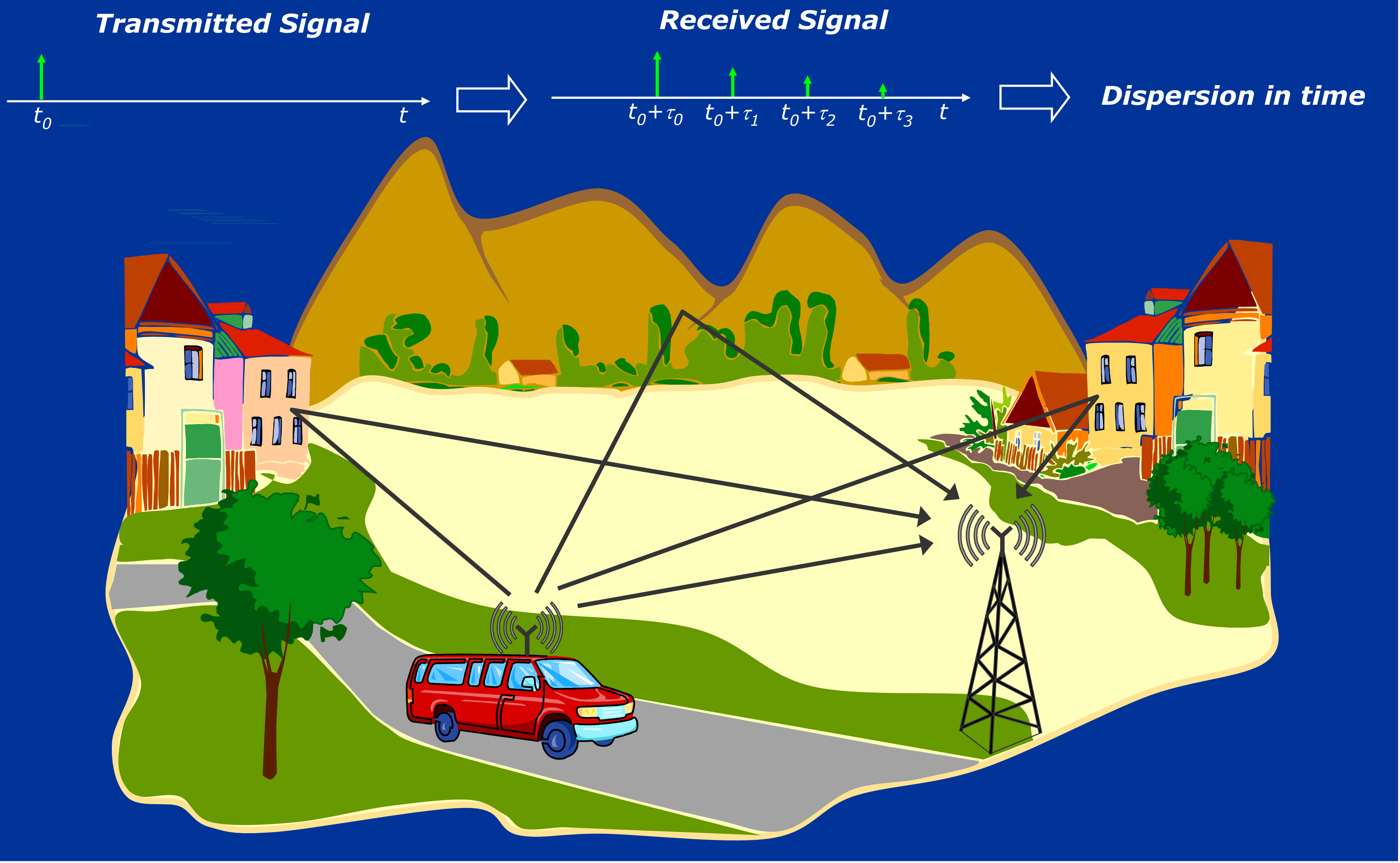
Laboratory for Communication Engineering
Department of Engineering, University of Cambridge

`E-mail: mrdr3@eng.cam.ac.uk`

Agenda

- The wireless propagation channel: A case for OFDM?
- Elements of a typical OFDM communications system
- OFDM generation/detection
- Coding/decoding and mapping/demapping techniques
- Channel estimation and synchronisation operations
- Advantages and disadvantages of OFDM
- Applications of OFDM
- Recent developments

The Wireless Propagation Channel

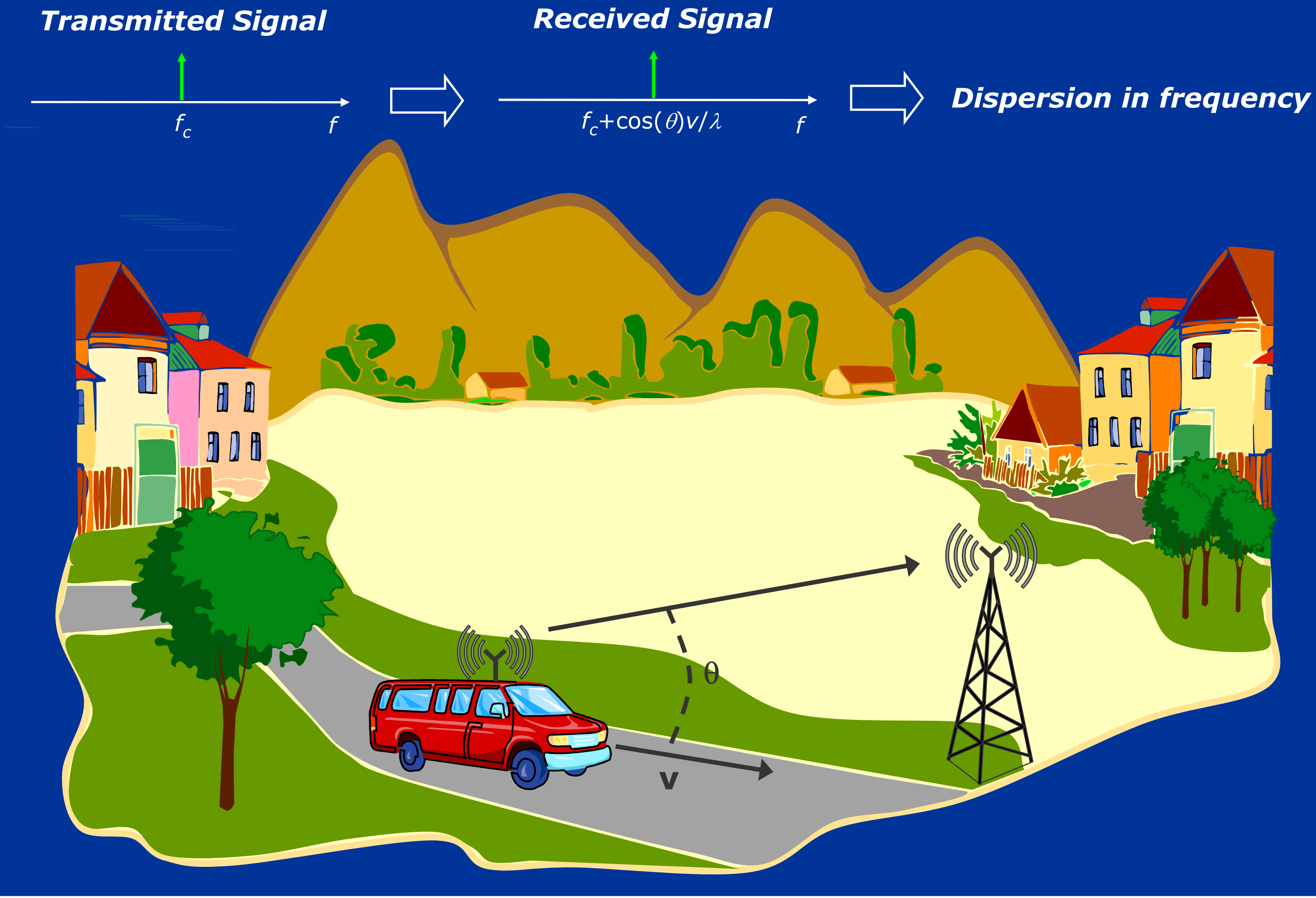


Transmitted Signal

Received Signal

Dispersion in time

The Wireless Propagation Channel

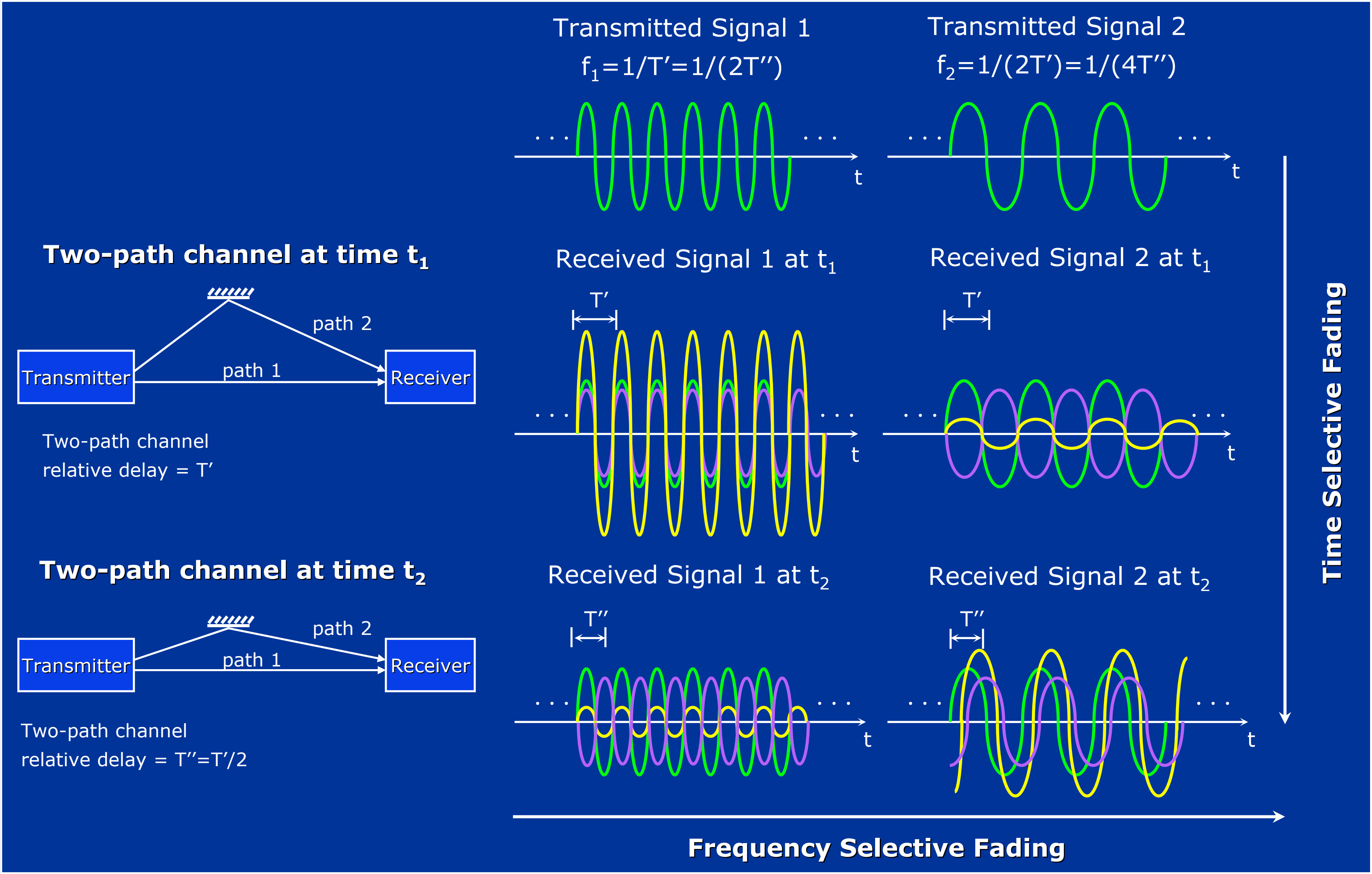


Transmitted Signal

Received Signal

Dispersion in frequency

The Wireless Propagation Channel



Two-path channel at time t_1

Transmitter

path 1

path 2

Receiver

Two-path channel
relative delay = T′

Two-path channel at time t_2

Transmitter

path 1

path 2

Receiver

Two-path channel
relative delay = T″=T′/2

Transmitted Signal 1
$f_1=1/T'=1/(2T'')$

Transmitted Signal 2
$f_2=1/(2T')=1/(4T'')$

Received Signal 1 at t_1

Received Signal 2 at t_1

Received Signal 1 at t_2

Received Signal 2 at t_2

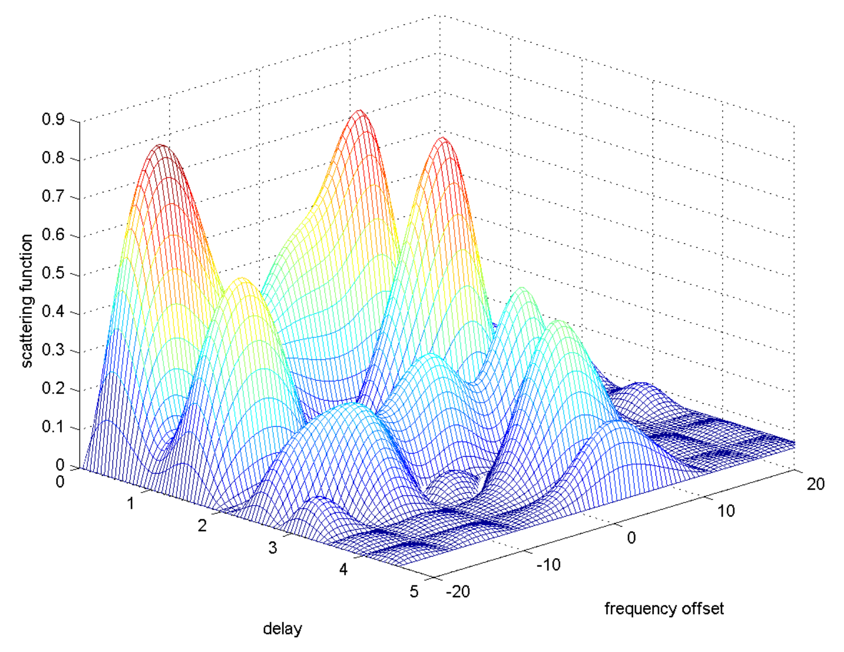
Time Selective Fading

Frequency Selective Fading

The Scattering Function

- The scattering function measures the power spectrum of the channel at delay τ and frequency offset λ with respect to the carrier frequency.

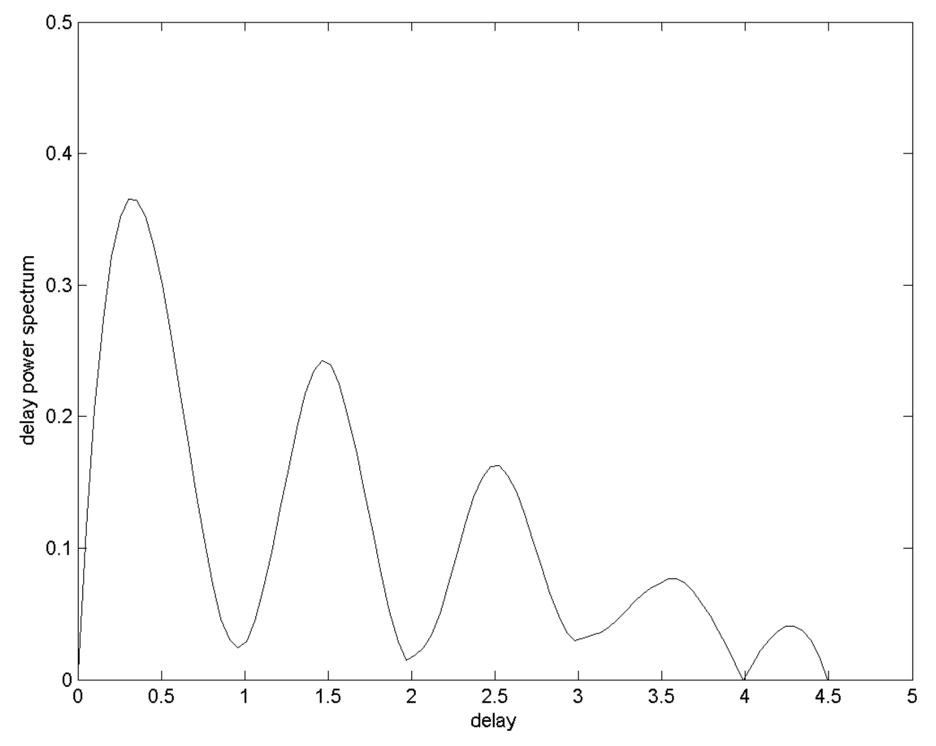
Typical Scattering Function



The Delay Power Spectrum and Delay Spread

- The delay power spectrum measures the power of the channel at delay τ.
- The range of values τ over which the delay power spectrum is essentially non-zero is defined as the delay spread T_m.
- The channel coherence bandwidth B_{coh} is the inverse of the delay spread T_m, i.e., $B_{coh}=1/T_m$.
- The channel coherence bandwidth measures the width of the band of frequencies over which the fading is highly correlated.

Typical Delay Power Spectrum

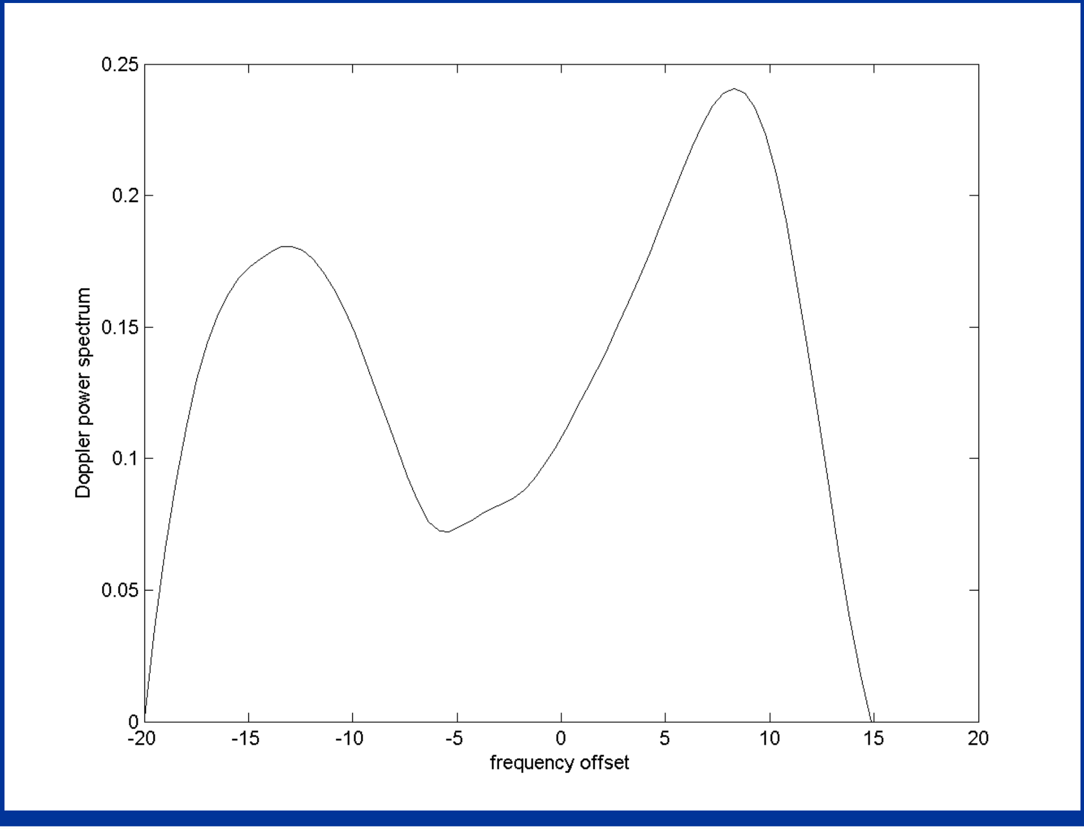

Typical Delay Power Spectrum

The Doppler Power Spectrum and Doppler Spread

- The Doppler power spectrum measures the power of the channel at frequency offset λ.
- The range of values λ over which the Doppler power spectrum is essentially non-zero is defined as the Doppler spread B_d.
- The channel coherence time T_{coh} is the inverse of the Doppler spread B_d, i.e., $T_{coh}=1/B_d$.
- The channel coherence bandwidth measures the width of the interval of time over which the fading is highly correlated.

Typical Doppler Power Spectrum

Typical Doppler Power Spectrum



Fading Statistics

- The fading characteristics of the wireless channel are described by probability distributions.

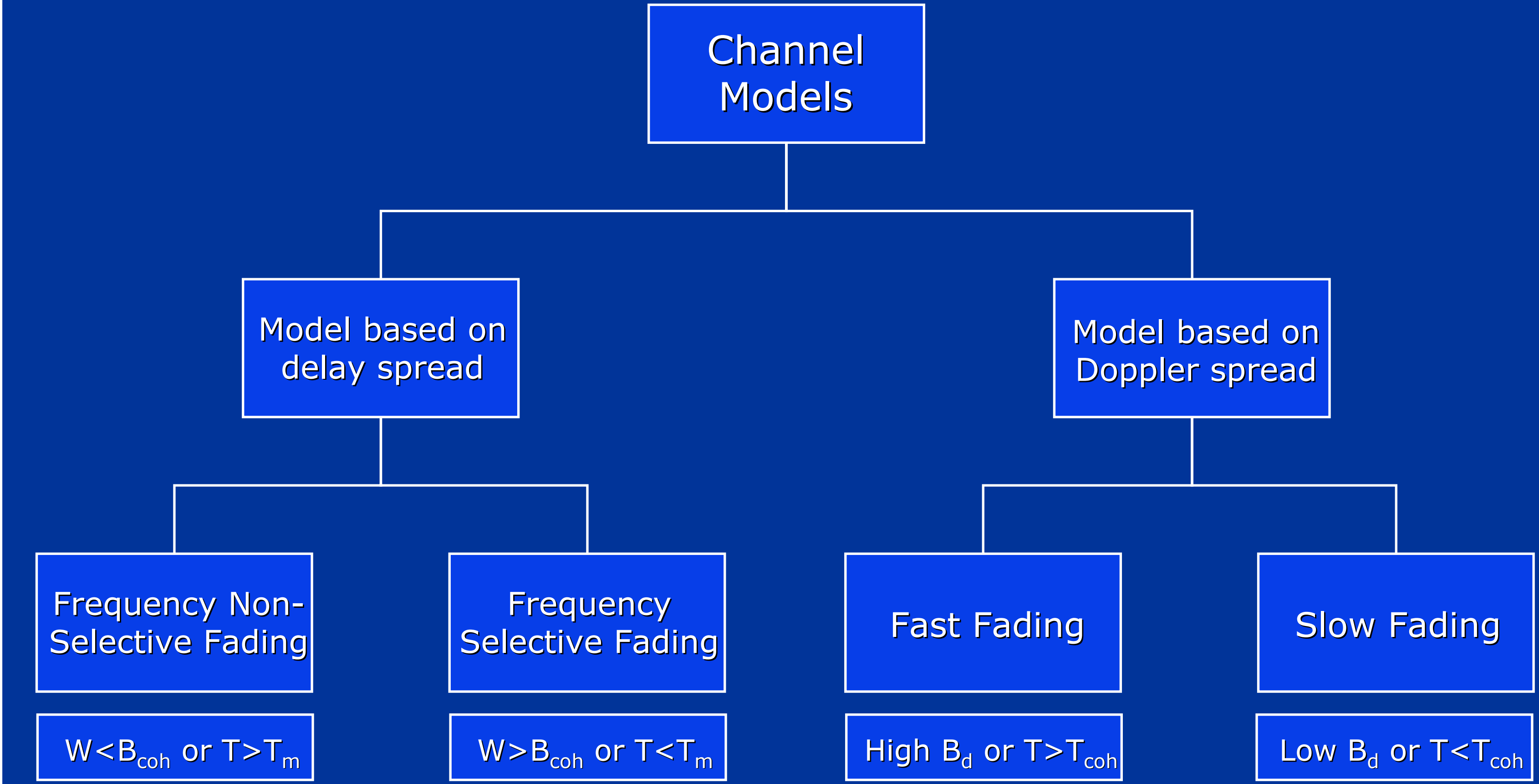
- Assuming that there are a large number of scatterers/reflectors in the medium, application of the central limit theorem leads to a complex Gaussian distribution model for the channel response.

- If there is no dominant scatterer in the medium (e.g. only NLOS components), the envelope of the channel response is described by the Rayleigh distribution.

- If there is a dominant scatterer in the medium (e.g. an LOS component), the envelope of the channel response is described by the Ricean distribution.

