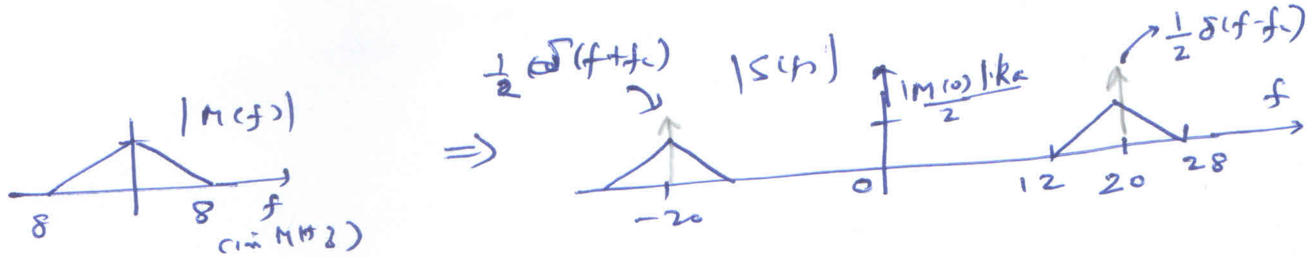
EE5151 - Quiz 1

Solutions
24/9/19

1. [2+2 = 4 marks]

$s(t) = (1 + k_a m(t)) \cos 2\pi f_c t$ → full AM signal

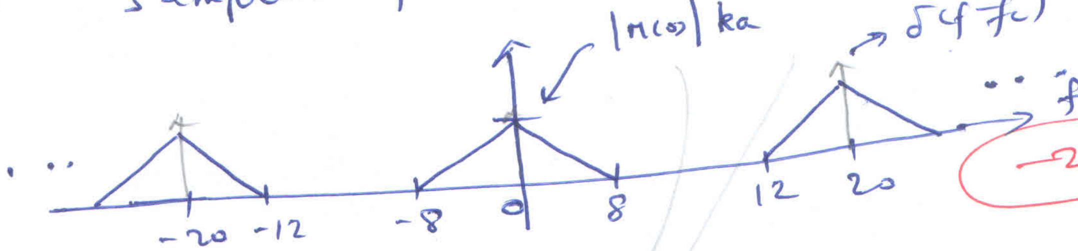
(a)



$$\left\lfloor \frac{20}{8 \times 2} \right\rfloor = 1 \Rightarrow 2W' = f_s = \frac{20}{1} = \boxed{20\ \text{MHz}}$$

−2−

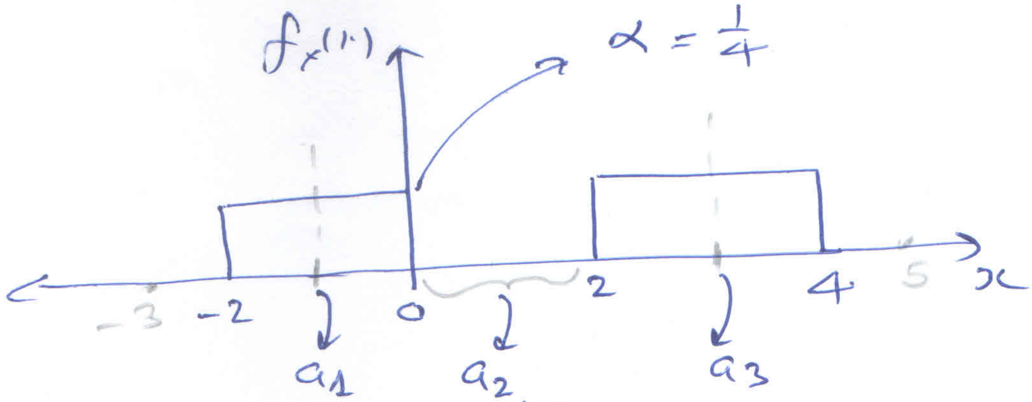
(b) Sampled Spectrum (Spectrum of Samples)



−2−

note that the ÷2 disappear because of the allowed aliasing in this case.

