Department of Electrical Engineering, IIT Madras

End Sem -- EE6002 : Multicarrier Communications

45 Marks **May 2, 2014**

1. **[1+2+2=5 marks]** Consider a Direct Sequence (DS) CDMA system with spreading factor T/Tc = 256. For each of the following scenarios, compute the number of uplink users this system can be simultaneously supported using pseudo-noise (PN) codes. Use the uplink capacity expression discussed in class. Neglect impact of other cell interference, and voice activity detection. SINR required is 6dB for the weakest link.
(a) Under strict equi-power control, all user signals arrive with power P. What is the pole capacity?
(b) All users arrive at same power P. Noise rise allowed is 6dB. How many users can be supported then?
(c) Half the users can arrive at power 2P, and the other half remain at power P. In this case, what is the pole capacity?

2. **[2+3=5 marks]** In a DS-CDMA downlink resource allocation problem, orthogonal variable spreading factor (OVSF) codes are being used to provide different rates to a 15 users currently connected to the base-station. Recall that in the OVSF tree, R_0 represents the rate at level 0 (which is the root of the tree, where no spreading is done). Next, R_1 represents the rate of each of the two code-streams at level 1, formed by using the spreading codes {11} and {10}. Clearly $R_1 = R_0/2$, and in general, $R_{i+1} = R_i/2$ where i+1 is one level lower than level i. Given that among the 15 users, 1 user is at rate R_2, 1 user is at R_3, 6 users at R_4, and 7 users are at R_5, answer the following:
(a) What is the sum rate supported across the 15 users? Express your answer as a function of R_0.
(b) Sketch the OVSF code tree up to level 5, and indicate how the above 15 users are allotted resources (codes) from this tree. Circle or tick the selected codes in the code tree.

3. **[1.5+1.5+3+2+2=10 marks]** A 10.24MHz OFDM system with FFT size of N=256 is to be designed. The sampling rate fs=10.24MHz, and the cyclic prefix (CP) length is taken to be $1/5^{th}$ of the useful OFDM symbol duration. For spectral shaping and for ensuring zero-DC, 16 of these 256 subcarriers are modulated with "zeros".
(a) What is the largest delay spread (in μsecs) that the system can handle without experiencing IBI?
(b) If a 20 symbol frame is constructed with one preamble symbol followed by 19 data-carrying OFDM symbols, what is the frame rate of the system?
(c) The link can have a maximum frequency error of 90KHz. Design a preamble such that it will allow the Schmidl-Cox Algorithm (SCA) to estimate the entire frequency error. Ensure that the number of subcarriers used in the preamble is maximized under the above constraint so as to enable the signaling of many base-station IDs using these preambles. Make a neat sketch of the preamble symbol (in the frequency domain), indicating the spacing, and the first and last indices (which lie between subcarrier index +128 and -127). How many subcarriers are used by the preamble?
(d) How much averaging is possible in the SCA given your design in part (c)? Give the expression, and also the total number of samples that can be used in this averaging. Assume uncorrelated noise samples.
(e) If every data-carrying OFDM symbol has 8% pilot overhead, and all the subcarriers carry QPSK symbols, what is the spectral efficiency of this OFDM system in bits/sec/Hz?

4. [1+1+1.5+1.5=5 marks] Consider an OFDM system with N=1024 subcarriers, where each symbol has 120 equi-spaced pilot subcarriers. The CP of the system is N_{CP}=60 samples, where the sampling rate is the reciprocal of the band-width. Recalling that the order of complexity of a N-point FFT is NlogN, and that of (cleverly) inverting a NxN matrix is N^2logN, answer the following:

(a) What is the complexity of an FFT-based channel estimation algorithm?

(b) What is the complexity of a modified Least Squares (mLS) algorithm?

(c) If the receiver knows the sample-spaced delay locations of the channel, and there are only 6 such multipath components, what will be the new complexity of the mLS algorithm? Compare your answers in (a), (b), and (c) and comment.

(d) Can you predict the impact this information of the delay localitions would have on the mean square error (MSE) of the mLS in (c) ? Express this gain (or loss) in the dB scale with respect to the mLS in (b).

5. [2+3+4+1=10 marks] A 8-point FFT is used to define an OFDM system which can use P = 1.5 Joules per symbol. After excluding the zero-tones for n=4 and -3, and the DC subcarrier at n=0, we are left with 5 tones over which data can be sent. The magnitude response of the channel is known to the transmitter, and is as follows: |H(3)| = 0.5; |H(2)| = 0.8; |H(1)| = 0.1; |H(-1)| = 0.5; and |H(-2)| = 0.4. Find the rate R (in number of bits/sec) that can be supported by each of the following 3 different power allocation schemes given that the noise variance in every sub-carrier is No=0.1.

(a) Uniform power allocation – R_{UNI}

(b) Zero-forcing power allocation where the power allocated on the n^{th} subcarrier is proportional to $\kappa/|H(n)|^2$ where is κ the factor which ensures that the total power does not exceed P per symbol -- R_{ZF}

(c) Water-pouring based power allocation giving the capacity – C

(d) Compare R_{UNI}, R_{ZF}, and C and comment. For what condition(s) will all the 3 schemes give an identical allocation result?

6. [5 marks] A generalized multicarrier system with low PAPR is defined by using the F-DOSS type symbol repetition scheme followed by appropriate frequency shifting in order to build an uplink multiple access system. Given that the total number of subcarriers is 512, and there are 4 active uplink users (labelled as user #1, #2, #3 and #4) requiring respective 50%, 25%, 12.5% and 12.5% of the resources, draw the transmit side block diagram. Label the diagram indicating the number of repetitions and the frequency shifting required for each user.

7. [1.5+1.5+2=5 marks] Consider a multiuser OFDM (OFDMA) downlink with two transmit antennas. Now, 2 mobile users are to be served such that user-A consumes the top 50% of the sub-carriers, while user-B is served on the bottom 50% of the subcarriers.

(a) Given that user-A is being served using closed-loop beam-forming where H_1[k,n] and H_2[k,n] are the channel gains available at the transmitter for imposing on the n^{th} subcarrier during the k^{th} OFDM symbol, draw a block diagram of the transmit chain from the FEC output to the antenna output.

(b) On the other hand, user-B is being served using open-loop spatial multiplexing where 2 different symbols are sent to the user using the 2 antennas. Add the block diagram for this user also to your figure in part (a). Assume the channel gains to be G_1[k,n] and G_2[k,n] in this case.

(c) Assuming that both the mobile stations have a single antenna only, write down the measurement equation after the FFT at the receiver on the n^{th} subcarrier, both for user-A and for user-B. Assume the noise variable to be v[k,n].

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