Channel Models: Classification and Characteristics

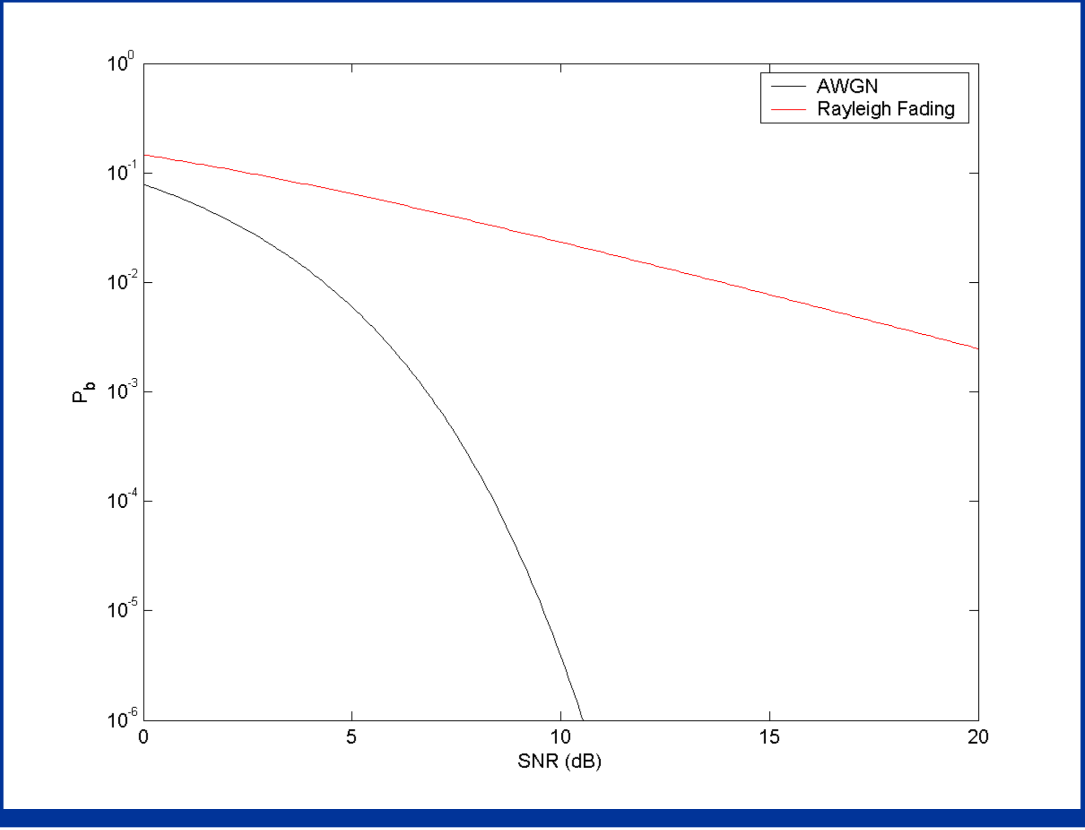
Channel Models Classification and Characteristics



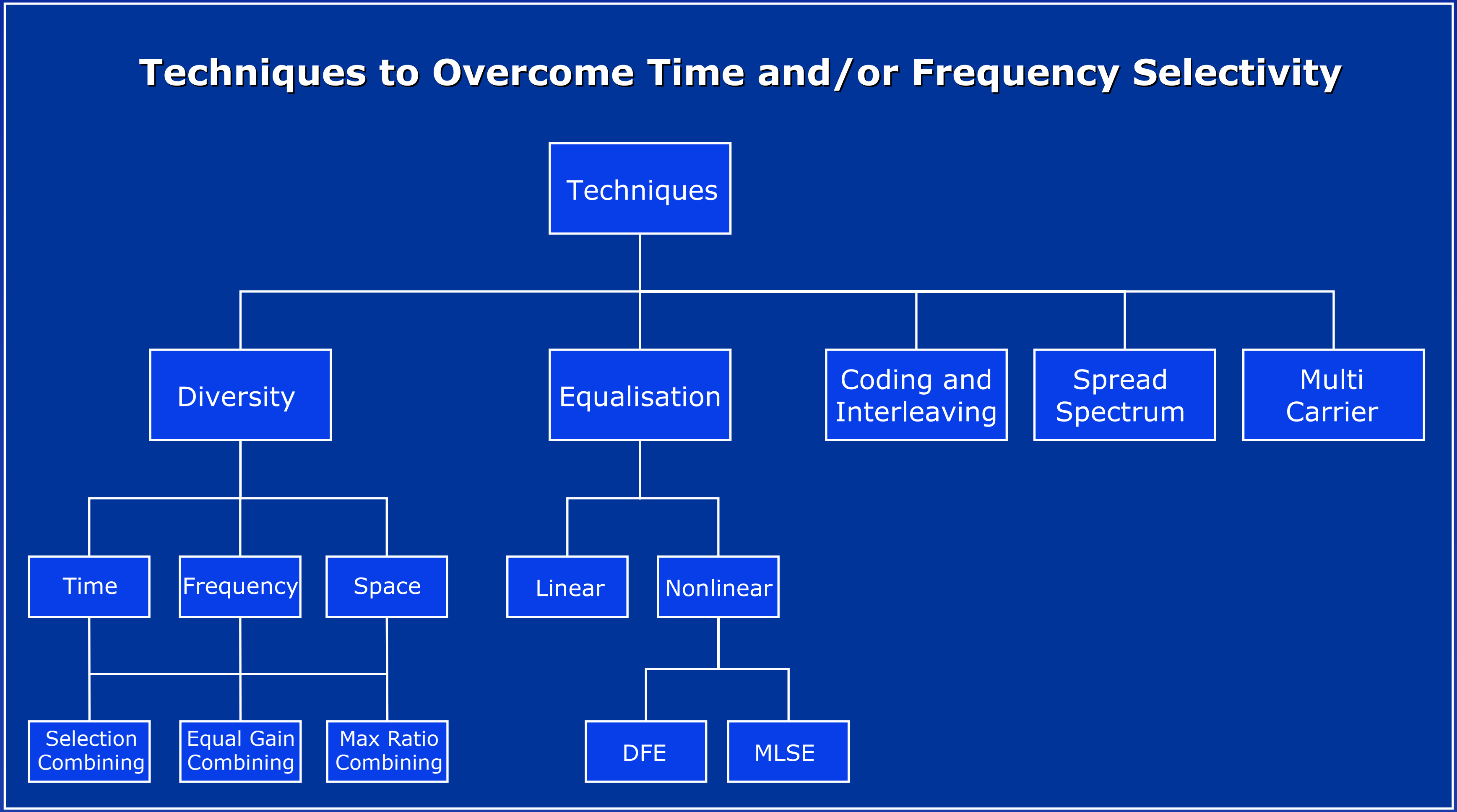
- Channel Models
  - Model based on delay spread
    - Frequency Non-Selective Fading
      - $W<B_{coh}$ or $T>T_m$
    - Frequency Selective Fading
      - $W>B_{coh}$ or $T<T_m$
  - Model based on Doppler spread
    - Fast Fading
      - High B_d or $T>T_{coh}$
    - Slow Fading
      - Low B_d or $T<T_{coh}$

The Frequency Non-Selective, Slow Rayleigh Fading Channel

P_b *vs*. SNR for a BPSK signal in an AWGN channel and in a Rayleigh fading channel



Techniques to Overcome Time and Frequency Selectivity



Techniques to Overcome Time and/or Frequency Selectivity

Techniques

Diversity

Equalisation

Coding and Interleaving

Spread Spectrum

Multi Carrier

Time

Frequency

Space

Linear

Nonlinear

Selection Combining

Equal Gain Combining

Max Ratio Combining

DFE

MLSE

Diversity, Coding and Interleaving

- Diversity techniques are often used to mitigate the the effects of the wireless channel.
- The principle of diversity is to transmit the same information on two or more sub-channels that fade independently from one another.
- Independently fading sub-channels are realised either in the time, frequency or space domains.
- Selection combining, equal gain combining or maximal ratio combining techniques recombine the information in the various sub-channels at the receiver.
- Coding and interleaving are also often used to mitigate the effects of the wireless channel. Coding and interleaving are an efficient form of (time) diversity.

Equalisation

- Equalisation is generally used to mitigate intersymbol interference in wireless channels.
- Linear equalisers are less complex than non-linear equalisers, but generally produce noise enhancement in severely distorted channels.
- Non-linear equalisers perform better than non-linear equalisers, but are generally more complex.
- Maximum likelihood sequence estimation (MLSE) equalisers are optimum, but highly complex.
- Decision feedback equalisers (DFE) are sub-optimum, but less complex.

Spread-Spectrum Techniques

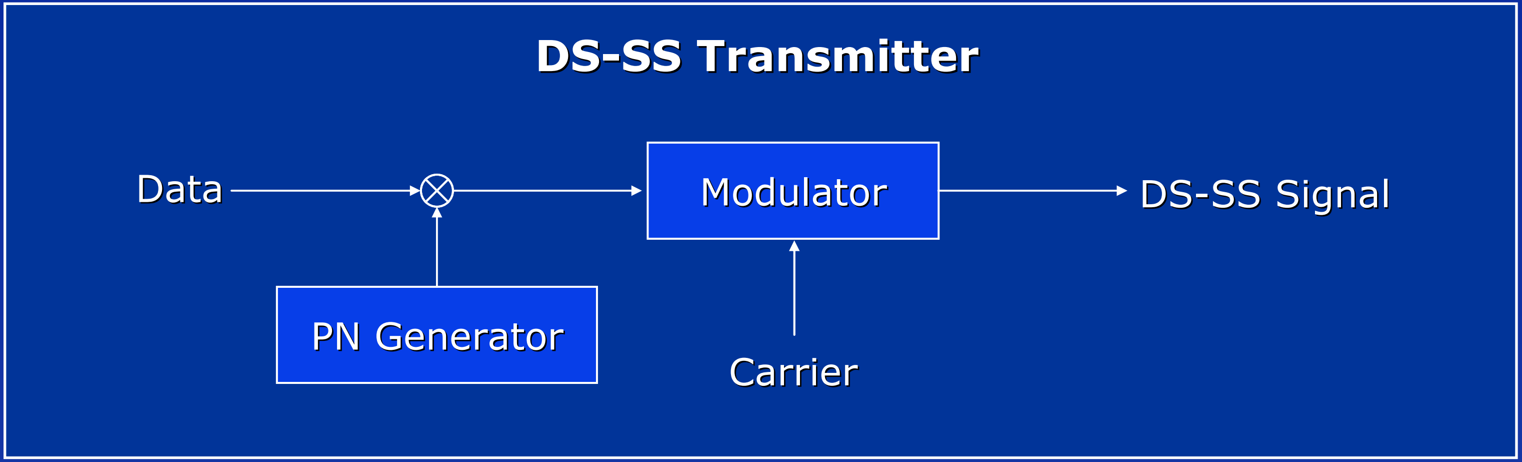
- In spread spectrum modulation the transmitted spectrum is spread over a range much greater than the message bandwidth.

- In direct sequence spread spectrum (DS-SS) the transmitted spectrum is spread by multiplying the signal by a wide-band pseudo-noise (PN) sequence.

- In frequency hopped spread spectrum (FH-SS) the transmitted spectrum is spread by modulating the signal onto a wide-band series of frequencies generated by a frequency synthesiser driven by a pseudo-noise (PN) sequence.

- The ratio of the transmitted signal spectrum to the message spectrum is the known as the bandwidth expansion factor or the processing gain.