2. [$\cancel{1}^{\frac{1}{2}} + 2 + 2 + \cancel{1}^{1\frac{1}{2}} = 6$ marks]



can be anywhere between 0&2 (>0 and <2)

$a_2 = 1$ is a sensible choice, but not necessary ~~the only choice~~

(a) $\alpha = 1/4$ $-\frac{1}{2}$

(b) Quantisation Boundaries

$a_1 = -1$ $-\frac{1}{2}$; $a_3 = 3$ $-\frac{1}{2}$;

$-\frac{1}{2}$ ← a_2 → any value between 0 (>0) and 2 (<2)

(*) This will be a uniform quantizer if $a_2 = 1$; else, it seems to ~~be~~ resemble a non-uniform quantizer (since the quantisation regions look unequal) $-\frac{1}{2}$

(c) Assume Quantisation Boundaries

$a_1 = -1$

$a_2 = 1$

$a_3 = 3$

Quantisation Regions (for uniform quantizer)

$(-3, -1)$, $(-1, 1)$, $(1, 3)$, and $(3, 5)$ -1

Corresponding quantization values

	x_i
$(-3, -1) \longrightarrow$	-1.5
$(-1, 1) \longrightarrow$	-0.5
$(1, 3) \longrightarrow$	2.5
$(3, 5) \longrightarrow$	3.5

-1

(d) $SQR = \dfrac{E[(X-\mu)^2]}{\sigma_q^2}$

2. <u>cont'd</u>

$$\mu = E[X] = 1 \quad \text{check this;}$$

$$\therefore E[(X-\mu)^2] = E[X^2] - \mu^2$$

$$= \frac{1}{4}\left[\left[\frac{x^3}{3}\right]_{-2}^{0} + \left[\frac{x^3}{3}\right]_{2}^{4} \right] - 1$$

$$\therefore E[(X-\mu)^2] = \frac{16}{3} - 1 = \boxed{\frac{13}{3}}; \quad -\frac{1}{2}$$

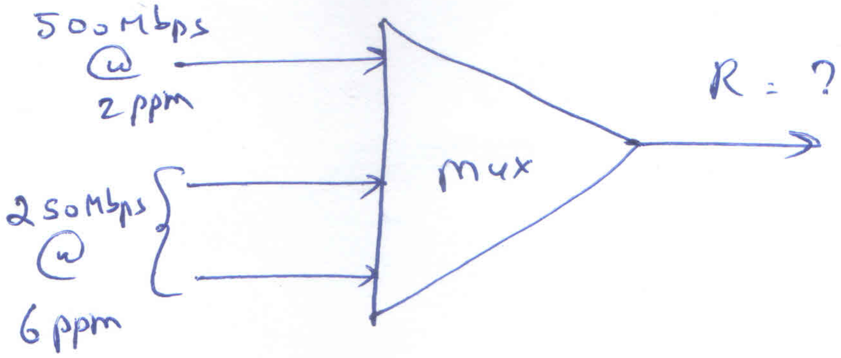
$$\sigma_q^2 = \frac{(b-a)^2}{12} \quad \text{for}$$

$$\Rightarrow \text{when } b = 1 \text{ and } = -1 \Rightarrow \sigma_q^2 = \frac{4}{12} = \boxed{\frac{1}{3}}; \quad -\frac{1}{2}$$

$$\therefore \text{SQR (in linear scale)} = \frac{13/3}{1/3} = \boxed{13}; \quad -\frac{1}{2}$$

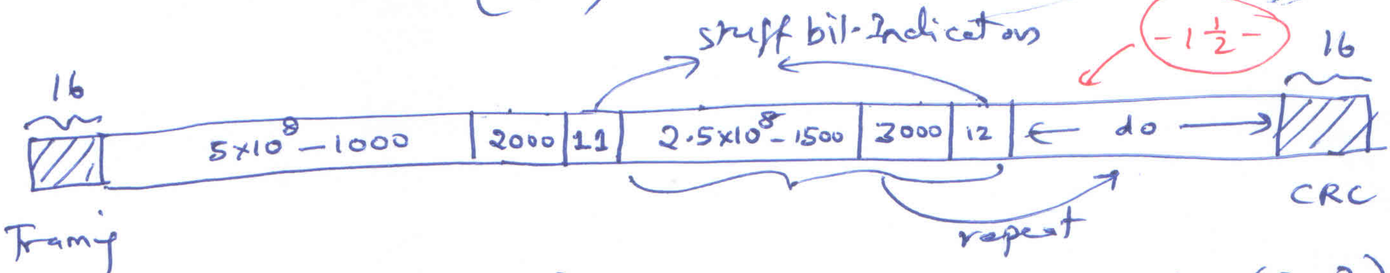
i.e. in dB scale $\text{SQR} = 11.14\ \text{dB}$;

3. [2.5 + 1.5 = 4 marks]



(a) for 1 sec frame :

- $500 \times (\pm 2) = \pm 1000$ bits
- $250 \times (\pm 6) = \pm 1500$ bits more per stream



