EE 5151: Communication Techniques

Sept. 2019 **Tutorial #2** KG / IITM

1. Find the "best" 2-bit uniform quantiser (i.e., specify the quantisation boundaries $\{a_1, a_2, a_3\}$ for quantizing a scalar random variable X given by the following probability distribution functions:
(a) Uniform pdf from -2 to 2; i.e., [-2, +2].
(b) Uniform pdf [-1, +3]
(c) Uniform pdf [-1, +2]
(d) Also, in each case, specify the SQR. Note that SQR = $E[(X-\mu)^2] / E[(X-X_q)^2]$ where $\mu = E[X]$ and the quantized value is X_q.
(e) In each case, specify if it is a mid-rise quantizer of a mid-tread quantizer.

2. The psd of a WSS process X(t) is given by

$$S_X(f) = \begin{cases} \dfrac{f+5000}{5000}, -5000 \le f \le 0 \\ \dfrac{-f+500}{5000}, 0 < f \le 5000 \\ \\ 0, otherwise \end{cases}$$

and the maximum amplitude of this process is 6.
(a) What is the power content of this process?
(b) If this process is sampled at fs to guarantee a guard band of 2000Hz, then what is fs?
(c) At this sampling rate, if we use a linear PCM system with 256 levels, what is the resulting SQNR in dB?
(d) What is the resulting bit rate?
(e) If we need to increase the SQNR by atleast 25dB, how many levels are required? What is the new bit rate?

3. Llyod-Max Non-uniform Quantization: Assume that a random signal x(t) with $x=x(t=t_0)$, follows a probability density function $f_X(x)$, and the objective is to find the N-1 signal boundaries $\{a_1, a_2, \cdots, a_{N-1}\}$ and the corresponding N quantized values $\{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N\}$ such that the quantization noise variance given by

$$E_q = E[(x-x_q)^2] = \int_{-\infty}^{a_1} (x-\hat{x}_1)^2 f_X(x)\,dx + \sum_{i=2}^{N-1} \int_{a_{i-1}}^{a_i} (x-\hat{x}_i)^2 f_X(x)\,dx + \int_{a_{N-1}}^{\infty} (x-\hat{x}_N)^2 f_X(x)\,dx$$

is minimized. In other words, differentiate E_q w.r.t $\{a_1, \cdots, a_{N-1}\}$ and $\{\hat{x}_1, \cdots, \hat{x}_N\}$, equate them to zero, and show the following:

$$(i)\, a_i = \frac{\hat{x}_i + \hat{x}_{i+1}}{2} \text{ and } (ii)\, \hat{x}_i = \frac{\int_{a_{i-1}}^{a_i} x\, f_X(x)\,dx}{\int_{a_{i-1}}^{a_i} f_X(x)\,dx}$$

4. The Llyod-Max (LM) quantizer indicates that the signal level should be the mid-point of the quantization interval (see (*i*)), and that the quantized value is the "centroid" of the corresponding interval (see (*ii*)). However, a practical method to construct this non-uniform quantizer is given by the iterative method below for a *k*-bit quantizer where $N=2^k$:

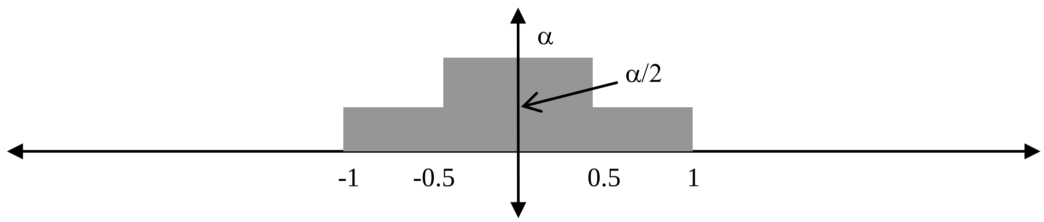
Step 1: choose *N*-1 uniform intervals $\{ a_1, a_2, \cdots, a_{N-1} \}$

Step 2: find the corresponding centroids $\{ \hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N \}$ using (*ii*)

Step 3: re-compute $\{a_i\}$ using (*ii*) in (*i*)

Iterate between steps 2 and 3 until "convergence"

Use the above procedure for 3 iterations to define the 2-bit LM quantizer for the below source pdf. Compare the E_q obtained between the uniform quantizer (mid-tread) and the LM quantizer.



5. From "Digital Telephony" J.C.Bellamy, 3rd Ed., pp. 158-160: Problems **3.1, 3.2, 3.3, 3.4, 3.5, 3.8*, 3.14*, 3.16*, 3.17*, 3.18,** and **3.19.** The ones marked "*" are tougher ones, presumably**.**

6. A WSS random process has an auto-correlation function given by $R_X(\tau) = \dfrac{A^2}{2} e^{-|\tau|} Cos(2\pi f_0 \tau)$.

Assume that the random process never exceeds 6 in magnitude, and that A=6.

(a) How many uniform quantisation levels are required to provide an SQNR of at least 40dB ?

(b) If we want to increase the minimum SQNR to 60dB, how should the required number of quantisation levels change?

(c) If f_0=1MHz, what is the bit rate you will require to send the quantised samples in both of the above cases?

7. If Z has the below pdf (specify α), and starting with the quantisation boundaries $\{a_1, a_2, a_3\}$= {-1, 0, +1} perform 3 LM iterations to find a non-uniform quantizer. What is the corresponding SQR? Compare with the SQR for the initial uniform quantizer.