Direct Sequence Spread Spectrum



DS-SS Transmitter

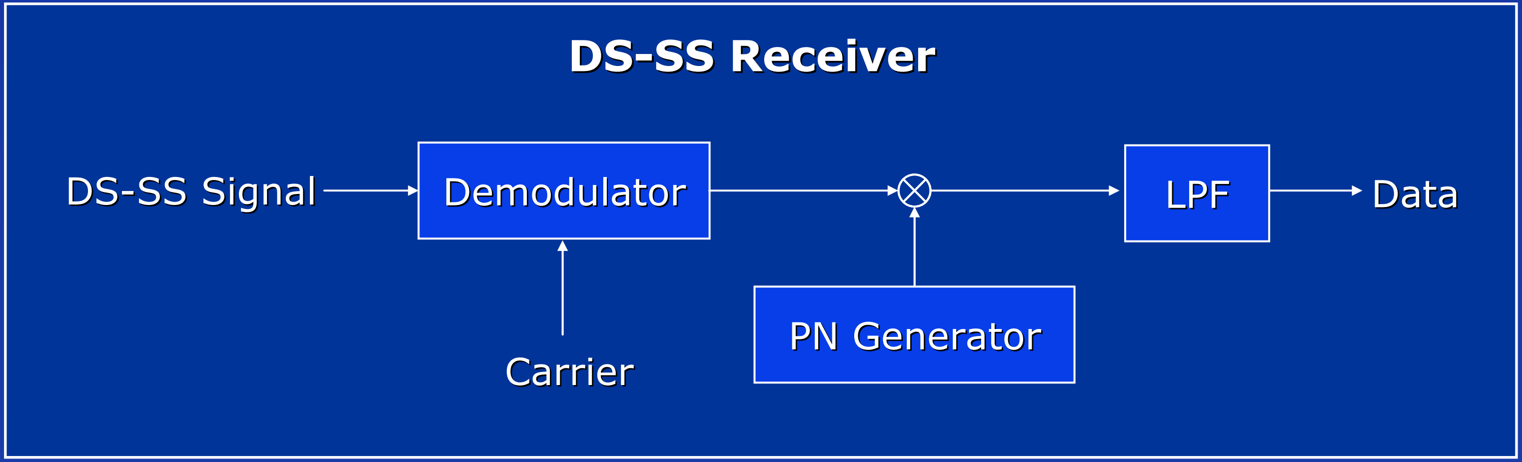
Data

PN Generator

Modulator

Carrier

DS-SS Signal



DS-SS Receiver

DS-SS Signal

Demodulator

Carrier

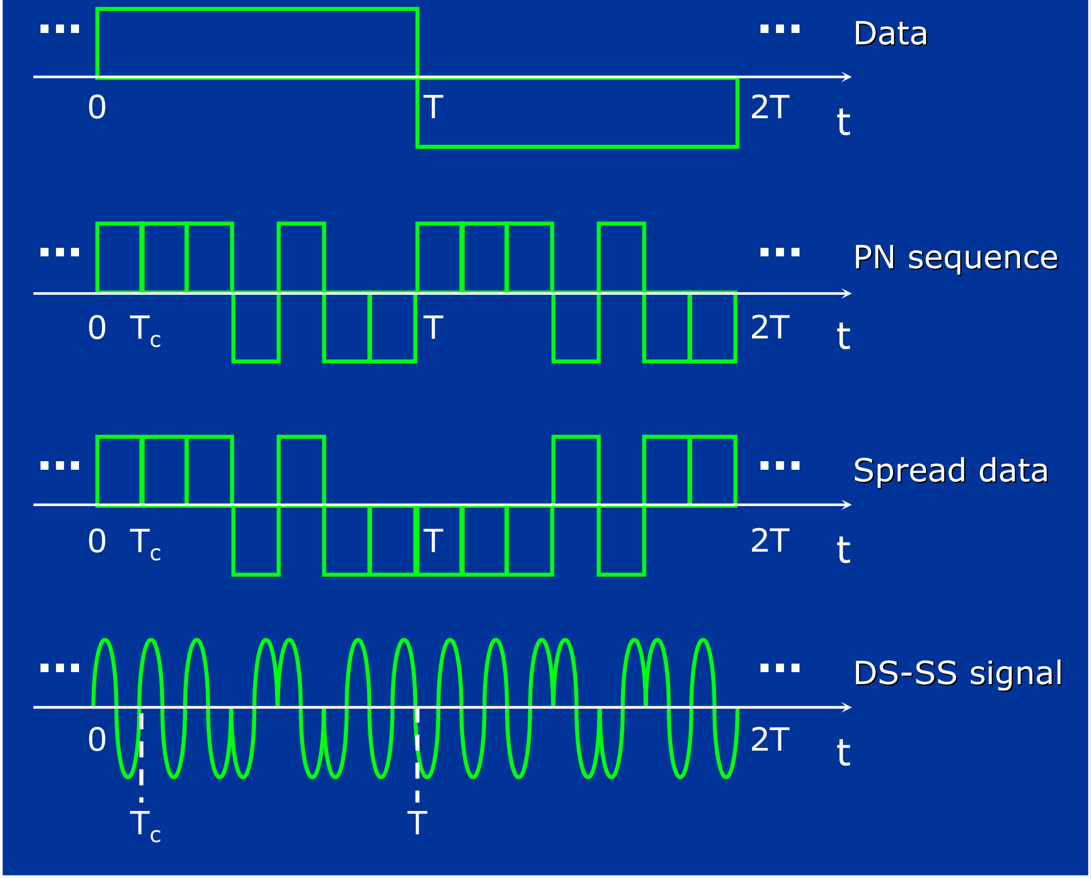
PN Generator

LPF

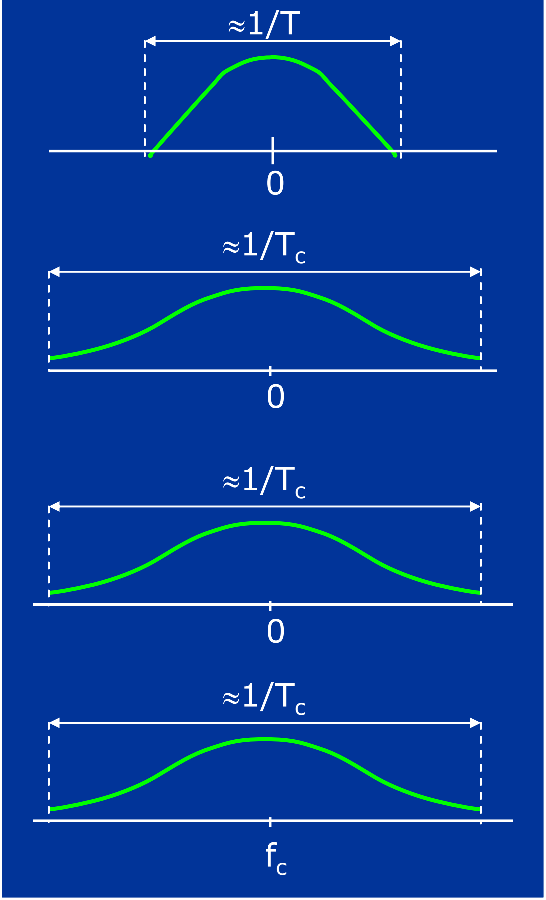
Data

DS-SS Transmitter Operation

Waveforms

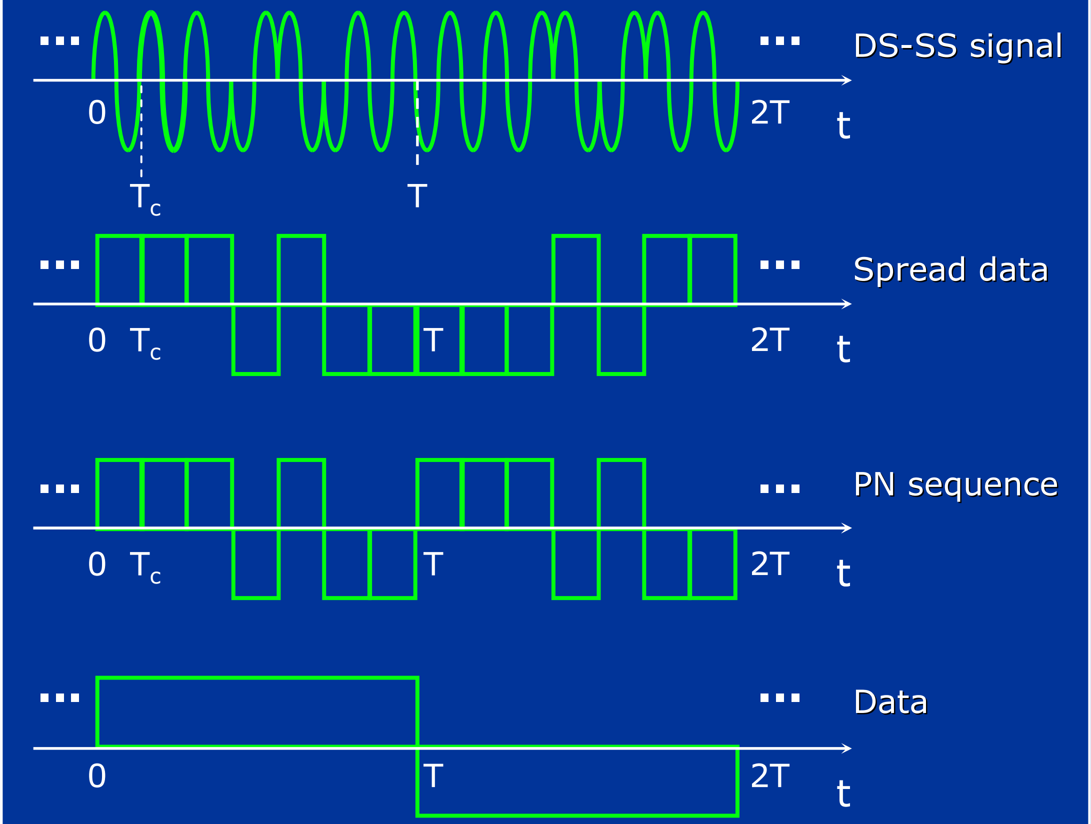


Spectra

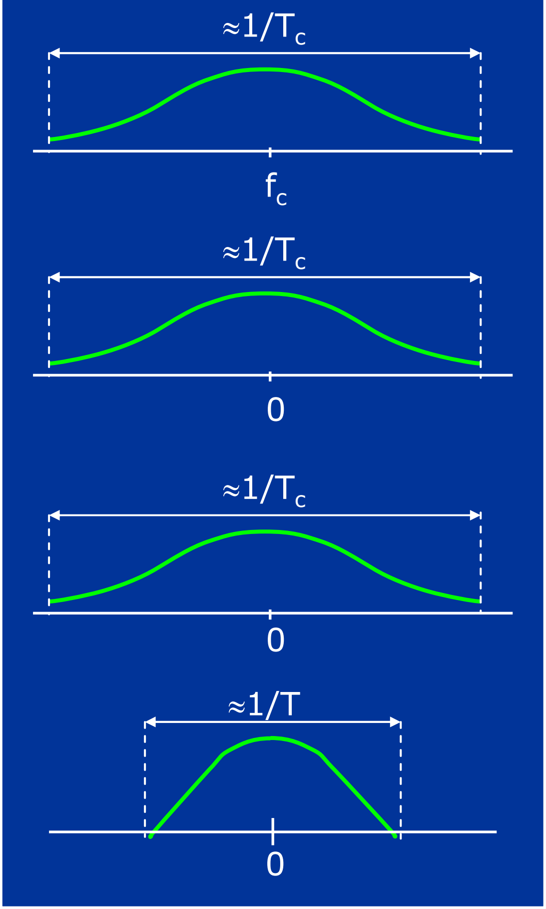


DS-SS Receiver Operation

Waveforms

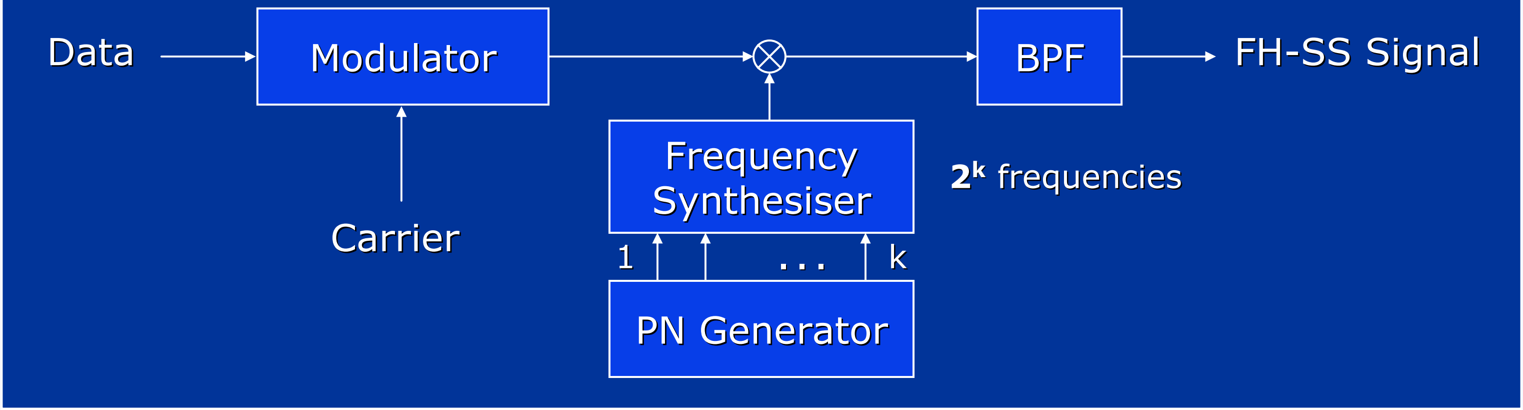


Spectra



Frequency Hopped Spread Spectrum

FH-SS Transmitter



Data

Modulator

Carrier

Frequency Synthesiser

1 . . . k

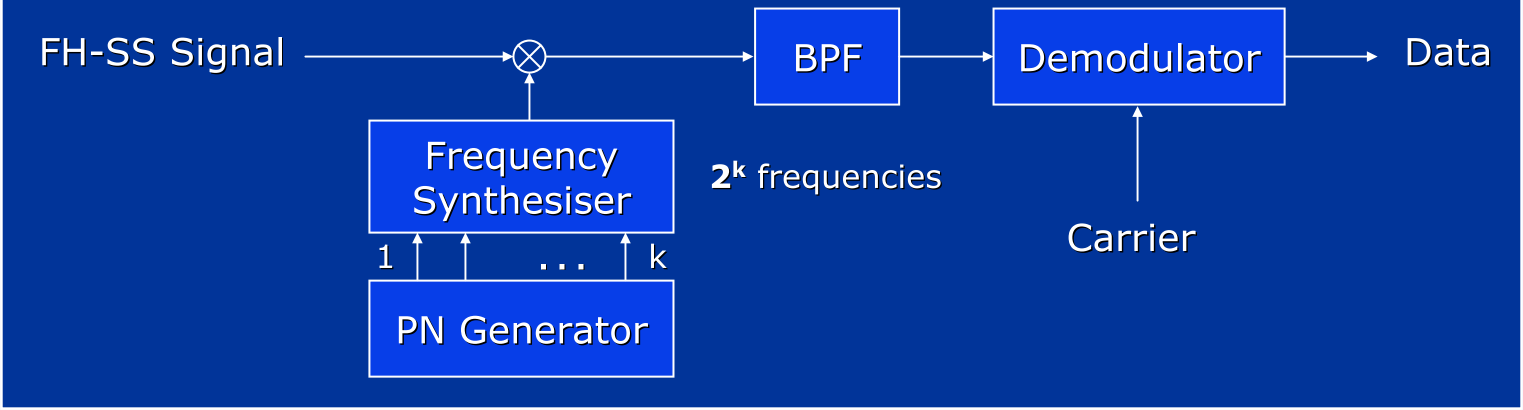
PN Generator

BPF

FH-SS Signal

2^k frequencies

FH-SS Receiver



FH-SS Signal

Frequency Synthesiser

1 . . . k

PN Generator

2^k frequencies

BPF

Demodulator

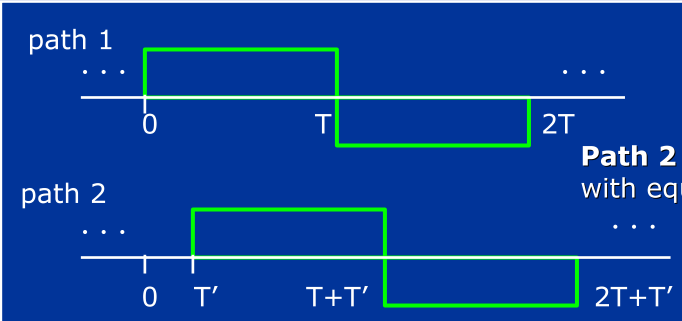
Carrier

Data

Spread Spectrum Techniques on Multipath Channels

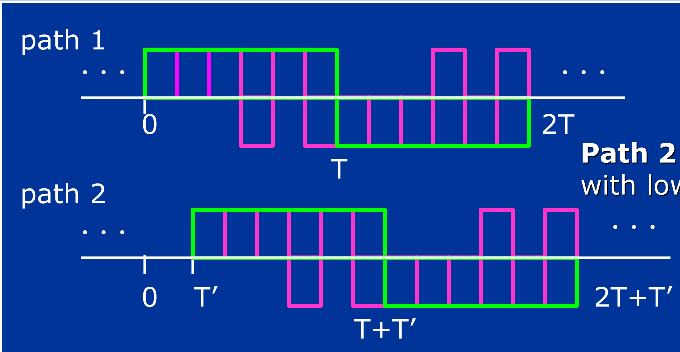
Two-path channel
relative delay = T′

- Time domain interpretation



Path 2 contributes
with equal power

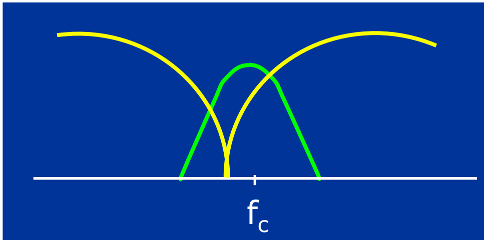
Ordinary modulation case



Path 2 contributes
with lower power

Spread Spectrum modulation case

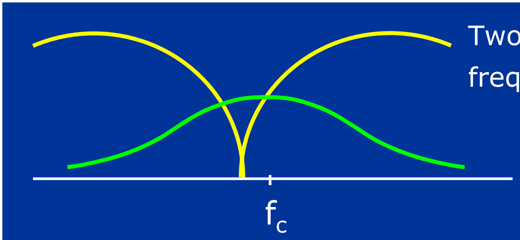
- Frequency domain interpretation



Two-path channel
frequency response

Significant distortion

Ordinary modulation case



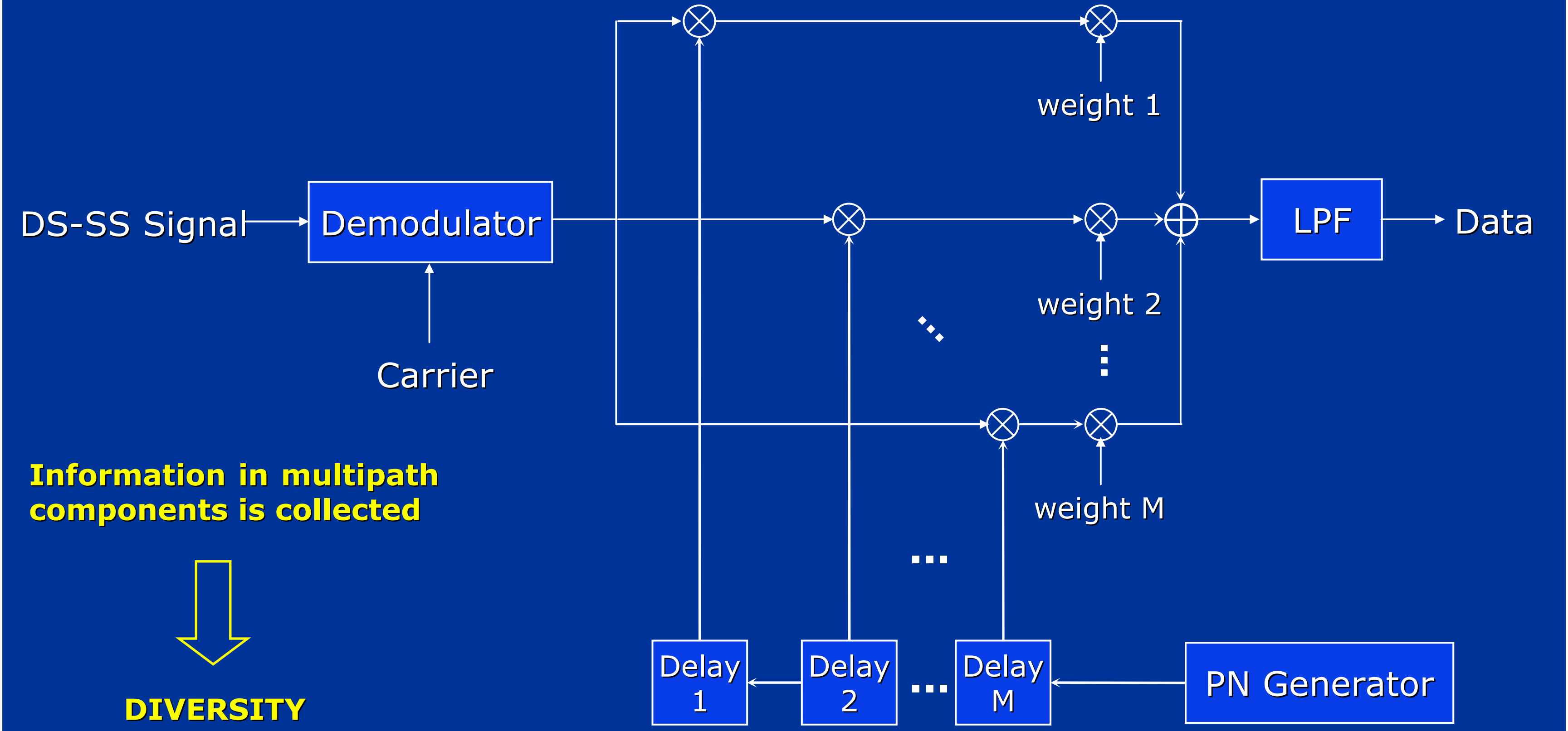
Two-path channel
frequency response

Negligible distortion

Spread Spectrum modulation case

Spread Spectrum Techniques on Multipath Channels

Spread spectrum modulation with Rake receiver



Information in multipath components is collected

⇩

DIVERSITY

Multi-Carrier Techniques

- In a single carrier modulation scheme each data symbol is transmitted sequentially on a single carrier ⇒ signalling interval equal to data symbol duration.

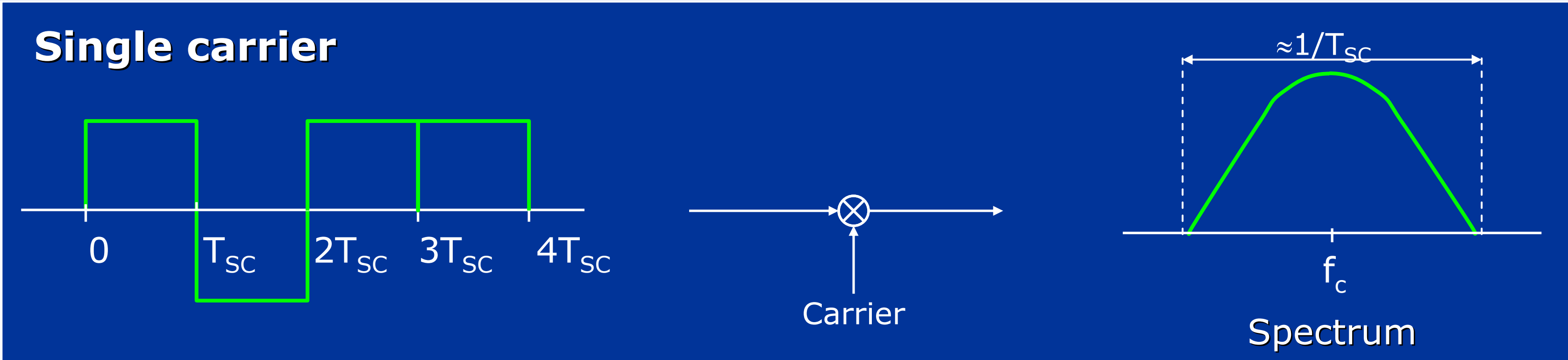
- In a single carrier modulation scheme the modulated carrier occupies the entire available bandwidth.

- In a multi-carrier modulation scheme N sequential data symbols are transmitted simultaneously on N multiple carriers ⇒ signalling interval equal to N times data symbol duration.

- In a multi-carrier modulation scheme each modulated carrier occupies only a small part of the entire available bandwidth.

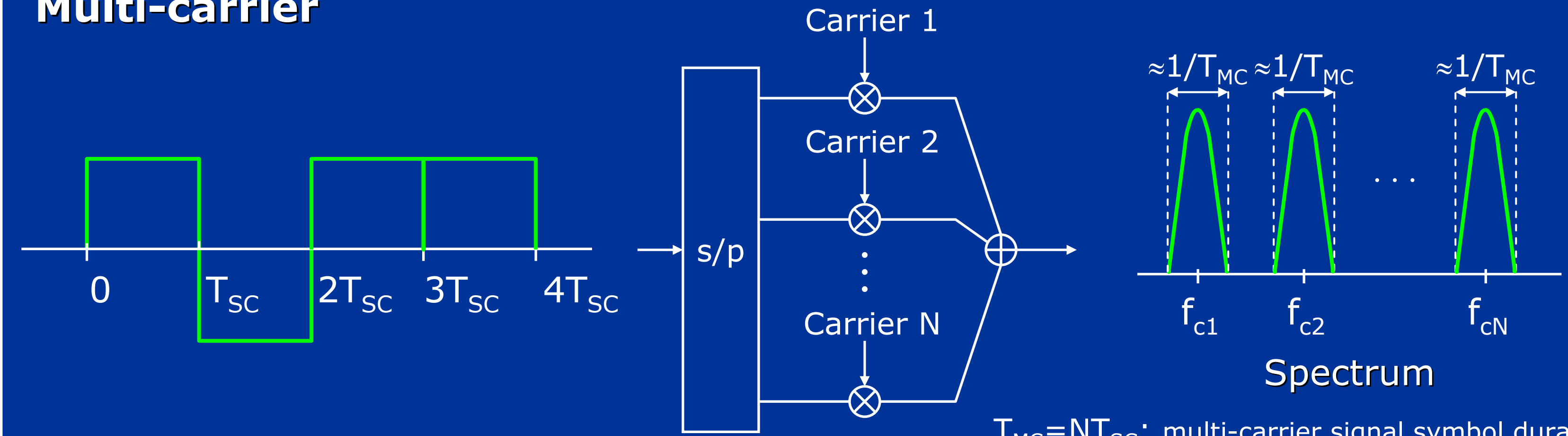
Multi-Carrier Techniques

Single carrier



T_{SC}: single carrier signal symbol duration

Multi-carrier

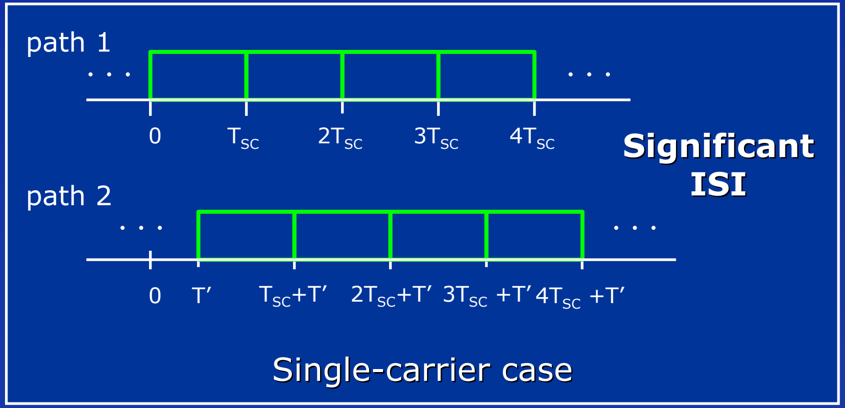


$T_{MC}=NT_{SC}$: multi-carrier signal symbol duration

Multi-Carrier Techniques on Multipath Channels

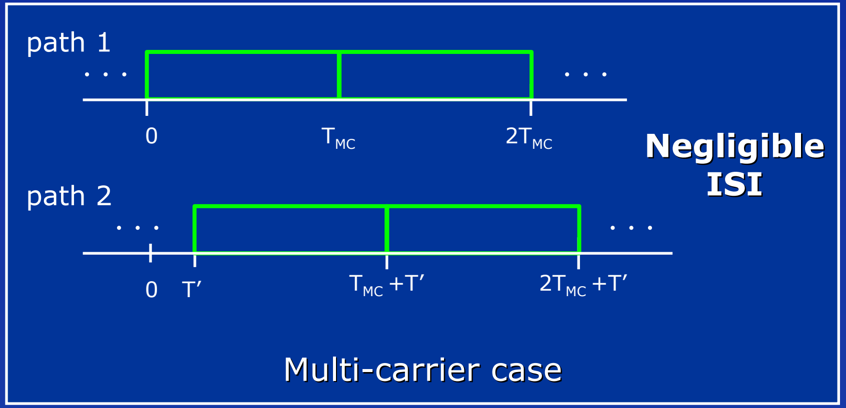
Two-path channel
relative delay = T′

- Time domain interpretation



Significant ISI

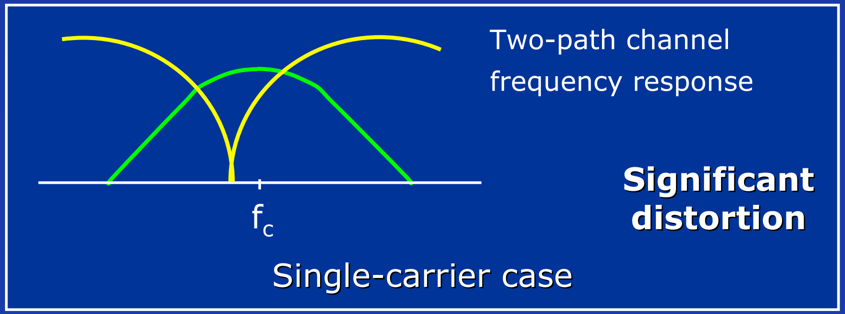
Single-carrier case



Negligible ISI

Multi-carrier case

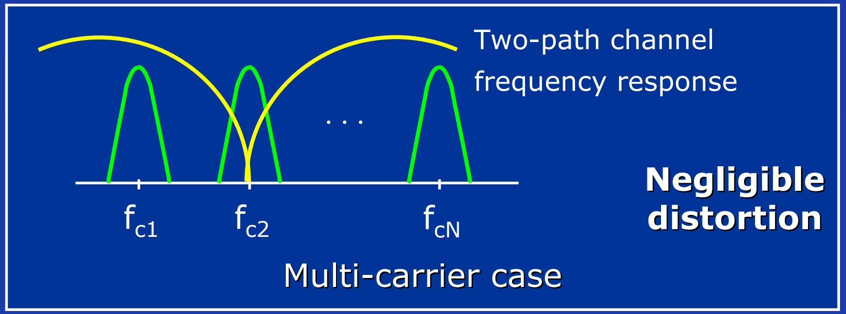
- Frequency domain interpretation



Two-path channel frequency response

Significant distortion

Single-carrier case



Two-path channel frequency response

Negligible distortion

Multi-carrier case

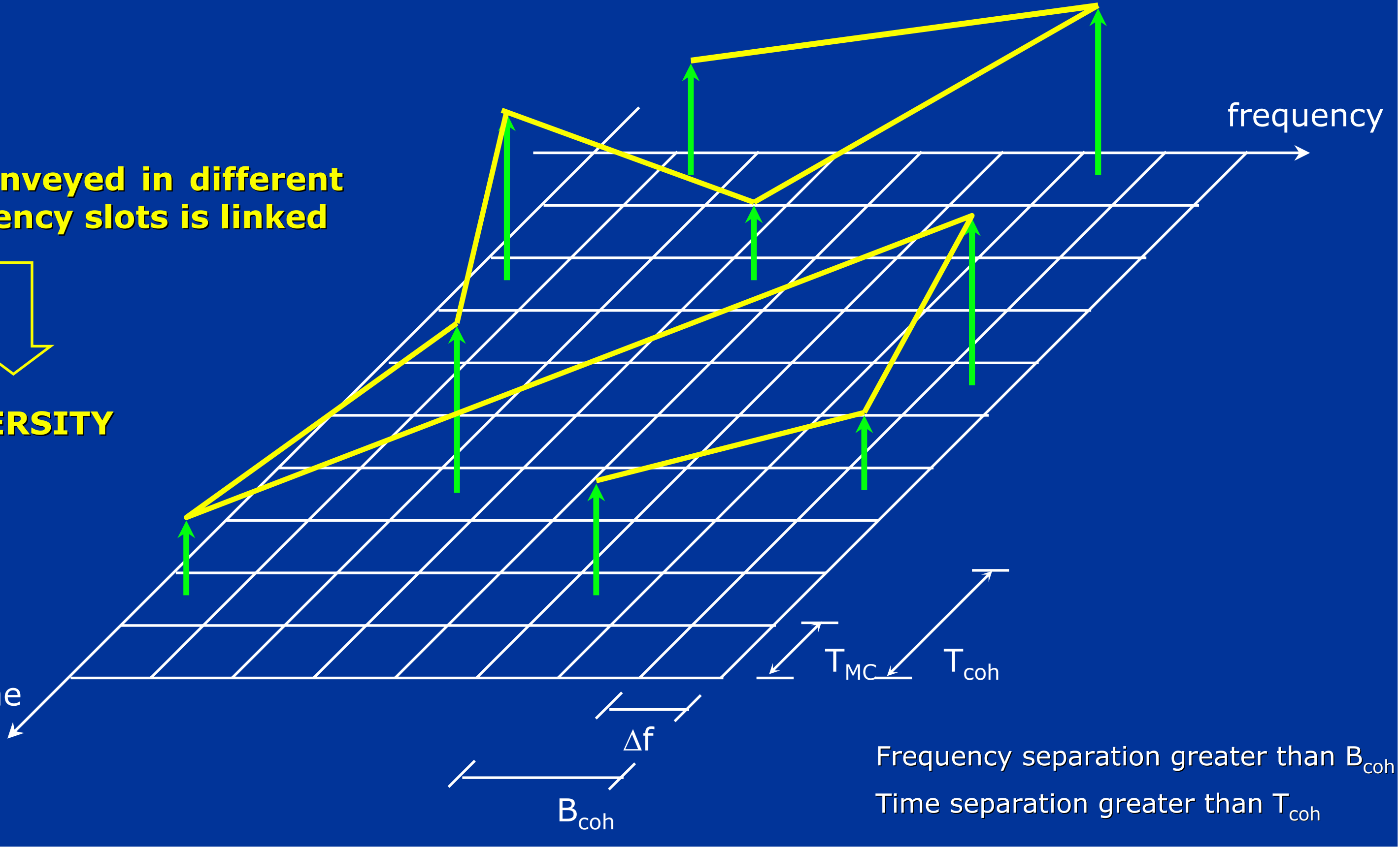
Multi-Carrier Techniques on Multipath Channels

Multi-carrier modulation with coding and interleaving

Information conveyed in different time and frequency slots is linked

⇩

DIVERSITY



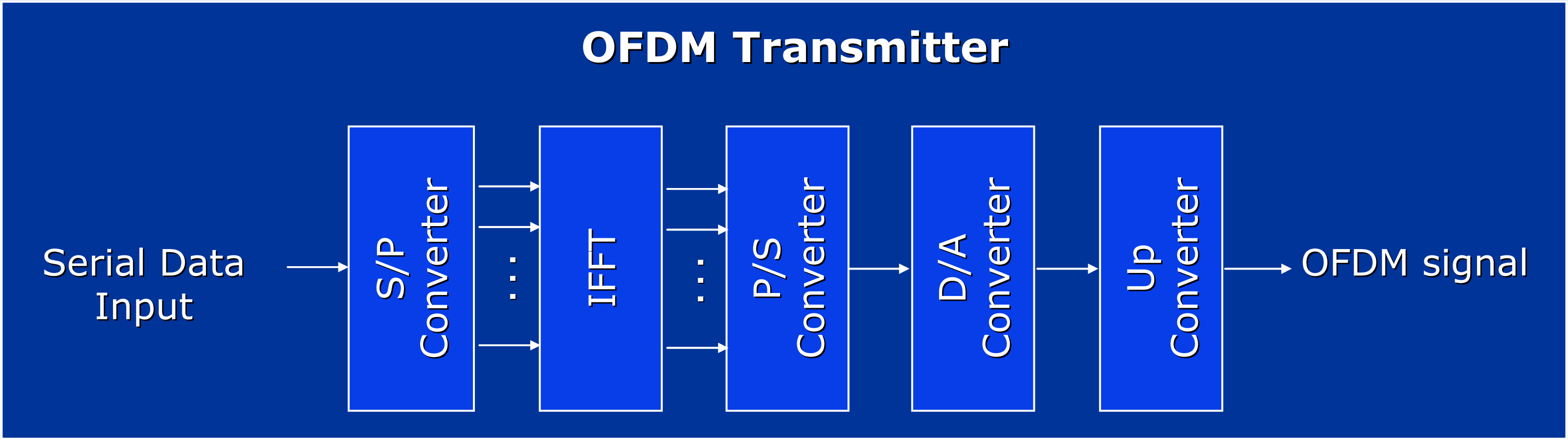
Frequency separation greater than B_{coh}

Time separation greater than T_{coh}

Orthogonal Frequency Division Multiplexing

- OFDM is a multi-carrier modulation scheme.
- In OFDM the frequency spacing between adjacent sub-carriers is $\Delta f = 1/T_{MC} = 1/(NT_{SC})$.
- $\Delta f = 1/T_{MC}$ is the minimum frequency separation that is necessary to ensure orthogonality between the sub-carriers over the signalling interval of length T_{MC}.
- In OFDM the frequency spectrum of each sub-carrier overlaps the frequency spectrum of adjacent sub-carriers.

