EE 5151: Communication Techniques

Sept. 2019 **Tutorial #4** KG / IITM

1. Given a population of N=20,000 users, each offering E_u =0.04 Erlangs of traffic, define a 3-stage blocking switch with k sub-arrays in the middle-stage, each containing 250x250 cross-points such that the blocking probability $P_b = 10^{-3}$ or less. Use Lee graph approach to find this least value of k.
(a) Determine the number of cross-points for the above switch.
(b) For the same size of the middle-stage sub-arrays (i.e., same size of n) as in (a), define a non-blocking switch. How does the complexity of this switch compare to (a)?
(c) Rework value of k and part (a) if we require $P_b \leq 10^{-5}$.

2. A total of N=4096 lines have to be switched, where each line offers Eu =0.05 Erlangs of traffic. All the 3 stages of the switch are to be built using sub-arrays of size 64x64 (where in the input and output stages, not all lines need be utilized if $k < 64$).
(a) Define a blocking switch such that blocking probability $P_b = 10^{-3}$ or less. What is the complexity (including any un-utilized cross-points) ?
(b) Is it possible to build a non-blocking 3-stage switch in this case? Specify.

3. The first 400 inlets carry users with Eu=0.05 Erlangs while the next 600 inlets carry users with Eu =0.01 Erlangs. Given that the users are grouped into blocks of n=50 each, define a 3-stage block switch with overall $P_b = 10^{-2}$ or less. What is the total number of cross-points in this switch? *Hint*: The overall blocking probability is computed by considering the 4 cases, namely user from set1 calls another user in set1, or user from set1 calls user from set2, etc.

4. For the switch considered in Problem 2 (a) part, use the blocking probability expression following the work of Jacobaeus (which does **not** assume that the paths from input-to-middle stage and paths from output-to-middle stage are independent) given in eqn. (5.10) in page 239 of the book. What will be the new value of k for this case? How does this compare with your answer in 2(a)? Comment.

5. Consider a population of N=4000 users, each of E_u=0.01 Erlangs. Design a 3-stage blocking switch of least complexity such that the blocking probability P_b=10^{-4} or less. What is k, and the total number of cross-points for this switch? ***Hint:*** To minimize the total number of cross-points, choose the input sub-array dimension n "appropriately" where $N/m=n$.

6. Consider the 5-stage switch in the book, first described in page 237, Fig. 5.9. Here, blocking is introduced also in the middle stage(s). The input has N/n_1 sub-arrays, each of dimension n_1x k_1, where N is the total population to be served by this switch. The middle-stage (which is actually a blocking switch with 3-stages) has k_1 sub-arrays, each of size N/n_1x N/n_1. Each of these sub-arrays has $N/(n_1$x $n_2)$ sub-arrays, of dimension n_2xk_2 where k_2 is the number of middle stage sub-arrays (each of dimension $N/(n_1$x $n_2)$ x $N/(n_1$x $n_2)$). Assume each user offers E_u Erlangs of traffic.

(a) Prove using the Lee-Graph approach that blocking probability of the 5-stage switch is given by $P_b = \left\{1 - q_1^2 \left[1 - (1 - q_2^2)^{k_2}\right]\right\}^{k_1}$ where $q_1 = (1 - p_1)$ with $p_1 = \frac{n_1 E_u}{k_1}$ and $q_2 = (1 - p_2)$ with $p_2 = \frac{n_2 p_1}{k_2}$.

(b) For N=50,000, and n_1=50 and n_2=50, find the 5-stage switch with minimum number of cross-points so that $P_b=10^{-8}$ or less. Assume E_u = 0.01 Erlangs each.

(c) Can you find a better choice of n_1 and n_2 for this case? (i.e., a choice that will minimize the number of cross-points further?)

7. Consider a population of N=50,000 users, each of E_u=0.02 Erlangs.

(a) Design a 3-stage, non-blocking switch of the least complexity. What is the dimension n of the input stage sub-arrays? What is the total complexity (in total number of cross-points)?

(b) Now, instead design a 3-stage blocking switch of least complexity such that the blocking probability $P_b=10^{-5}$ or less. What is k, the number of middle-stage sub-arrays, and the total number of cross-points for this switch for each of the following choices: (i) n = 500; (ii) n = 125. Explain the reason for your answer (if any) by comparing with your corresponding answers in (a).

8. In a village, a base-station with M = 2 servers is used to serve with 5% blocking, the users who offer Eu=0.01 Erlangs of load each. Now, if one more server is added to this base-station, what will be the new blocking percentage?