$\Rightarrow$ output Rate $= 5\times10^8 - 1000 + 2000 + 2\times\left(2.5\times10^8 - 1500 + 3000\right) + 11 + (12\times 2) + 32$

$= \boxed{1000.004067 \text{ Mbps}}$ -1-

(b) with 500 msec frame :

- ~~500 x~~ ± 500 bits ← SB

$\Rightarrow$ SBIs is only 10

For 1 sec Frame :

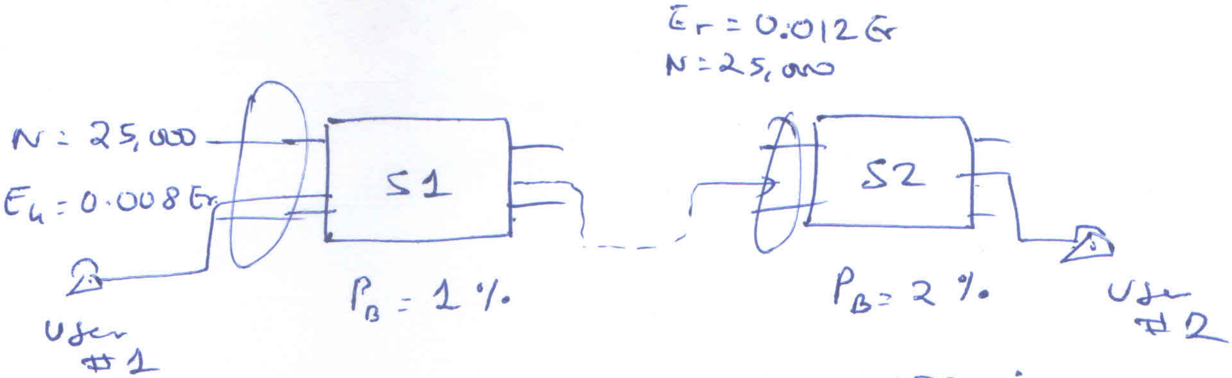
SBIs $= 11 + 12 + 12 = 35 > 32$ ← CRC + Framing

For 500 msec Frame :

SBIs $= 10 + 11 + 11 = 32$ ← acceptable -1-

$\therefore$ output frame length with maximum efficiency will be | 500 msec -0.5-

7. [1+3+2 = 6 marks]



& $m \times m = 100 \times 100$;

$\Rightarrow n = \frac{N}{m} = \boxed{250}$;

(a) $P\left[\begin{array}{c}\text{user \#1 can make} \\ \text{a call to user \#2}\end{array}\right] = (1-0.01)(1-0.02)$

$\Rightarrow P\left[\begin{array}{c}\text{\#1 cannot make} \\ \text{a call to \#2}\end{array}\right] = 1 - (0.99 \times 0.98)$

$= \boxed{0.0298}$; −1−

$(2.98 \approx 3\%)$

(b) For S1:

$q_1 = \frac{n \times 0.008}{k_1} = \frac{2}{k_1}$ $\left(\frac{1}{2}\right)$; $q_2 = \frac{n \times 0.012}{k_1} = \frac{3}{k_2}$ $\left(\frac{1}{2}\right)$

$P_{b1} = 1\% = 0.01 \underset{(\geq)}{=} (1-(1-q_1)^2)$, $q_1 = \frac{2}{k_1}$

||ly For S2:

$P_{b2} = 2\% = 0.02 \underset{(\geq)}{=} (1-(1-q_2)^2)^{k_2}$, $q_2 = \frac{3}{k_2}$

Solving we get

$k_1 = 6 \Rightarrow P_{b1} = 0.0294$

$k_1 = 7 \Rightarrow P_{b1} = 0.0067$ ✓ (< 0.01) −1−

$k_2 = \cancel{7} \Rightarrow P_{b2} = 0.0628$

$k_2 = 8 \Rightarrow P_{b2} = 0.019$ ✓ (< 0.02) −1−

$\boxed{\begin{array}{l} k_1 = 7 \\ k_2 = 8 \end{array}}$

4. [contd.]

$$(N \times R_1) \qquad k_1 \times m^2$$

$$S1 = 2 \times (\underset{25000}{\cancel{250}} \times 7) + 7 \times (100 \times 100)$$

$$= \cancel{73{,}500} \quad -1-$$

$$S2 = 2 \times (\overset{25000}{\cancel{256}} \times \overset{k_2}{8}) + k_2 \times m^2$$

$$8 \times 100^2$$

$$= \cancel{84{,}000} \quad -1-$$

$S1 \rightarrow$ is cheaper (lower complexity)