Orthogonal Frequency Division Multiplexing



OFDM Transmitter

Serial Data Input

S/P Converter

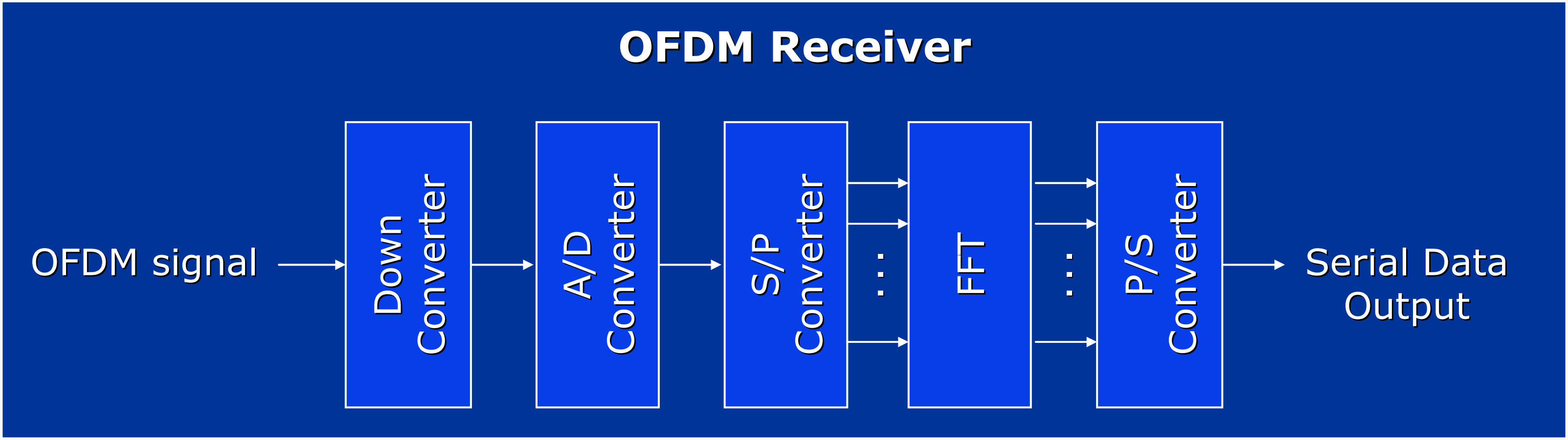
IFFT

P/S Converter

D/A Converter

Up Converter

OFDM signal



OFDM Receiver

OFDM signal

Down Converter

A/D Converter

S/P Converter

FFT

P/S Converter

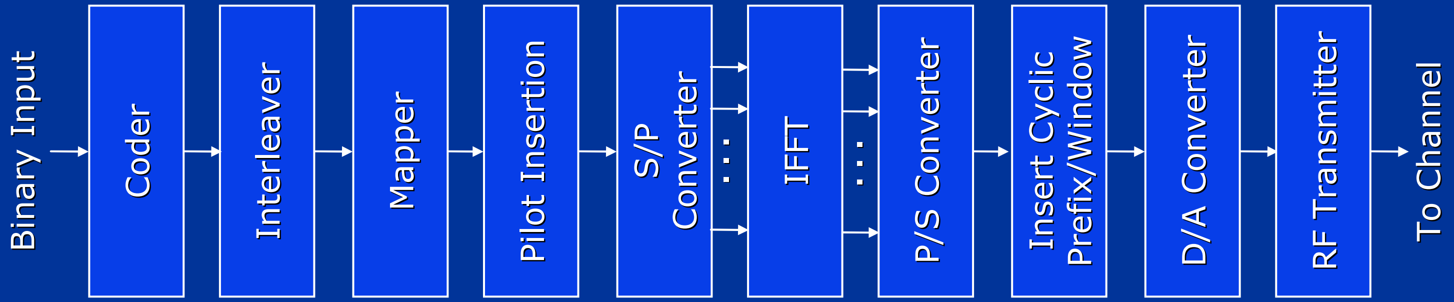
Serial Data Output

Orthogonal Frequency Division Multiplexing

- Advantages of OFDM:
  - Good performance under delay spread/frequency selective fading conditions;
  - Bandwidth efficiency;
  - Efficient digital signal processor based generation/ detection techniques.
- Disadvantages of OFDM:
  - Poor performance under Doppler spread/time selective fading conditions;
  - Sensitive to non-linear distortion;
  - Sensitive to timing and frequency offsets as well as phase noise.

Elements of a Typical OFDM Communications System

OFDM Transmitter



Binary Input

Coder

Interleaver

Mapper

Pilot Insertion

S/P Converter

IFFT

P/S Converter

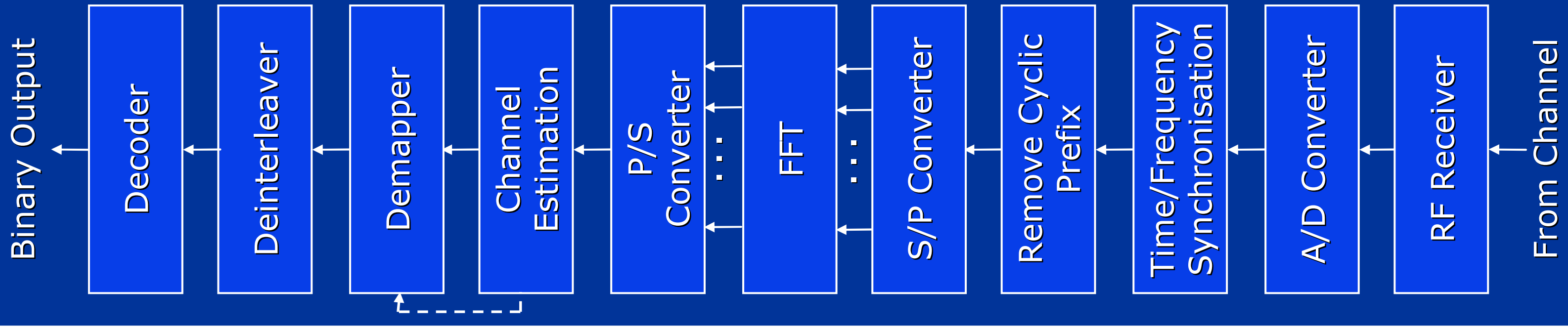
Insert Cyclic Prefix/Window

D/A Converter

RF Transmitter

To Channel

OFDM Receiver



Binary Output

Decoder

Deinterleaver

Demapper

Channel Estimation

P/S Converter

FFT

S/P Converter

Remove Cyclic Prefix

Time/Frequency Synchronisation

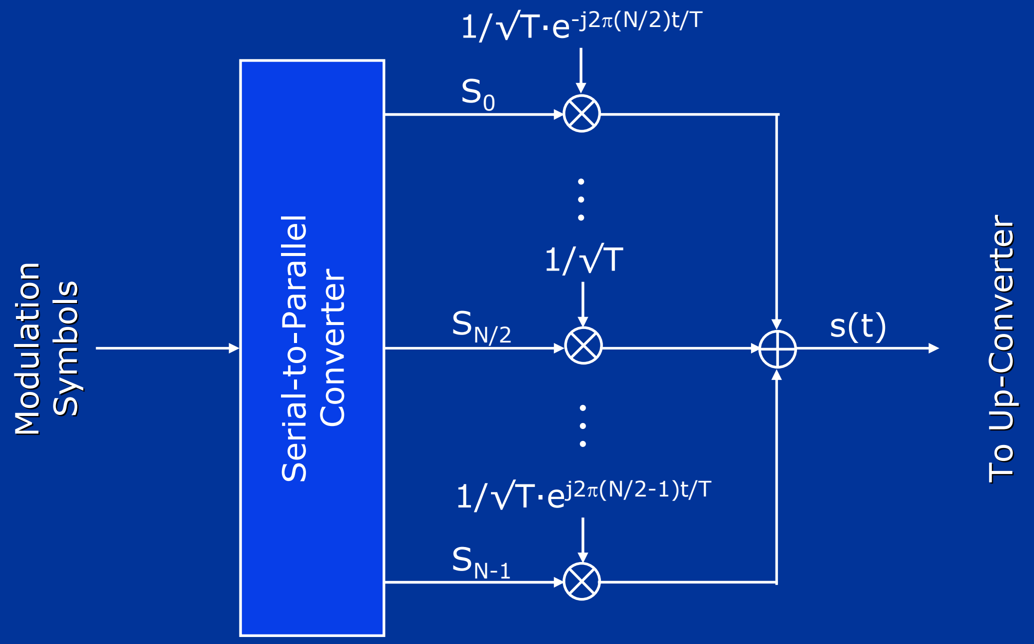
A/D Converter

RF Receiver

From Channel

Oscillator Based OFDM Generation

Oscillator Based OFDM Generation



$1/\sqrt{T}\cdot e^{-j2\pi(N/2)t/T}$

S_0

$1/\sqrt{T}$

$S_{N/2}$

$1/\sqrt{T}\cdot e^{j2\pi(N/2-1)t/T}$

S_{N-1}

Modulation Symbols

Serial-to-Parallel Converter

$s(t)$

To Up-Converter

Equation:

$$s(t)=\sum_{n=0}^{N-1} S_n\, 1/\sqrt{T}\, e^{j\frac{2\pi(n-N/2)t}{T}}$$

N is the number of sub-carriers

T is duration of signalling interval

Oscillator Based OFDM Generation

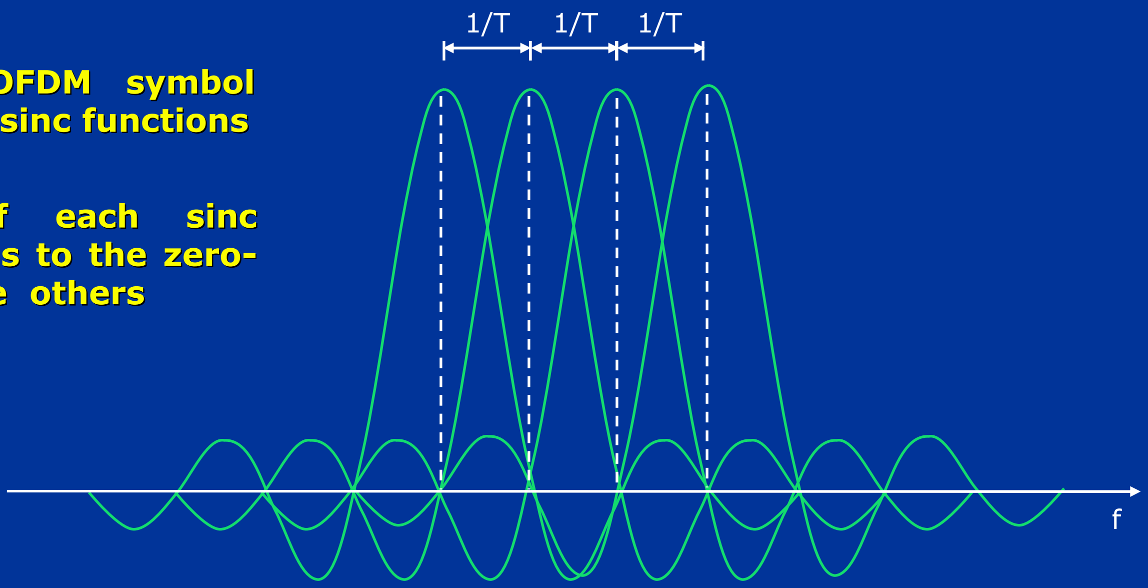
- In OFDM the frequency separation between adjacent sub-carriers is 1/T.
- This is the minimum frequency separation between adjacent sub-carriers necessary to achieve orthogonality and hence detectability.
- In OFDM the frequency spectrum of each sub-carrier overlaps the frequency spectrum of adjacent sub-carriers.

Oscillator Based OFDM Generation

Spectrum of an OFDM Symbol

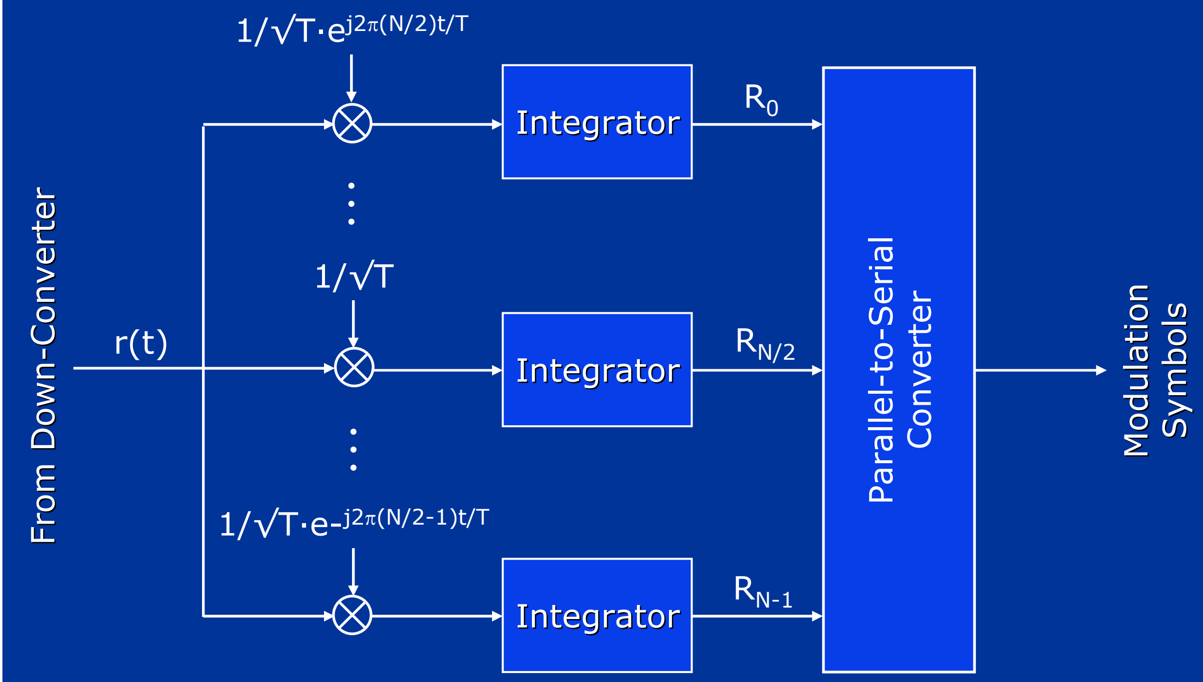
Spectrum of an OFDM symbol consists of several sinc functions

The maximum of each sinc function corresponds to the zero-crossings of all the others



Oscillator Based OFDM Detection

Oscillator Based OFDM Detection



From Down-Converter

$r(t)$

$1/\sqrt{T} \cdot e^{j2\pi(N/2)t/T}$

$1/\sqrt{T}$

$1/\sqrt{T} \cdot e^{-j2\pi(N/2-1)t/T}$

Integrator

Integrator

Integrator

R_0

$R_{N/2}$

R_{N-1}

Parallel-to-Serial Converter

Modulation Symbols

Equation:

$$R_n = \int_0^T r(t) 1/\sqrt{T} e^{-j\frac{2\pi(n-N/2)t}{T}} dt$$

Oscillator Based OFDM Detection

- The operation of an OFDM receiver can be viewed from two different perspectives.
- From one view point, the receiver correlates the OFDM symbol with a local version of each complex sub-carrier.
- Orthogonality implies that the correlation between any two different complex sub-carriers is zero but when the complex sub-carriers have the same frequency.
- Therefore, the correlation operation yields the information conveyed by each sub-carrier without ICI.

Oscillator Based OFDM Detection

- From another view point, the receiver evaluates the Fourier transform of an OFDM symbol at specific frequencies.

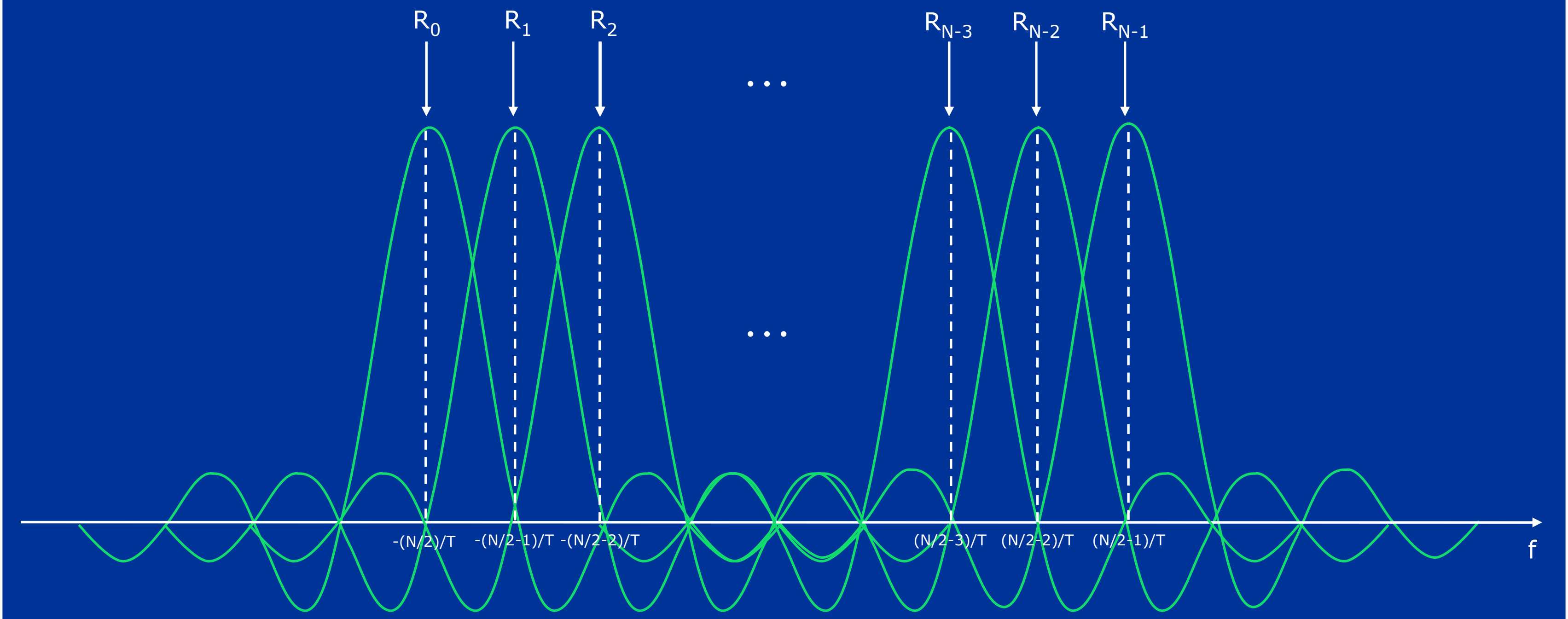
- The spectrum of an OFDM symbol consists of a series of sinc() functions where the maximum of each function corresponds to the zero-crossings of all the other sinc() functions.

- Or, the spectrum of an OFDM symbol fulfils Nyquist's criterion for an ISI free pulse shape, where in the OFDM case the pulse shape is present in the frequency domain instead of the time domain.

- Therefore, the Fourier transformation operation yields the information conveyed by each sub-carrier without ICI.

Oscillator Based OFDM Detection

Information Recovery



FFT Based OFDM Generation/Detection

- OFDM generation/detection is achieved using the IDFT and the DFT, or, the IFFT and the FFT, respectively.

- The scaled samples $s_k=\sqrt{(T/N)}\cdot s(kT/N), k=0,\ldots,N-1$, of the transmit OFDM symbol are generated by taking the IDFT of the “re-ordered” modulation symbols,

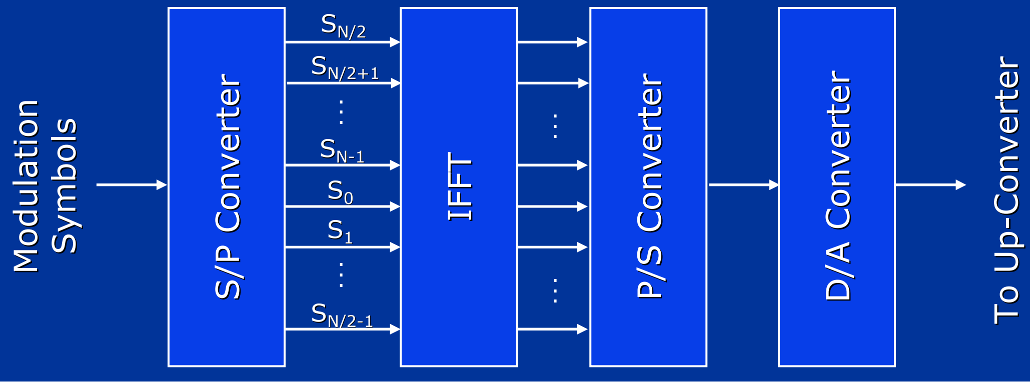
$$s_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} S_{(n+N/2)_N+} e^{j\frac{2\pi kn}{N}}, k = 0,1,\ldots,N-1$$

- The “re-ordered” modulation symbols are detected by taking the DFT of the scaled samples $r_k=\sqrt{(T/N)}\cdot r(kT/N), k=0,\ldots,N-1$, of the receive OFDM symbol,

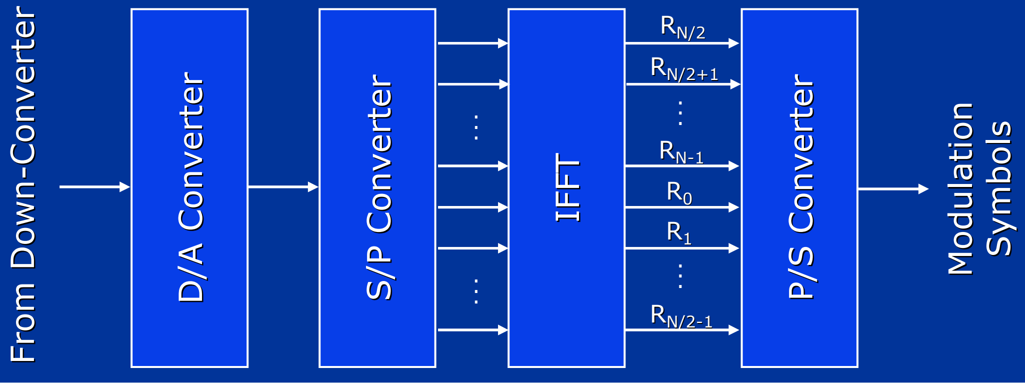
$$R_{(n+N/2)_N} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r_k e^{-j\frac{2\pi kn}{N}}, n = 0,1,\ldots,N-1$$

FFT Based OFDM Transceiver

FFT Based OFDM Generation



FFT Based OFDM Detection

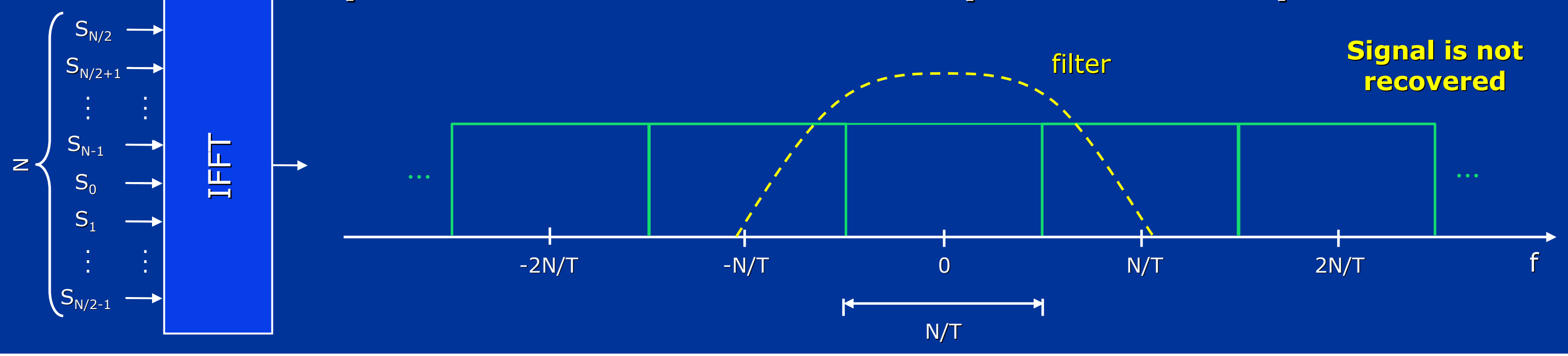


Zero Padding

- With an N point IDFT, it is only possible to generate an N sub-channel OFDM symbol sampled at the Nyquist rate.
- In this case, it is difficult to recover the continuous time signal from the sampled signal with filters with realisable passband-to-stopband transition regions.
- With an N’>N point IDFT, it is possible to generate an N sub-channel OFDM symbol sampled at a rate higher than the Nyquist rate.
- In this case, it is easier to recover the continuous time signal from the oversampled signal using filters with realisable passband-to-stopband transition regions.
- The zero padding technique achieves oversampling.

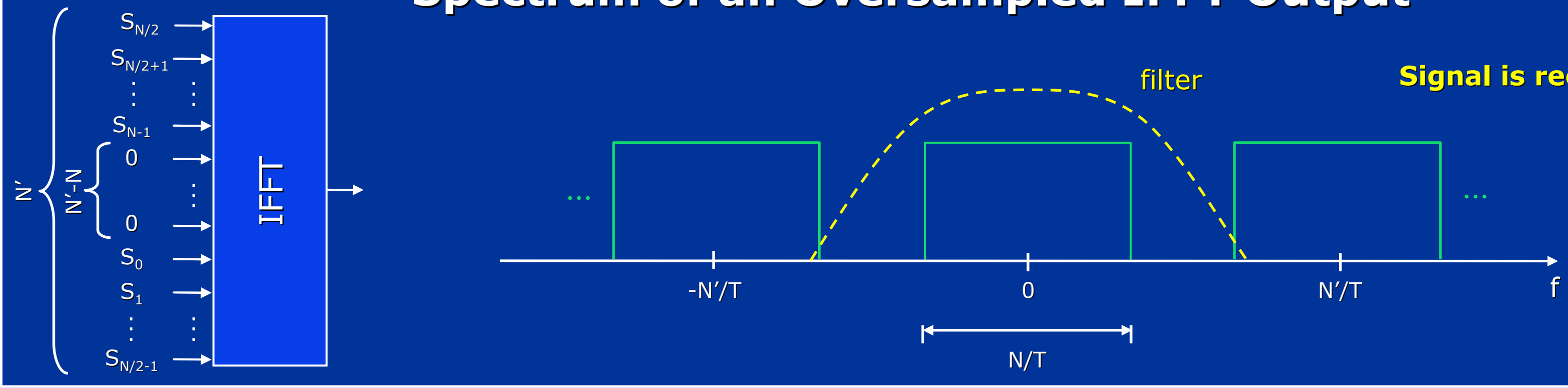
Zero Padding

Spectrum of a Non-Oversampled IFFT Output



Signal is not recovered

Spectrum of an Oversampled IFFT Output

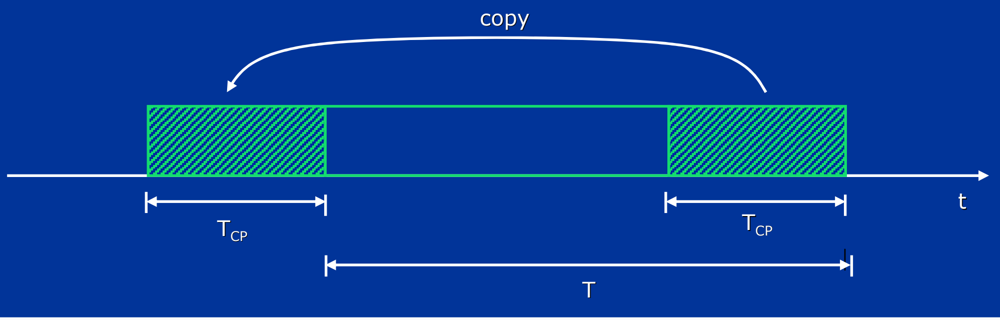


Signal is recovered

Cyclic Prefix

- To eliminate ISI in OFDM a guard time is inserted with a duration longer than the multipath channel maximum delay.
- Moreover, to eliminate ICI in OFDM the guard time is cyclically extended.
- Note that in a multipath channel an appropriate guard time avoids ISI but not ICI, unless it is cyclically extended.

Cyclic Extension of an OFDM Symbol

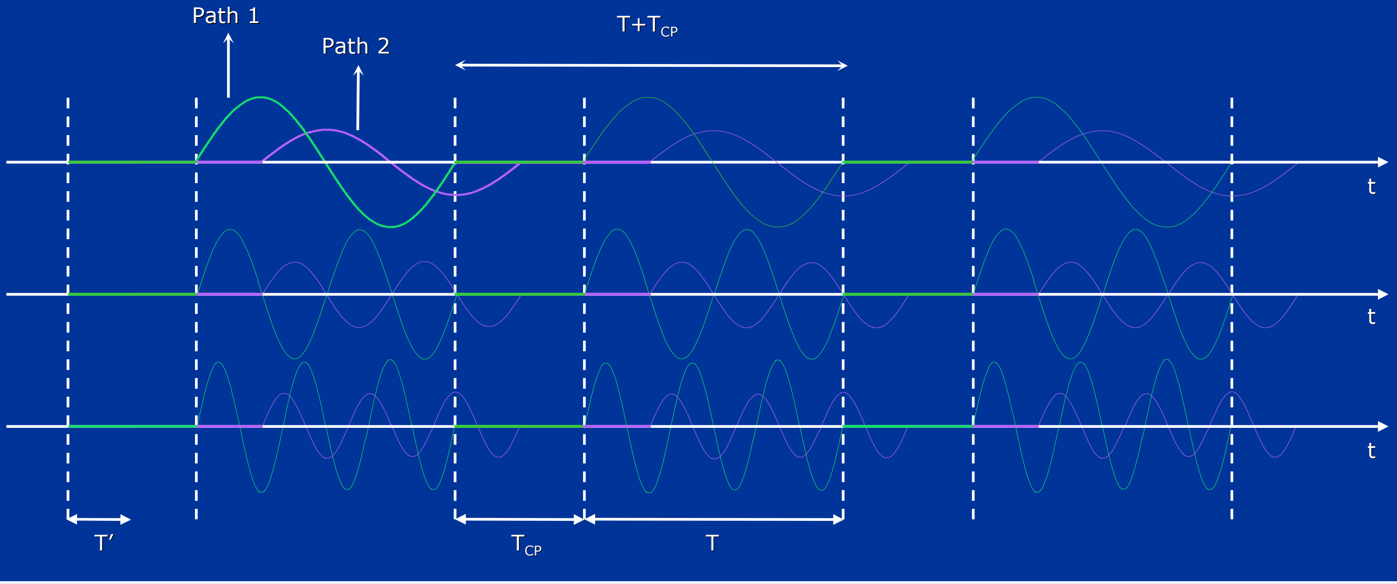


Cyclic Prefix

OFDM Signal with "Empty" Guard Time

Two-path channel
relative delay = T′

ISI is eliminated but ICI is not

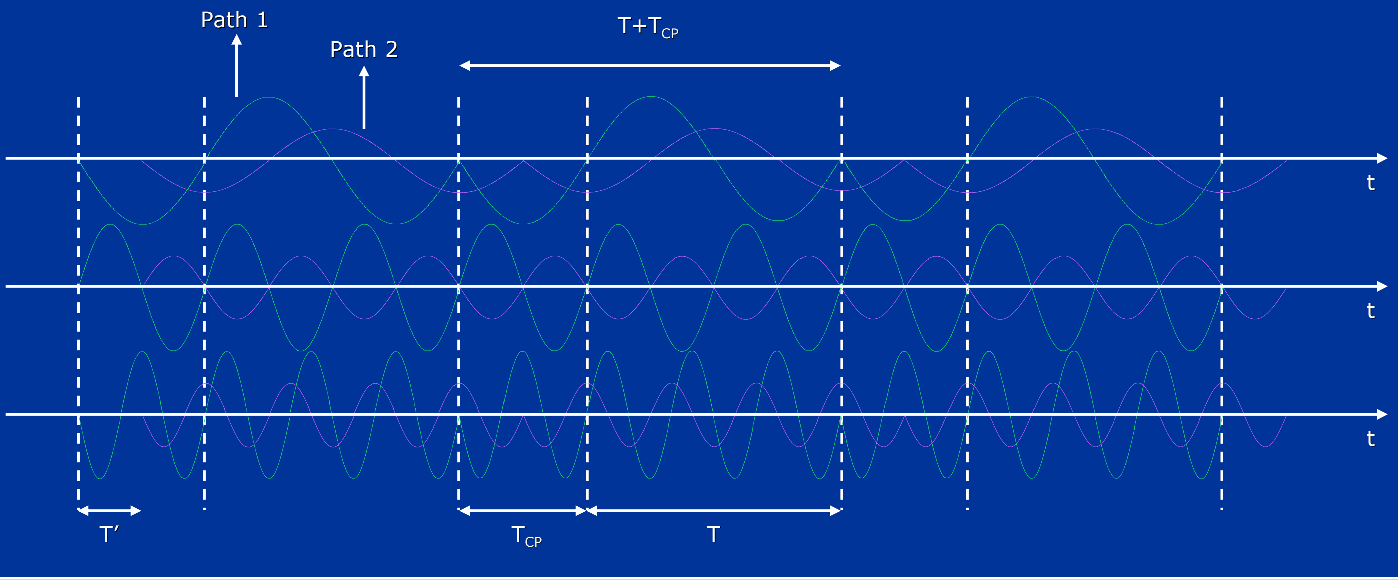


Cyclic Prefix

OFDM Signal with Cyclic Extended Guard Time

Two-path channel
relative delay = T′

Both ISI and ICI are eliminated



Cyclic Prefix

- With a cyclic prefix, the received symbol in frame k and sub-channel n, $R_{k,n}$, is related to the transmitted symbol in the same frame and sub-channel, $S_{k,n}$, by

  $$R_{k,n}=H_{k,n}S_{k,n}+N_{k,n}$$

  where $H_{k,n}$ is the channel frequency response in frame k and sub-channel n and $N_{k,n}$ is the noise.

- Insertion of an appropriate cyclic prefix eliminates ISI and ICI in a multipath channel but it also introduces a loss in the SNR and data rate.

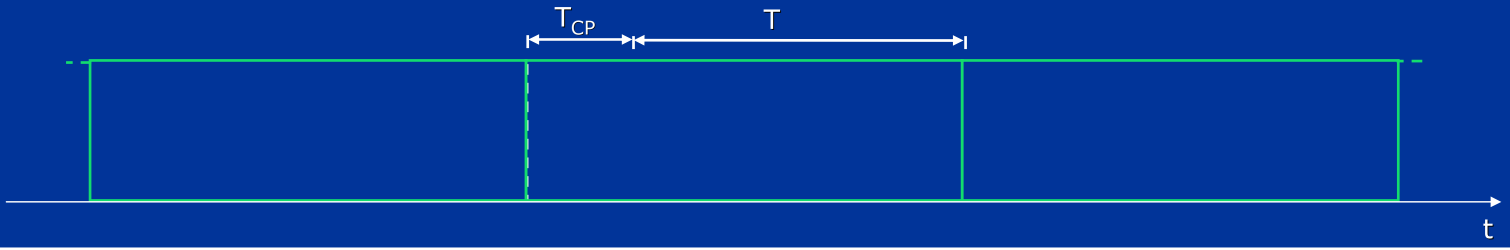
- The SNR loss is equal to $10\times\log_{10}(1+T_{CP}/T)$ (dB)

- The data rate loss is equal to $(1+T_{CP}/T)^{-1}$ $(\times 100\%)$

Windowing

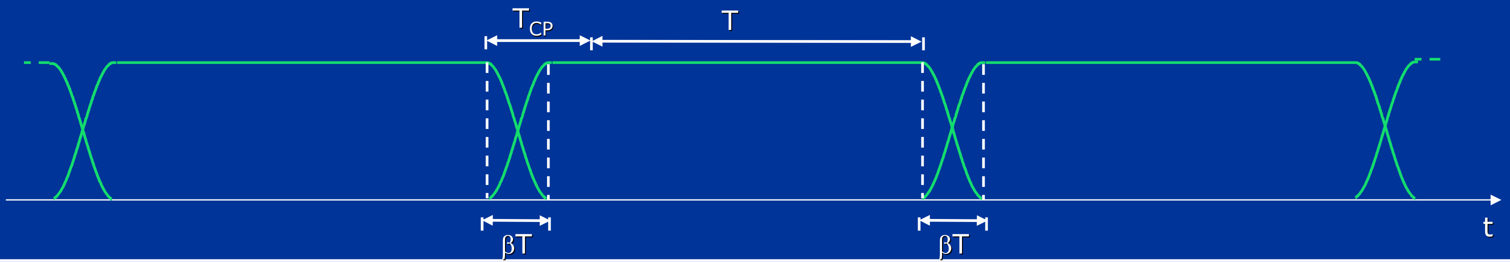
OFDM Signal with Rectangular Window

Sharp phase transitions ⇒ **High out-of-band radiation … Delay spread tolerance=T_{CP}**



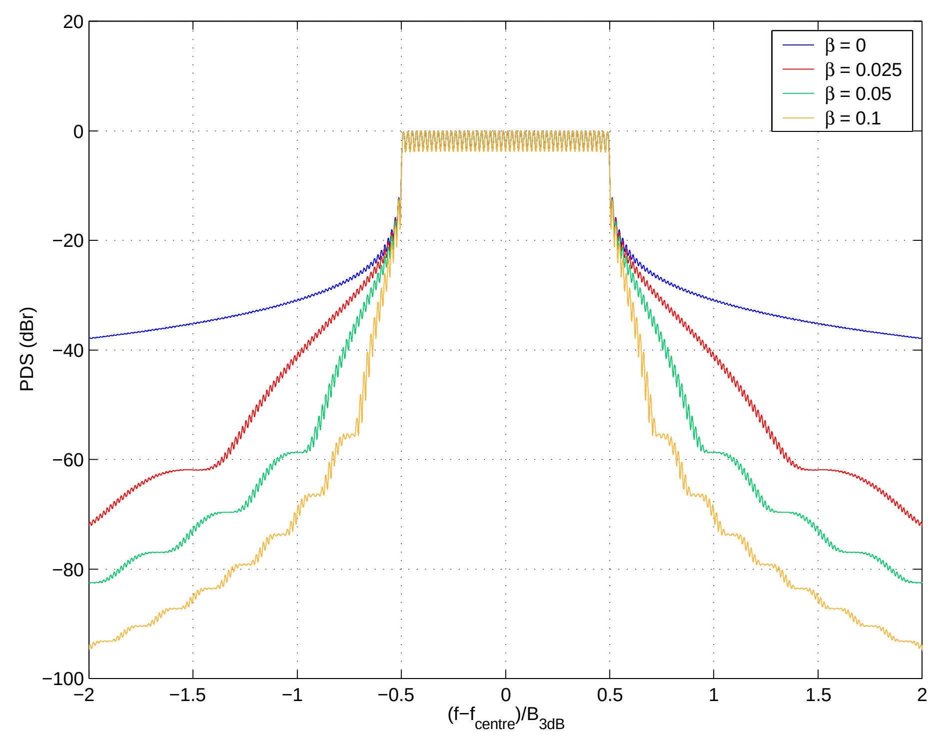
OFDM Signal with Raised Cosine Window

Smooth phase transitions ⇒ **Low out-of-band radiation … Delay spread tolerance=T_{CP}-βT**



Windowing

Power Density Spectrum of "Windowed" OFDM Signals



Coding and Interleaving Techniques

- In an OFDM system, the received symbol in frame k and sub-channel n, $R_{k,n}$, is related to the transmitted symbol in the same frame and sub-channel, $S_{k,n}$, by

  $$R_{k,n}=H_{k,n}S_{k,n}+N_{k,n}$$

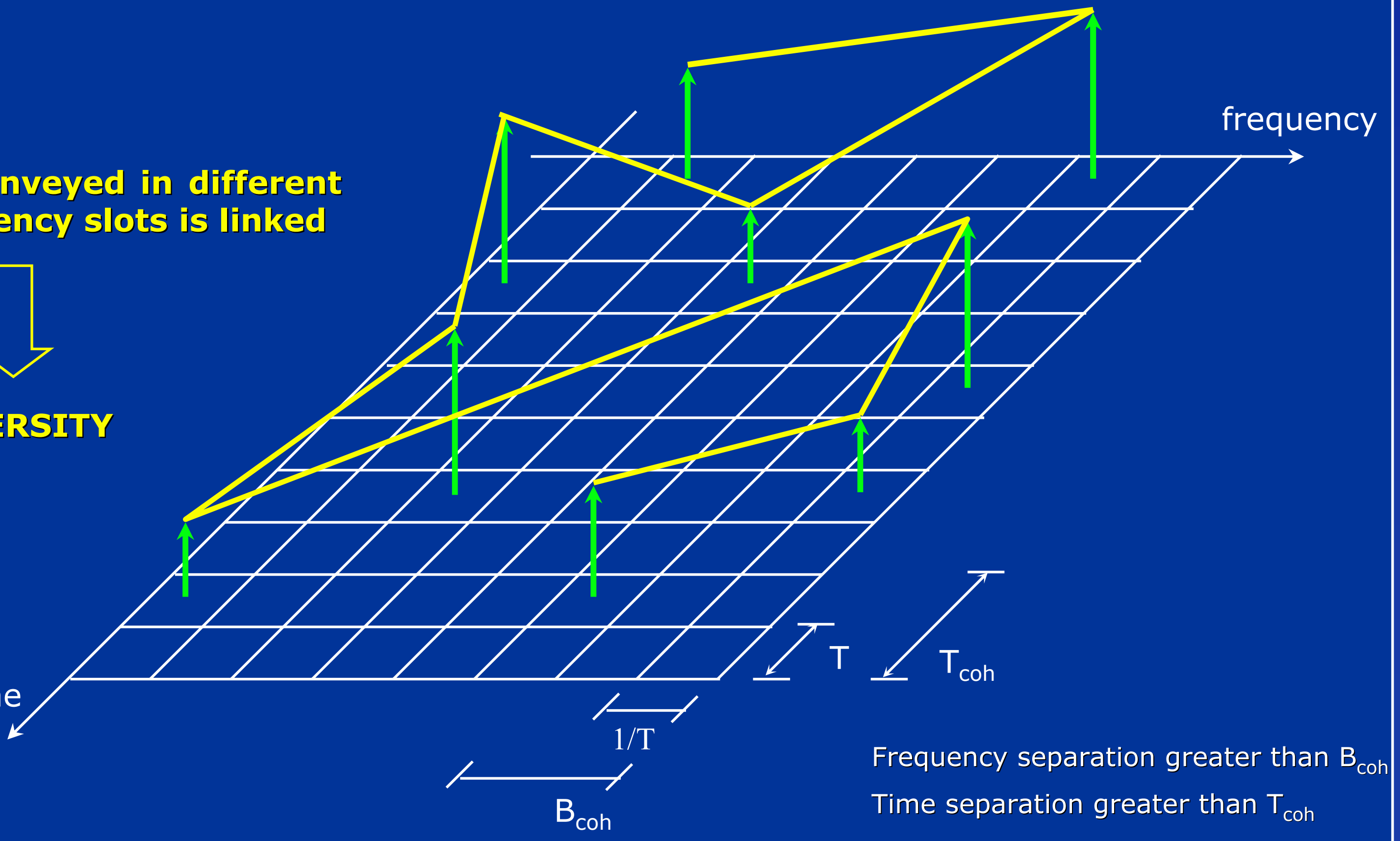
  where $H_{k,n}$ are the channel transfer factors and $N_{k,n}$ is the noise.

- Occasional deep fades in the channel cause groups of adjacent frames/sub-channels to be less reliable than other groups and hence errors to occur in burst rather than independently.

- Coding and interleaving provides a link between independently fading frames/sub-channels, so that strongly received ones correct for weakly received ones.

Coding and Interleaving Techniques

OFDM with coding and interleaving



Information conveyed in different time and frequency slots is linked

⇩

DIVERSITY

Frequency separation greater than B_{coh}

Time separation greater than T_{coh}

Coding Methods

- A number of coding methods have been proposed for OFDM systems including block, convolutional, concatenated and turbo coding.

- Trellis-coded modulation methods have also been proposed for OFDM systems, where the coding and modulation operations are merged together.

- Hard and soft decision decoding techniques can be used.

- Hard decision decoding performs worse than soft decision decoding. However, the former is less complex than the later.

Interleaving Methods

- A number of interleaving methods have been proposed for OFDM systems including block and convolutional interleaving.
- In block interleaving, the bits/symbols are written into a matrix column by column.
- These are subsequently read out from the matrix row by row to produce the interleaved bits/symbols.
- In convolutional interleaving, the bits/symbols are cyclically written into one of K shift registers that introduce a delay of 0 to K-1.
- These are subsequently read out cyclically to produce the interleaved bits/symbols.

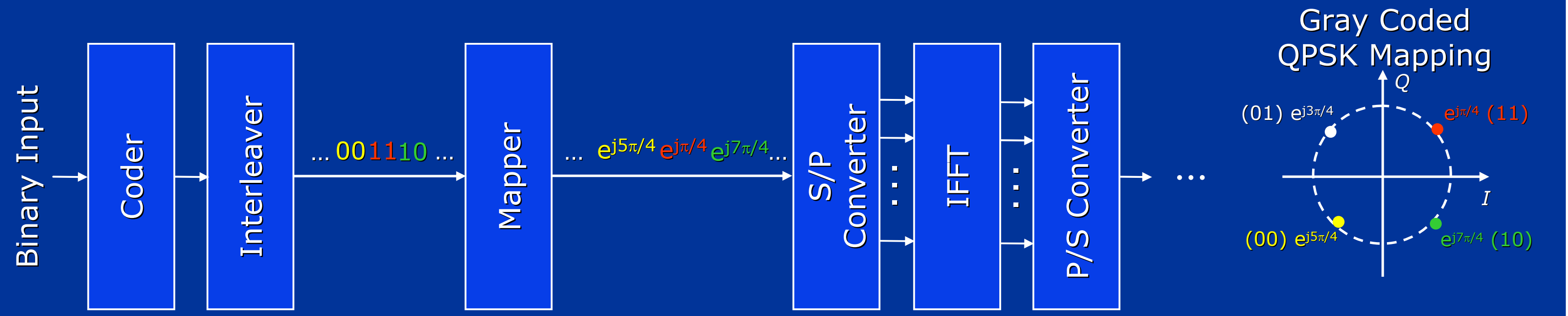
Mapping/Demapping Techniques

- At the transmitter, the bits at the output of the encoder and interleaver are mapped to modulation symbols, which will constitute the input to the IFFT.
- The transmitter uses either non-differential or differential encoding to map bits to modulation symbols.
- At the receiver, the modulation symbols at the output of the FFT are demapped to bits or “soft” bits, which will constitute the input to the deinterleaver and the decoder.
- The receiver uses coherent of differential detection to demap modulation symbols to bits/soft bits, depending on the mapping scheme used at the transmitter.

Non-Differential Encoding

- For mappings with no differential encoding, the encoded and interleaved bits are directly mapped to modulation symbols $S_{k,n}$.
- Examples of this mapping technique include M-ary phase shift keying (M-PSK) or M-ary quadrature amplitude modulation (M-QAM).

No Differential Encoding

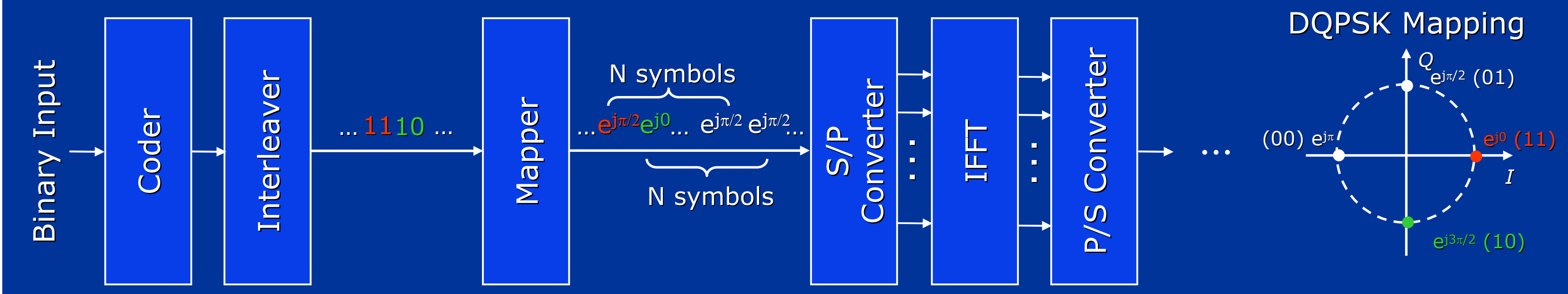


Differential Encoding

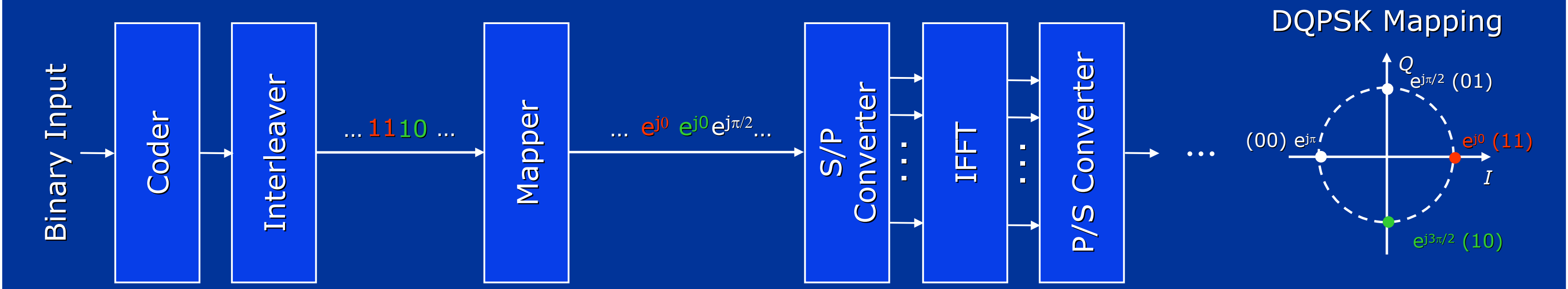
- For mappings with differential encoding, the encoded and interleaved bits are mapped to the quotient $B_{k,n}$ of two successive modulation symbols.
- If performed in the time direction, the modulation symbol $S_{k,n}=S_{k-1,n}B_{k,n}$., and each sub-carrier of the first OFDM symbol conveys a known/reference value.
- If performed in the frequency direction, the modulation symbol $S_{k,n}=S_{k,n-1}B_{k,n}$, and the first sub-carrier of each OFDM symbol conveys the known/reference value.
- Examples of this mapping technique include M-ary differential phase shift keying (M-DPSK), where $B_{k,n} \in \{e^{j2\pi m/M};\ m=0,\ldots,M-1\}$.

Differential Encoding

Differential Encoding in Time



Differential Encoding in Frequency



Coherent Detection

- For mappings with no differential encoding coherent detection is used at the receiver, whereby the decision is based on the quotient $D_{k,n}$ given by

$$D_{k,n}=R_{k,n}/\hat{H}_{k,n}=(H_{k,n}S_{k,n}+N_{k,n})/\hat{H}_{k,n}=S_{k,n}+N_{k,n}/\hat{H}_{k,n}$$

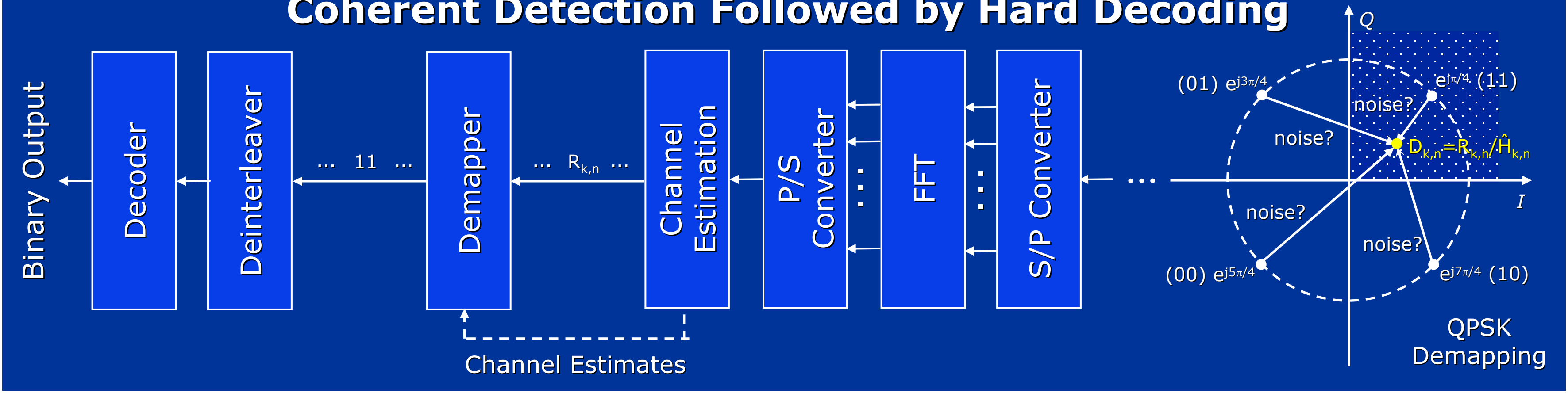
  where $\hat{H}_{k,n}$ is an estimate of the channel transfer factor $H_{k,n}$.

- Note that in OFDM systems an equaliser corresponds to a bank of complex multipliers.

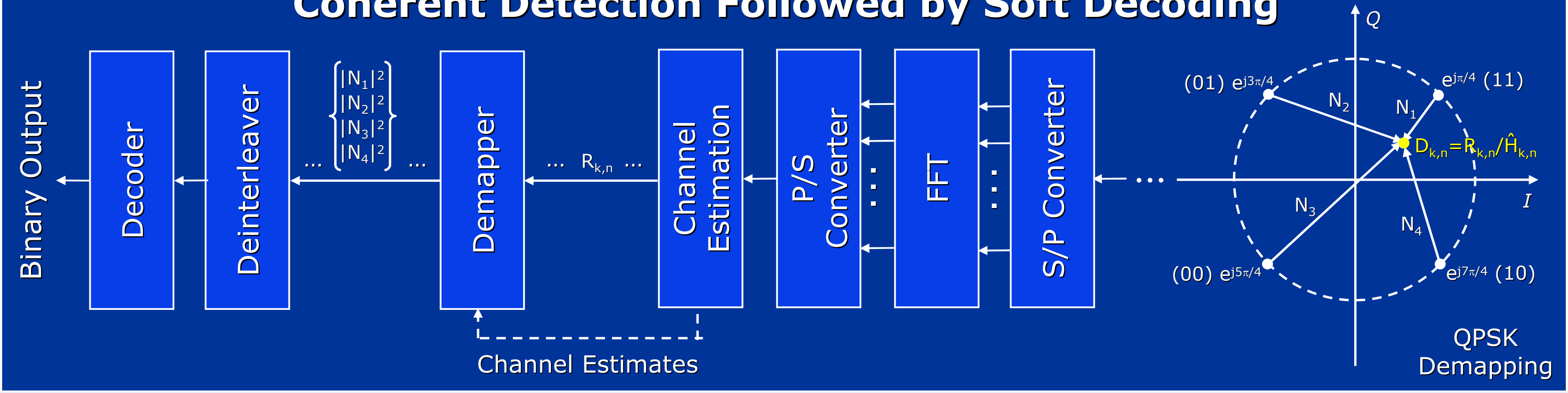
- The principal advantage of OFDM systems follows from this simple equalisation operation.

Coherent Detection

Coherent Detection Followed by Hard Decoding



Coherent Detection Followed by Soft Decoding



Differential Detection

- For mappings with differential encoding differential detection is used at the receiver.
- If differential encoding is performed in the time direction, the decision is based on the quotient:

$$D_{k,n}=R_{k,n}/R_{k-1,n}=(S_{k-1,n}B_{k,n}H_{k,n}+N_{k,n})/(S_{k-1,n}H_{k-1,n}+N_{k-1,n})$$

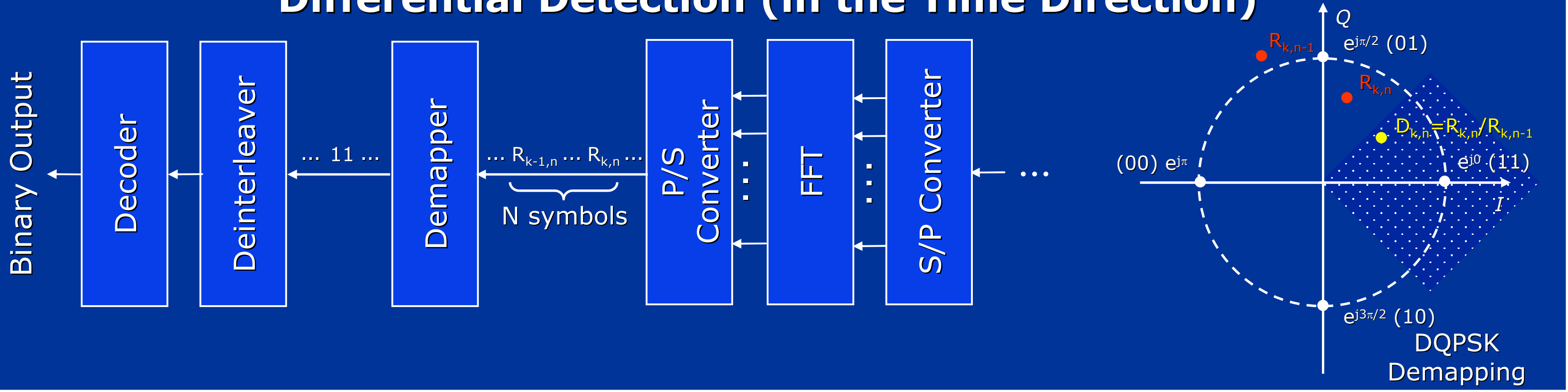
- If differential encoding is performed in the frequency direction, the decision is based on the quotient:

$$D_{k,n}=R_{k,n}/R_{k,n-1}=(S_{k,n-1}B_{k,n}H_{k,n}+N_{k,n})/(S_{k,n-1}H_{k,n-1}+N_{k,n-1})$$

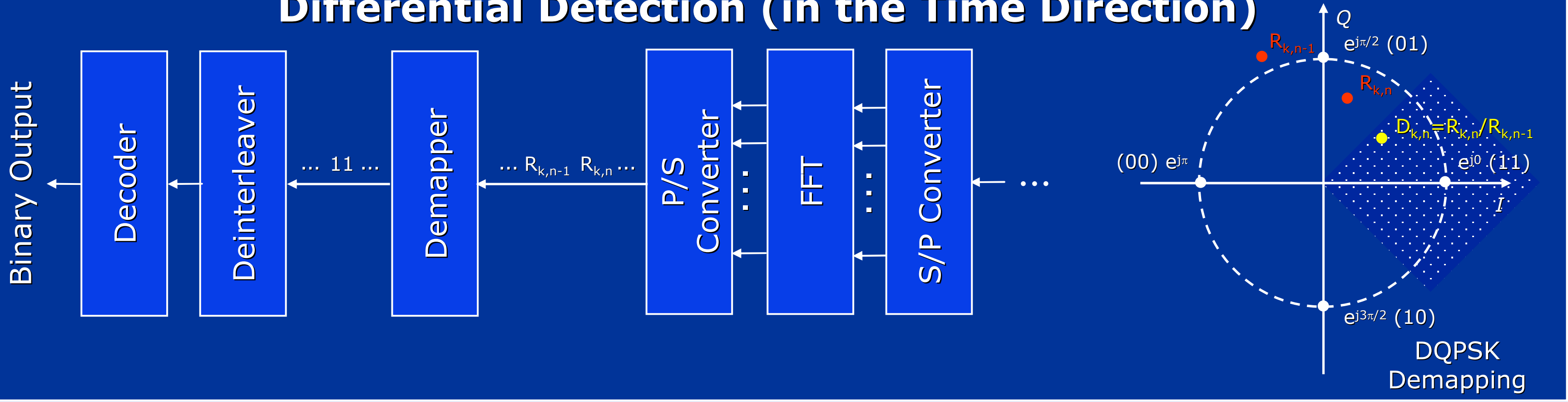
- In the absence of noise, the symbol containing the information is recovered provided that $H_{k,n}\approx H_{k-1,n}$ or $H_{k,n}\approx H_{k,n-1}$, i.e. signalling interval is smaller than T_{coh} or frequency separation is smaller than B_{coh}.

Differential Detection

Differential Detection (in the Time Direction)



Differential Detection (in the Time Direction)



Merits/Demerits of Differential and Non-Differential Schemes

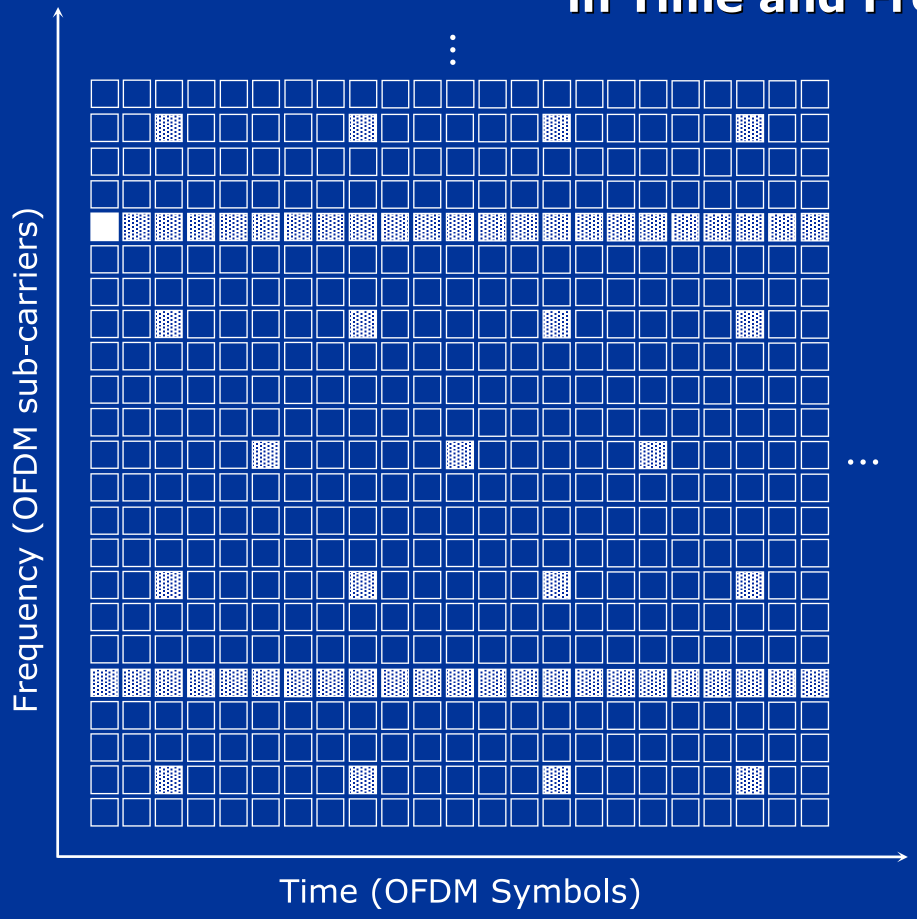
- Differential schemes require less complex receivers than non-differential schemes, as channel estimation is not necessary.
- Differential schemes are more robust than non-differential schemes to residual phase offsets caused by imperfect synchronisation.
- However, non-differential techniques require up to 3 dB less power than differential techniques to achieve a specific target error rate in noise.

Channel Estimation Operations

- In coherent detection schemes, information recovery requires an estimate of the channel response. Estimation of the channel consists of two steps:
- The first step involves the insertion of known symbols or a pilot structure into the OFDM signal, that yield point estimates of the channel frequency response.
- The second step involves an interpolation operation, that yields the remaining points of the channel frequency response from the point estimates.
- The performance of channel estimation operations depends both on the pilot structure and on the interpolation method.

Channel Estimation Operations

Example of a Pilot Structure to Track Channel Variations in Time and Frequency



High density of pilots gives good channel estimation accuracy, but higher loss in bandwidth efficiency/SNR

Low density of pilots gives lower loss in bandwidth efficiency/SNR, but poorer channel estimation accuracy

Maximum pilot separation in frequency= =Channel coherence bandwidth, B_{coh}

Maximum pilot separation in time= =Channel coherence time, T_{coh}

Synchronisation Operations

- Information recovery also requires accurate symbol timing and carrier frequency synchronisation between the OFDM transmitter and the OFDM receiver.
- Synchronisation in OFDM systems is performed before detection and involves two phases: acquisition and tracking.
- In the acquisition phase, the frequency and timing errors are coarsely estimated and corrected.
- In the tracking phase, only small short-term deviations are estimated and corrected.

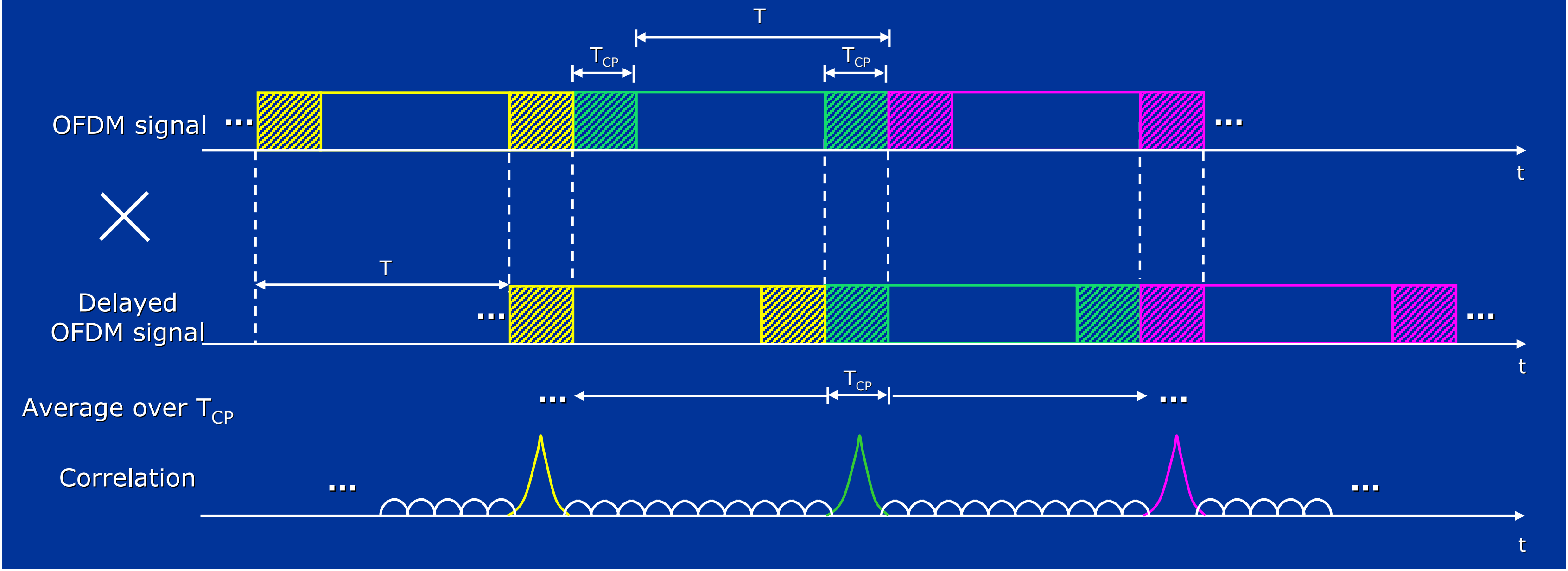
Synchronisation Operations

- A number of techniques to perform symbol timing and carrier frequency synchronisation have been proposed.
- Synchronisation techniques based on the cyclic prefix rely on the computation of the correlation of the received OFDM signal with a delayed version of the same received OFDM signal over an interval equal to the cyclic prefix interval.
- Synchronisation techniques based on the transmission of special OFDM training symbols rely on the computation of the correlation of the received OFDM signal with the known OFDM training symbols.
- The peaks of the correlation function yield the timing information and the phase of the peaks of the correlation function yield the frequency information.

Synchronisation Operations

Synchronisation Based on the Cyclic Prefix

Correlation main peak power varies from symbol to symbol **AND** **Correlation sidelobes are not negligible** ⇨ **Use special training symbols**



Choice of OFDM Signal Parameters

- How do we choose the parameters of an OFDM signal (e.g., T, T_{cp}, N) given specific data rate and bandwidth constraints and a channel with a specific delay and/or Doppler spread?
- Channel delay spread T_m dictates length of the OFDM cyclic prefix T_{CP}…length of the OFDM cyclic prefix is approx. two to four times channel delay spread to limit ISI/ICI.
- Length of the OFDM cyclic prefix T_{CP} dictates total length of the OFDM symbol $T+T_{CP}$…total length of the OFDM symbol is approx. five times length of OFDM cyclic prefix to limit SNR loss to approx. 1dB.
- The number of sub-carriers N, code rate and modulation are selected to meet the data rate and bandwidth constraints.
- Channel Doppler spread B_d may dictate a maximum number of sub-carriers N.

Choice of OFDM Signal Parameters

- Design an OFDM system with the following requirements: data rate=6, 9, 12, 18, 24, 36, 48 and 54Mbps, bandwidth< 20MHz and delay spread=200ns.
- Choose $T_{CP}=4\times200\text{ns}=800\text{ns}$ and $T+T_{CP}=5\times800\text{ns}=4000\text{ns}$, so $T=3200\text{ns}$ and $\Delta f=1/T=312.5\text{kHz}$.
- Note that the maximum number of sub-carriers that can be fit into the allocated bandwidth is 64.
- Choose size of IFFT (FFT) to be equal to 64 (power of 2) and number of (data) sub-carriers to be equal to 48.
- Note that the remaining IFFT (FFT) values are used for pilots and/or zero-padding.
- Finally, choose code rate and modulation to meet data rate requirements.

Choice of OFDM Signal Parameters

Choice of OFDM Signal Parameters

Data Rate	Bandwidth	N	Code Rate	Modulation
6 Mbps	15 MHz	48	1/2	BPSK
9 Mbps	15 MHz	48	3/4	BPSK
12 Mbps	15 MHz	48	1/2	QPSK
18 Mbps	15 MHz	48	3/4	QPSK
24 Mbps	15 MHz	48	1/2	16-QAM
36 Mbps	15 MHz	48	3/4	16-QAM
48 Mbps	15 MHz	48	2/3	64-QAM
54 Mbps	15 MHz	48	3/4	64-QAM

Advantages/Disadvantages of OFDM

- Advantages of OFDM:
  - Good performance under delay spread/frequency selective fading conditions;
  - Bandwidth efficiency;
  - Efficient digital signal processor based generation/ detection techniques.
- Disadvantages of OFDM:
  - Poor performance under Doppler spread conditions/time selective fading conditions;
  - Sensitive to non-linear distortion;
  - Sensitive to timing and frequency offsets as well as phase noise.

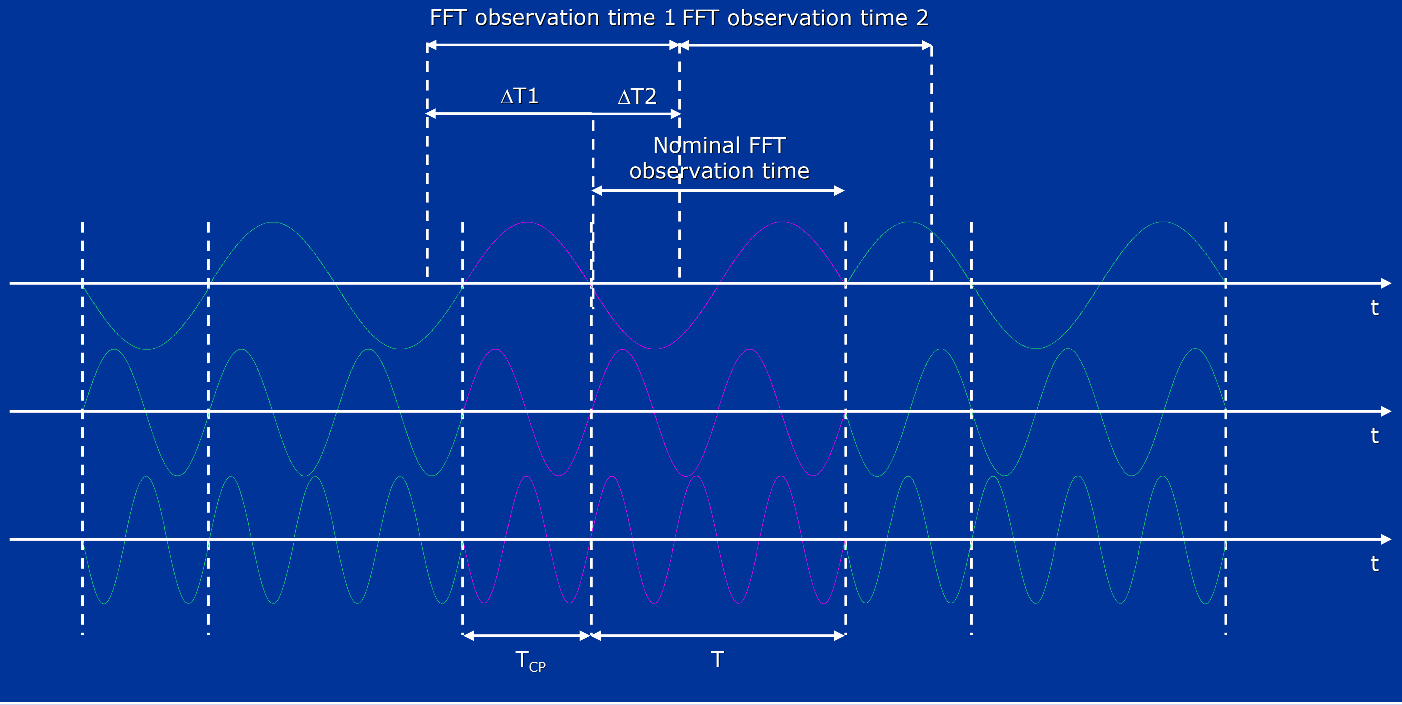
Timing and Frequency Offsets

- Timing offsets originate due to uncertainties to establish the OFDM symbol boundaries.
- Timing offsets give rise to intersymbol interference and interchannel interference or simply a phase offset in desired data.
- Carrier frequency offsets originate from frequency differences in the local oscillators at the transmitter and the receiver used to convert the baseband signal to a bandpass signal and vice versa.
- Frequency offsets give rise to interchannel interference and a reduction in power in the desired data.

Timing Offset

Effects of Timing Offset

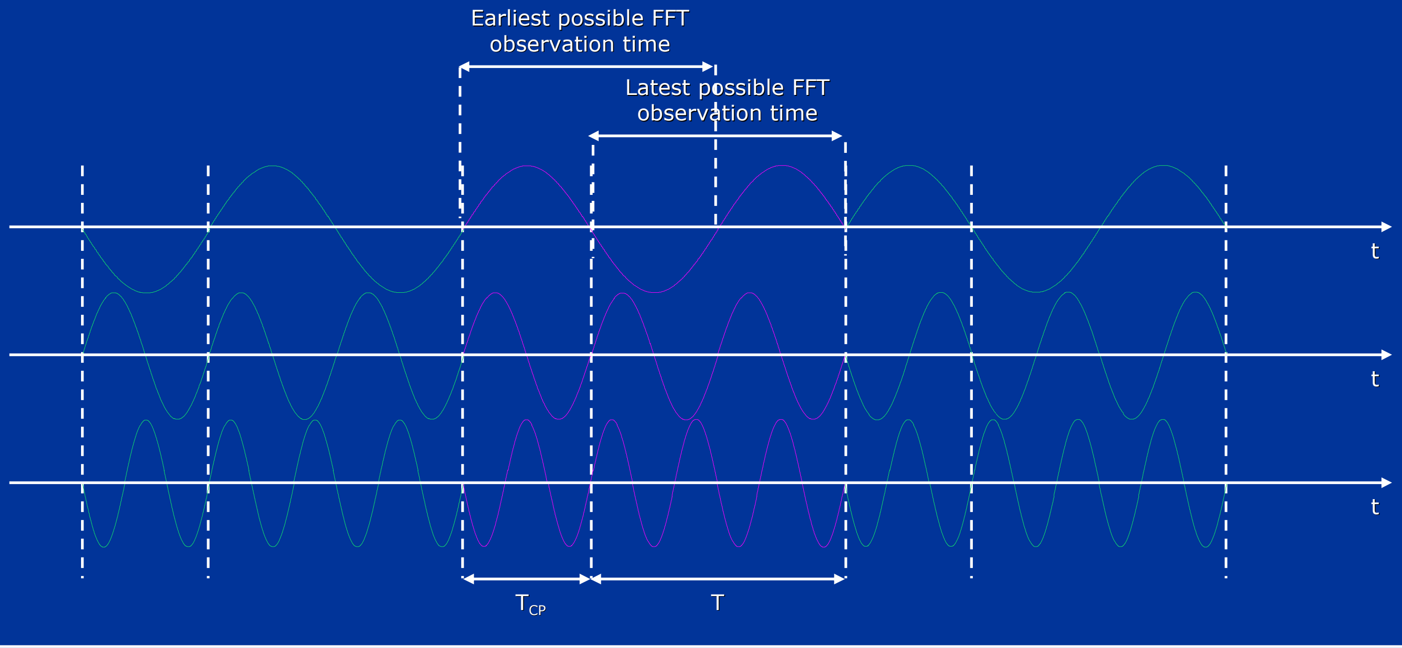
Timing offsets where FFT observation time overlaps adjacent symbols ⇨ **Intersymbol and intercarrier interference**



Timing Offset

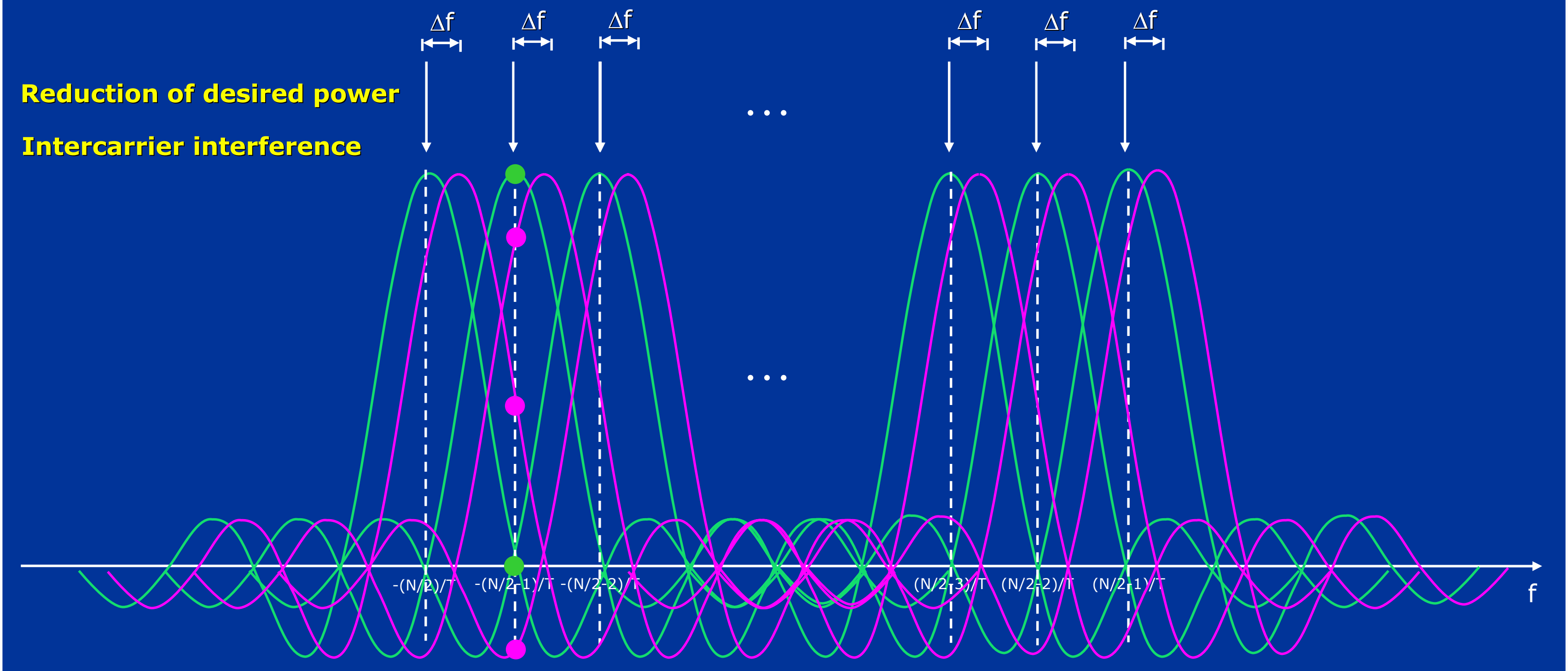
Effects of Timing Offset

Timing offsets where FFT observation time does not overlap adjacent symbols ⇒ **Phase offset $\varphi_n = 2\pi f_n \Delta t$, $n = 0, \ldots, N-1$**



Frequency Offset

Effects of Frequency Offset



Reduction of desired power

Intercarrier interference

Phase Noise

- In practice, the up-converter and down-converter oscillators in the OFDM system do not produce a carrier at exactly one frequency.
- Instead, these oscillators in the OFDM system produce a carrier at a nominal frequency with a time-varying frequency offset, i.e., phase noise.
- Phase noise introduces a phase offset common to all sub-carriers as well as intercarrier interference.

The PAPR

- OFDM signals are sensitive to non-linear distortion due to its high peak-to-average power ratio (PAPR) or, its multi-carrier nature.

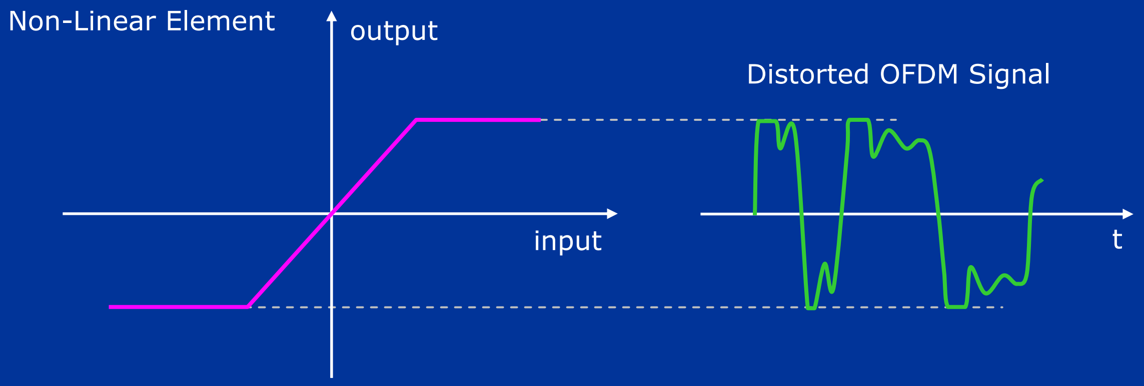
- From one view point, the saturation region of any non-linear element in the system occasionally clips the signal due to its high PAPR.

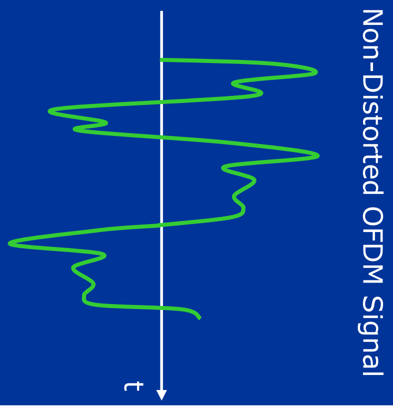
- From another view point, any non-linear element in the system introduces severe intermodulation distortion (IMD) due to the multi-carrier nature of the signal.

- These effects result in signal error probability degradation and signal spectral spreading.

The PAPR

Effects of Non-Linearities



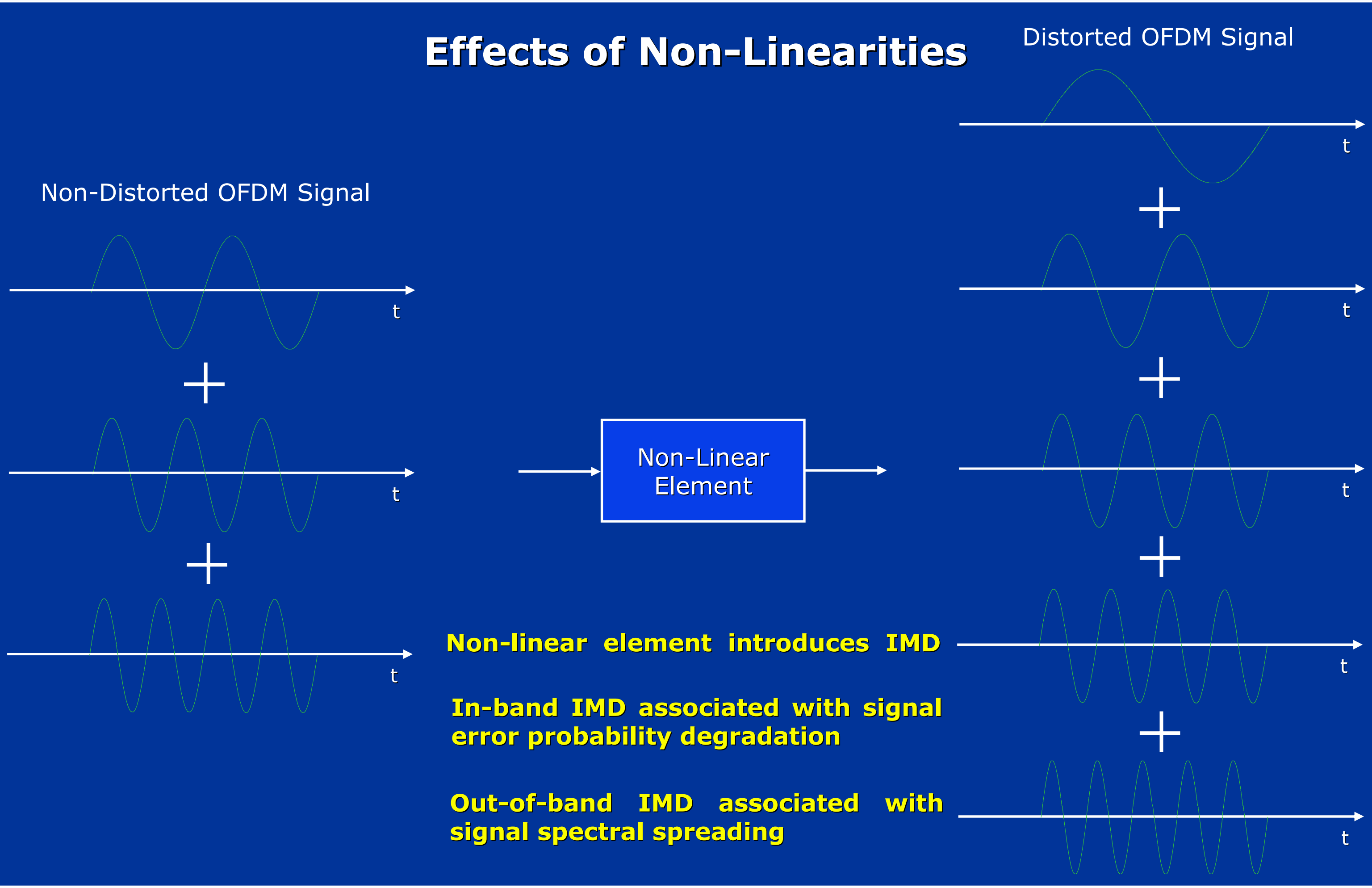


Large signal excursions hit saturation region of non-linear element

Output signal is a distorted replica of input signal

The PAPR

Effects of Non-Linearities



Non-linear element introduces IMD

In-band IMD associated with signal error probability degradation

Out-of-band IMD associated with signal spectral spreading

The PAPR

- A number of techniques have been proposed to improve the performance of non-linearly distorted OFDM signals, e.g., PAPR reduction and pre-distortion.
- PAPR reduction techniques rely on the reduction of the OFDM signal variability.
- Pre-distortion techniques rely on an appropriate prior distortion of the OFDM signal such that the effect of the non-linearity is undone.

Applications of OFDM

- OFDM has been proposed for a number of systems including wired and wireless applications.

- For example, OFDM has been selected for assymetric digital subscriber line (DSL) systems under the acronym of discrete-multitone (DMT).

- OFDM has also been selected for digital audio broadcasting (DAB) and digital video broadcasting (DVB) as well as wireless local area networks (LAN) – in IEEE 802.11/HIPERLAN- and wireless metropolitan area networks (MAN) – in IEEE 802.16/HIPERMAN.

Digital Subscriber Lines

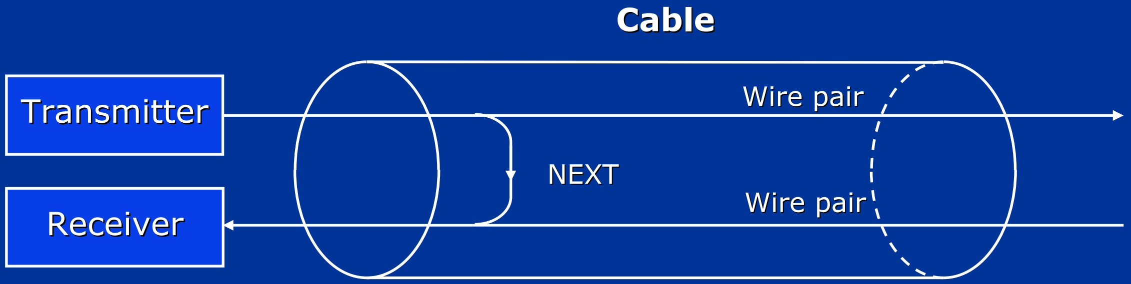
- OFDM under the acronym of DMT has been adopted for ADSL.
- ADSL is a scheme for high speed communication in the telephone access network or the subscriber line, where the bit rate offered in the downstream direction (to the subscriber) is larger than the bit rate offered in the upstream direction (to the central office).
- For example, in the USA the ADSL standard supports downstream bit rates from 1.54 to 6.1 Mbit/s and upstream bit rates from 9.6 to 192 kbit/s.
- ADSL is suitable for applications like video on demand, games, virtual shopping and internet surfing.

Digital Subscriber Lines

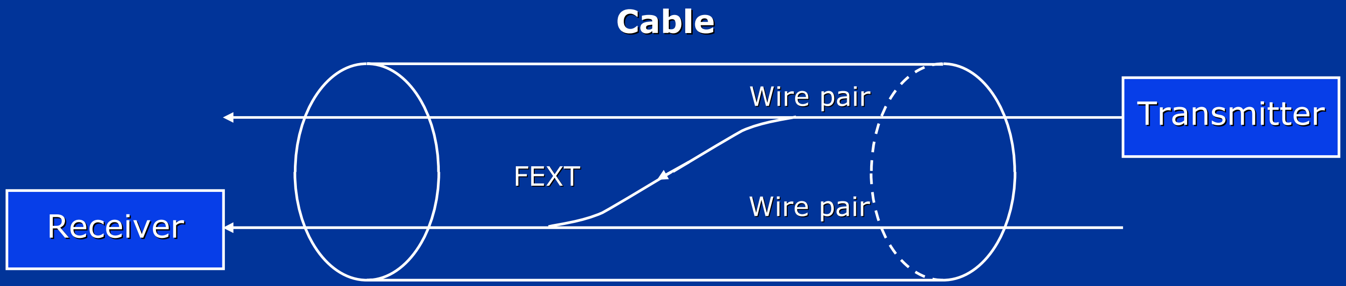
- Copper wire pairs are the dominating medium type in telephone access networks or subscriber lines.
- Impairments in this environment include the "highly spectrally shaped" frequency response of a typical copper wire pair and various noise sources.
- Noise sources include crosstalk from other copper wire pairs in the same cable, RF noise and impulse noise.
- Near-end crosstalk occurs at the central office when the weak upstream signal is disturbed by strong downstream signals.
- Far-end crosstalk is crosstalk from one transmitted signal to another in the same direction and appears both at the central office and the subscriber.

Digital Subscriber Lines

Near-End Crosstalk (NEXT)



Far-End Crosstalk (FEXT)



Digital Subscriber Lines

- OFDM in conjunction with bit loading techniques has been selected for ADSL for it efficiently combats the adverse effects in the telephone access network or the subscriber line.

- Bit loading is a scheme whereby higher-order constellations are assigned to high signal-to-noise ratio sub-channels and lower-order constellations are assigned to low signal-to-noise ratio sub-channels.

- Essentially, OFDM in conjunction with bit loading techniques offers a practical solution to achieve transmission rates close to the capacity of a “highly frequency shaped” linear channel.

Digital Broadcasting

- OFDM systems have been proposed to assist the migration from old analogue broadcasting systems to new digital broadcasting systems.

- For example, in Europe OFDM has been adopted for DAB and terrestrial DVB.

- OFDM systems have been proposed for digital broadcasting systems mainly due to their capability to combat multi-path propagation and narrowband interference as well as their capability to allow for the implementation of single frequency networks.

Digital Broadcasting

- Narrowband interference originates from the coexistence of analogue and digital broadcasting systems.
- Narrowband interference can be combated efficiently using spectrum shaping techniques.
- Essentially, sub-carriers experiencing high signal-to-interference ratios are transmitted whereas sub-carriers experiencing low signal-to-interference ratios are not transmitted.

Digital Broadcasting

- In a conventional broadcasting network, geographically adjacent transmitters transmit the same signal/program on distinct frequencies.
- In a single frequency broadcasting network several geographically dispersed transmitters transmit the same signal/program synchronously on the same frequency.
- A receiver therefore observes several replicas of the same signal possibly attenuated and delayed with respect to one another.

Digital Broadcasting

- A single frequency network can greatly enhance the spectrum and power efficiency of a broadcasting system.

- Spectrum efficiency is improved because only a single frequency instead of multiple frequencies is used to cover an entire region or country.

- Power efficiency is improved because the coverage area served for example by two transmitters operating simultaneously is greater than the sum of the coverage areas for each of the two transmitters operating independently.

- Essentially, the phenomena occurring in a single frequency network is analogous to the phenomena occurring in a multipath fading channel and hence it can be combated efficiently with OFDM.

Wireless LAN

- OFDM has been selected as the basis for the physical layer of a number of packet based indoor wireless LAN standards such as IEEE 802.11a for the 5 GHz band, IEEE 802.11g for the 2.4 GHz band and HiperLAN2.
- For example, IEEE 802.11a is a standard to interconnect portable devices to broadband networks.
- IEEE 802.11 offers data rates ranging from 6 Mbit/s to 54 Mbit/s.

Wireless LAN

- A nominal channel bandwidth of 20 MHz is used.
- A 64-point IFFT/FFT is used where 48 sub-carriers are allocated for data, 4 sub-carriers are allocated for pilots and the remaining sub-carriers are used for zero-padding.
- The cyclic prefix duration is set to be equal to 800 ns and the useful symbol duration is set to be equal to 3200 ns.
- Modulation schemes used include BPSK, QPSK, 16-QAM and 64-QAM.
- Forward error correction used includes the industrial standard convolutional code with rate 1/2, constraint length 7 and generator polynomials (133,171) octal. Higher rates of 2/3 and 3/4 are obtained by puncturing the convolutional code.

Wireless LAN

OFDM Parameters in IEEE 802.11

Cyclic Prefix Duration	800ns
Useful Symbol Duration	3200ns
Total Symbol Duration	4000ns
Sub-carrier Spacing	312.5kHz
Number of data sub-carriers	48
Number of pilot sub-carriers	4
Total number of sub-carriers	52
Bandwidth	16.56MHz

Wireless LAN

OFDM Parameters in IEEE 802.11

Data Rate	Code Rate	Modulation
6 Mbps	1/2	BPSK
9 Mbps	3/4	BPSK
12 Mbps	1/2	QPSK
18 Mbps	3/4	QPSK
24 Mbps	1/2	16-QAM
36 Mbps	3/4	16-QAM
48 Mbps	2/3	64-QAM
54 Mbps	3/4	64-QAM

Wireless MAN

- OFDM has been selected as the basis for the physical layer of a number of wireless MAN standards such as IEEE 802.16a in the 2 to 11 GHz range and HiperMAN.

- For example, IEEE 802.16a specifies a wireless MAN which provides an alternative to cable, DSL or T1 level services for last mile broadband access, and backhaul for 802.11 hotspots.

- IEEE 802.16a supports low latency applications such as voice and video.

- Moreover, IEEE 802.16a provides broadband connectivity without requiring direct line of sight between the subscribers terminal and the base station.

Wireless MAN

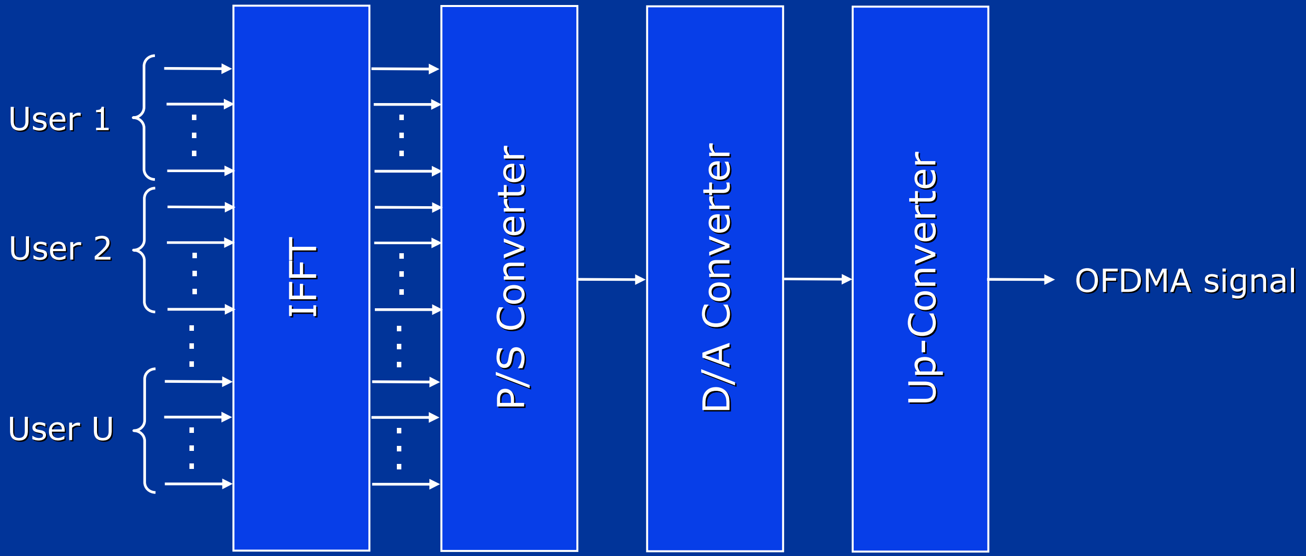
- Flexible channel sizes of e.g. 3.5, 5, 10 MHz, etc. are used.
- A 256-point IFFT/FFT is used where 192 sub-carriers are allocated for data, 8 sub-carriers are allocated for pilots and the remaining sub-carriers are used for zero-padding.
- Here the ratio of the cyclic prefix duration to the useful symbol duration is equal to 1/4, 1/8, 1/16 or 1/32.
- Modulation includes QPSK, 16-QAM or optionally 64-QAM.
- Forward error correction includes an outer Reed-Solomon code concatenated with an inner convolutional code, or optionally block turbo coding or convolutional turbo coding. Variable forward error correction capability is obtained by puncturing.
- Space-time coding can be optionally used to further improve performance.

Recent Developments

- Recent developments in the area of OFDM include orthogonal frequency division multiple access (OFDMA) and multi-carrier code division multiple access (CDMA).
- OFDMA is a multiple access technique based on OFDM technology.
- Multi-carrier CDMA is a multiple access technique based on the combination of OFDM and CDMA.
- The most popular multi-carrier CDMA techniques are MC-DS-CDMA and MC-CDMA.

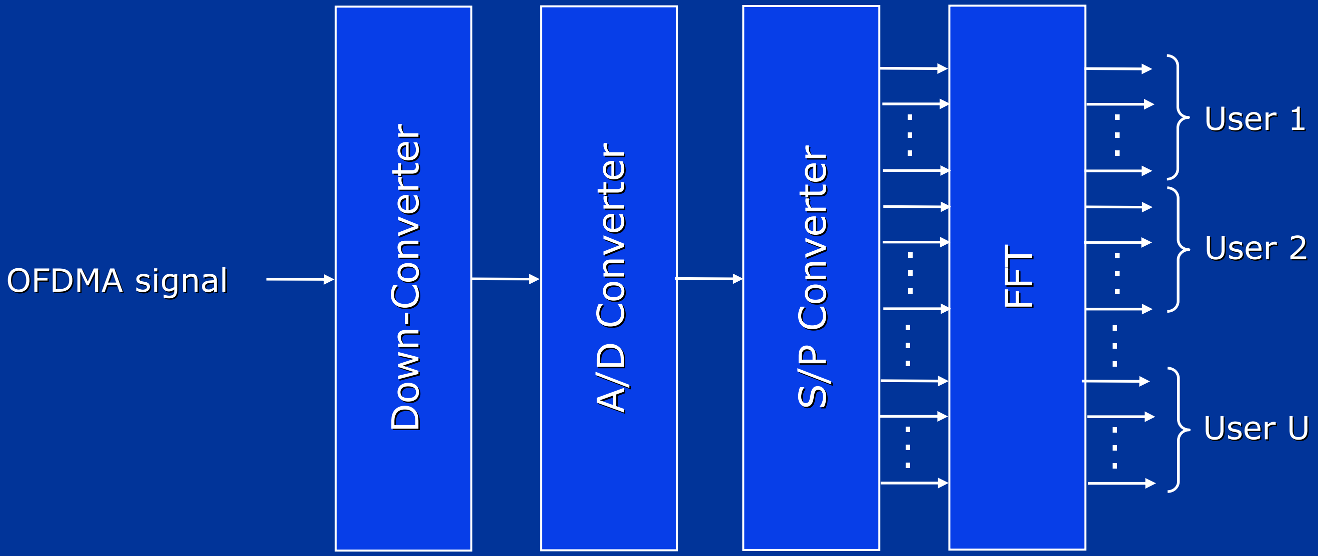
OFDMA

OFDMA Transmitter



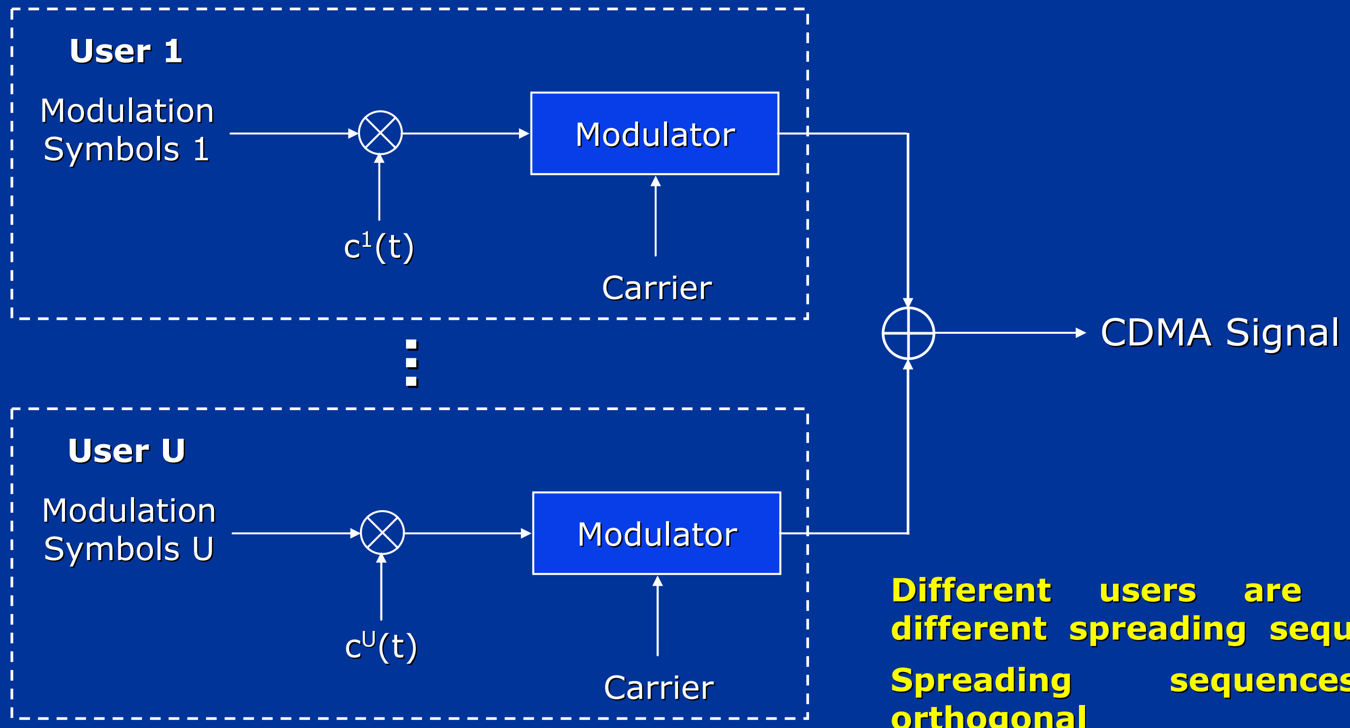
OFDMA

OFDMA Receiver



CDMA

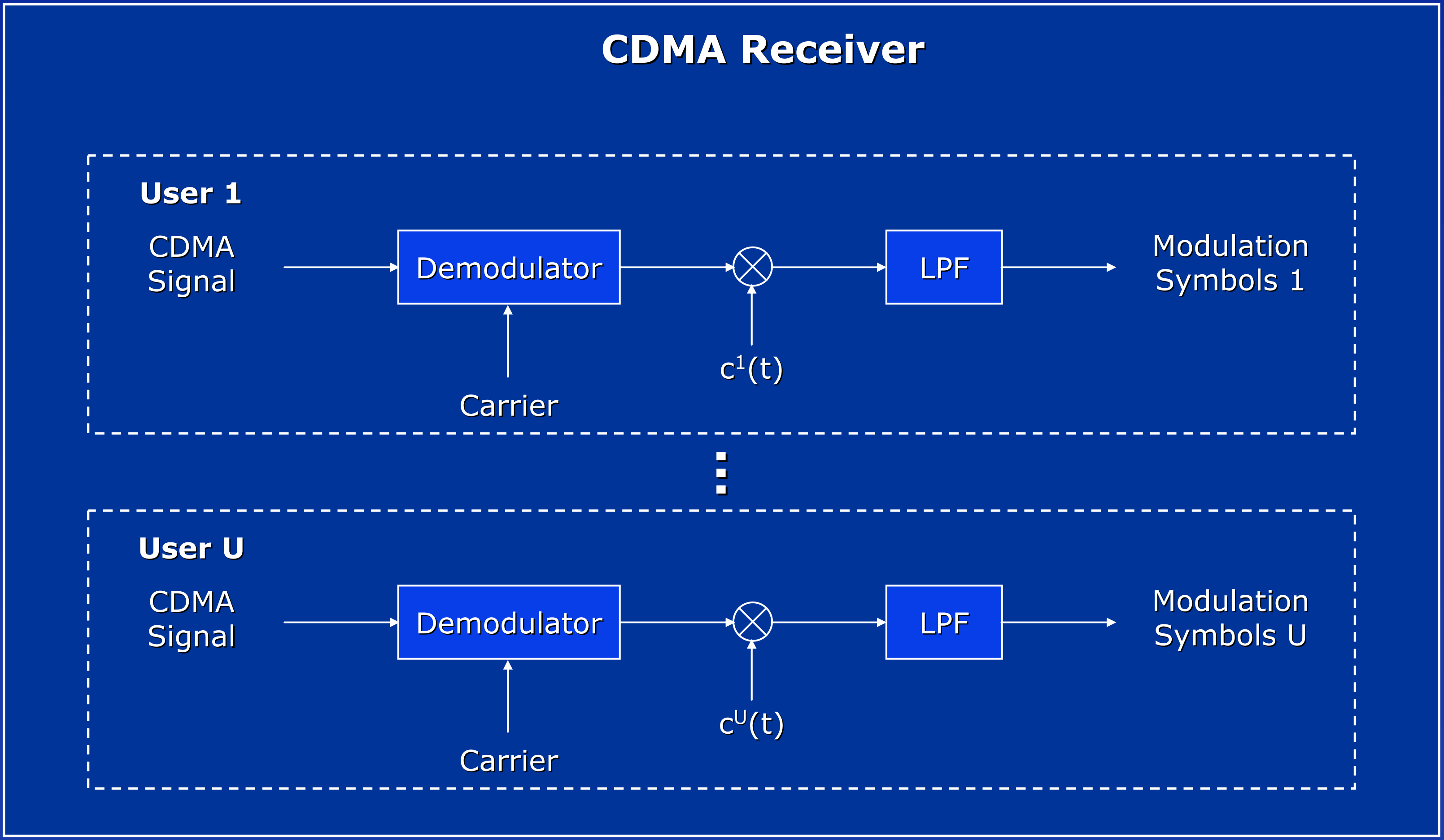
CDMA Transmitter



Different users are allocated different spreading sequences

Spreading sequences are orthogonal

CDMA


CDMA Receiver
User 1
CDMA Signal
Demodulator
Carrier
$c^1(t)$
LPF
Modulation Symbols 1
⋮
User U
CDMA Signal
Demodulator
Carrier
$c^U(t)$
LPF
Modulation Symbols U

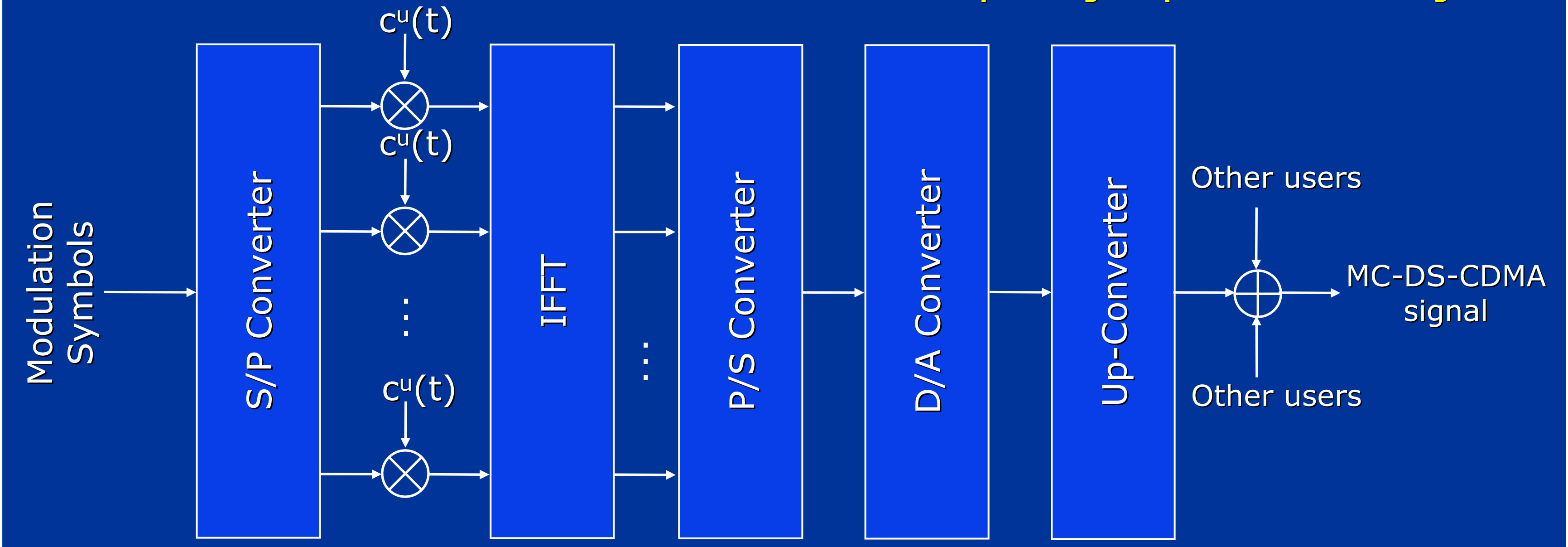
MC-DS-CDMA

MC-DS-CDMA Transmitter

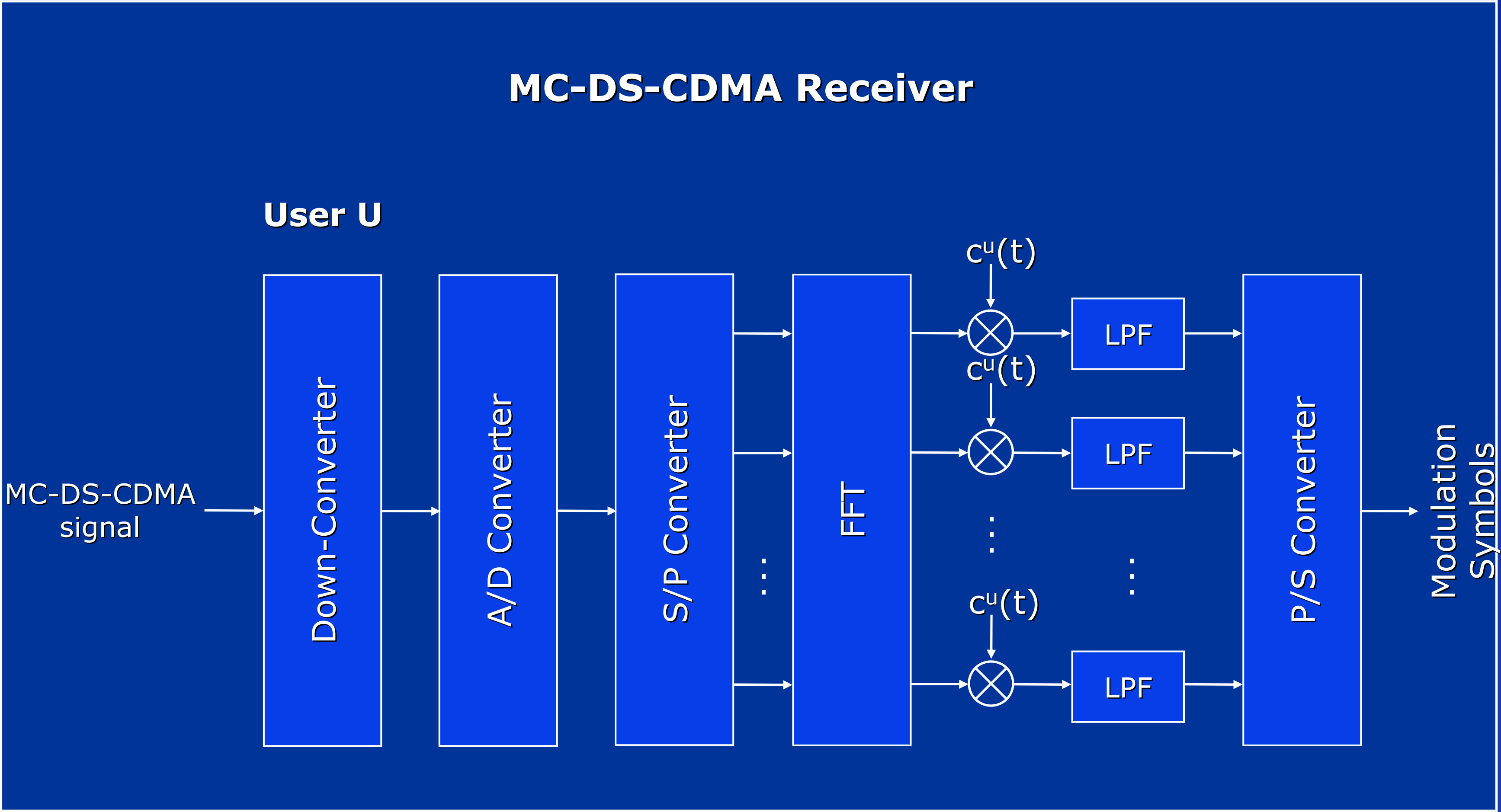
Different users are allocated different spreading sequences

Spreading sequences are orthogonal

User U



MC-DS-CDMA



MC-DS-CDMA Receiver

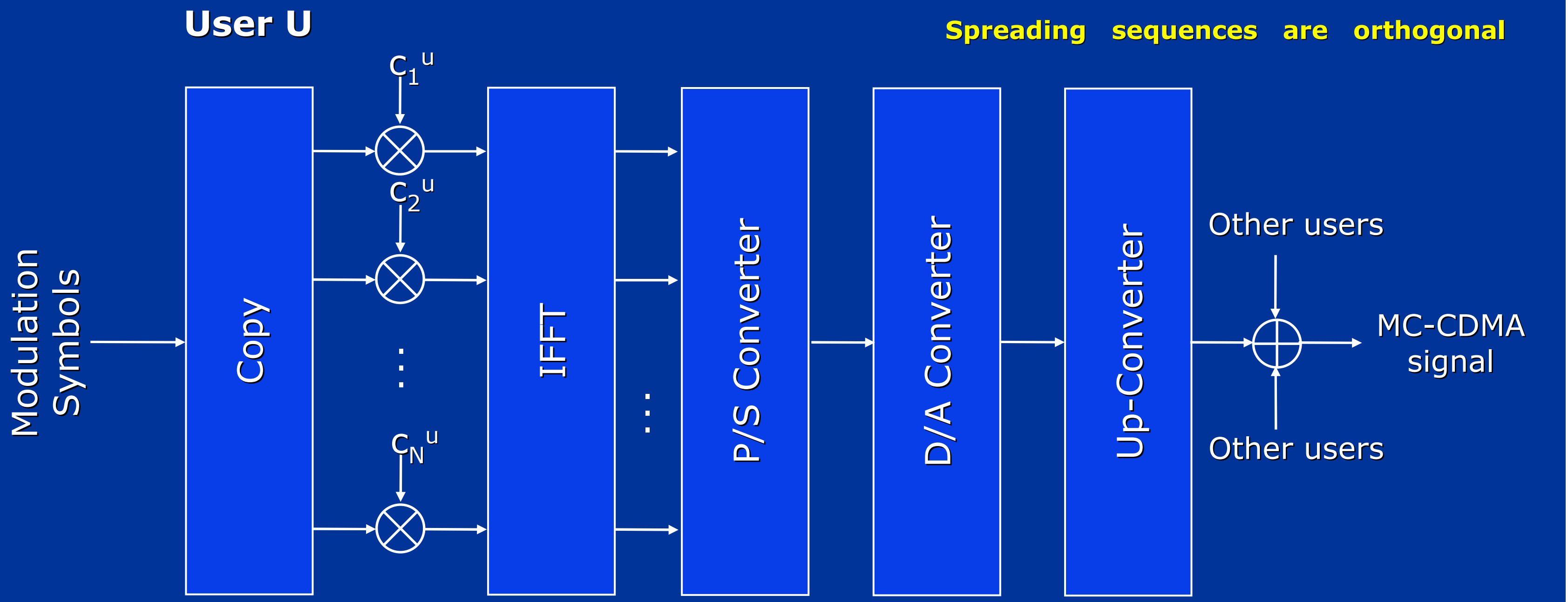
User U

MC-CDMA

MC-CDMA Transmitter

Different users are allocated different spreading sequences

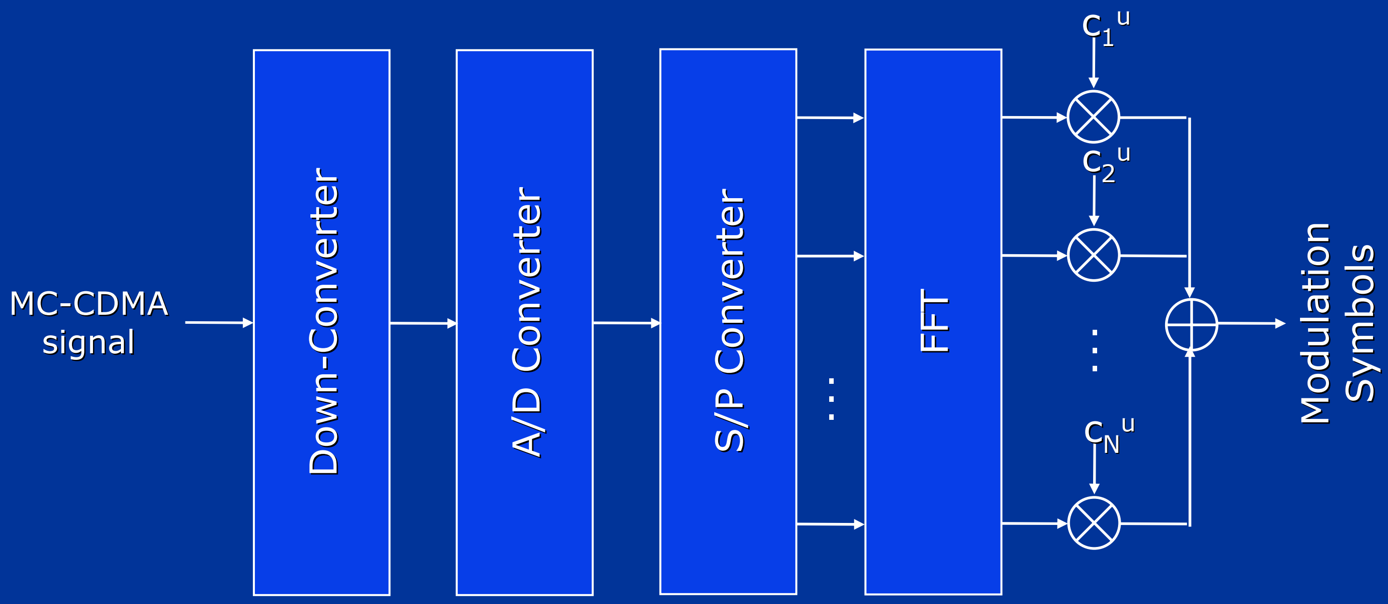
Spreading sequences are orthogonal



MC-CDMA

MC-CDMA Receiver

User U



MC-CDMA signal

Down-Converter

A/D Converter

S/P Converter

FFT

c_1^u

c_2^u

c_N^u

Modulation Symbols

Summary

- The characteristics of the wireless channel and its effects on communications systems were reviewed.
- In this context, OFDM is proposed to overcome the problems associated with wireless propagation.
- The elements of a typical OFDM communications system were described.
- Emphasis was given to generation/detection of OFDM signals, coding and interleaving methods, mapping and demapping techniques as well as channel estimation and synchronisation operations.
- The effects of timing and frequency offsets, phase noise and non-linearities on OFDM signals were also described.

Summary

- A range of applications of OFDM were reviewed including digital subscriber lines, broadcasting systems (DAB and DVB), wireless LAN and wireless MAN.
- A range of recent developments in the area of OFDM were also reviewed including OFDMA, MC-DS-CDMA and MC-CDMA.