Department of Electrical Engineering, IIT Madras

EE5141 : Fundamentals of Wireless and Cellular Communications

Tutorial #2 **Nov. 2014**

1. From D.Tse & P.Viswanath Chapter-2: (i) 2.2 (ii) 2.3 (iii) 2.4 (iv) 2.5 (v) 2.10 (vi) 2.14 (vii) 2.15* (viii) 2.16, and (ix) 2.17* (questions with "*" are a bit tougher, and are optional)

2. A communication link between a fixed base-station and a mobile-station uses a bandwidth of 20MHz at a carrier frequency of f_c=1.5GHz. The mobile which is moving at 50kmph, experiences a delay-spread of 4μsec. Find the following:

(a) What is the Doppler spread f_D in Hz? What is the coherence time?

(b) What is the normalized Doppler spread $f_D T$ (where T is the symbol duration)?

(c) What is the coherence band-width?

(d) At that mobile speed will be $f_D T$=0.001 ?

3. Recall that in the derivation of the power spectral density S(f) for the Clarke's model, the fraction of the power p(α) reaching the receiver in angle α is assumed to be uniform. Suppose instead we have a directional antenna with $p(\alpha) = \beta(1 + Cos(\alpha)), \alpha \epsilon [0,2\pi]$, with β appropriately chosen. What will be the modified expression for S(f) then?

4. From D.Tse & P.Viswanath Chapter-5: (i) 5.1 (ii) 5.2 (iii) 5.3 (iv) 5.6 (v) 5.10 (vi) 5.11 (vii) 5.13*

5. Consider a 1xN_R SIMO link with zero-mean, Rayleigh fading channel gains which are frequency flat. Let the complex channel gains be given by h_i, i=1,2,…,N_R, where $E[|h_i|^2]=\sigma^2$ for all i. Also, since they are i.i.d, we also have $E[h_i^* h_j]=0$, for j≠i. Given the measurement model **y**(k) = **h** $\sqrt{E}$s(k) + **n**(k), where **h** = $[h_1\, h_2, \dots, h_{N_R}]$, with $E[|s(k)|^2]=1$, and the transmit signal energy is E Joules. Also, the additive noise vector **n**(k) is white, Gaussian, and mutually uncorrelated with the signal with $E[|n_i|^2]=\sigma_n^2$ for all i, answer the following:

(a) Show that the expression for the *average* SNR when maximal ratio combining is done on **y**(k) is given by $SNR_{SIMO} = N_R\, E\, \sigma^2/\sigma_n^2$.

(b) Assuming Gaussian signaling, what is the expression for the capacity C (in bits/sec/Hz) for this SIMO system?

(c) By how much should N_R increase in order to double the value of C ?

6. Repeat Q**5.** part (a) for a N_T x1 MISO link when maximal ratio transmission is employed, where each transmit antenna puts out E/N_T Joules. Among these 2 schemes, which is preferable in general and why?

7. Consider the Alamouti space-time block code described below for a 2x1 MISO link:

(i) The received vector, constructed over 2 consecutive symbol intervals, is given by

$$\begin{bmatrix} y(k) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{bmatrix} \begin{bmatrix} h \\ g \end{bmatrix} + \begin{bmatrix} n(k) \\ n(k+1) \end{bmatrix}$$

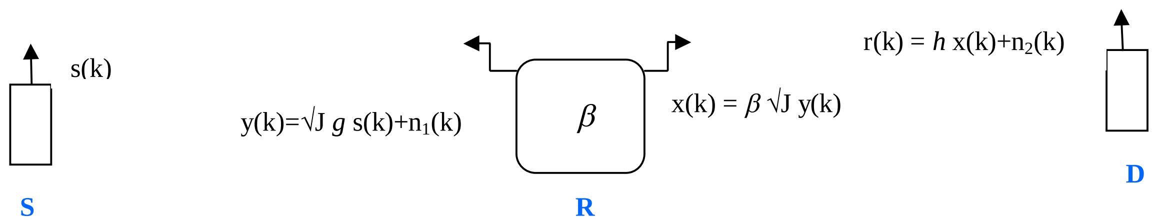
(ii) After conjugating the 2nd measurement, the above can be re-written as

$$\begin{bmatrix} y(k) \\ y^*(k+1) \end{bmatrix} = \begin{bmatrix} h & g \\ -g^* & h^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n(k) \\ n^*(k+1) \end{bmatrix} = \mathbf{P} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n(k) \\ n^*(k+1) \end{bmatrix}$$

(iii) Noticing that the 2x2 channel matrix **P** in the above is Unitary, a simple linear operation to separate the 2 transmit symbols is given by $\mathbf{P}^H$, which operates on $[y(k) \;\; y^*(k+1)]^T$ from the left.

Derive the SNR expression for this block coding scheme.

8. Consider a 2-phase relay-based link between a source node **S** to destination node **D**, through a relay node **R**. In phase 1, **S** sends a symbol s(k) to **R**. In phase 2, **R** sends (after some processing) the symbol to **D**. Let the SISO S-to-R link have channel gain g, and the SISO R-to-D link have gain h. The energy per symbol from each transmit node is limited to J Joules. Assume that knowledge of the channel gains is available at **R**, and that the AWGN noise terms $n_1(k)$ and $n_2(k)$ are uncorrelated and with equal variance $E[|n_i|^2]=\sigma_n^2$, for i=1,2. All the channel, signal, and noise terms are also mutually uncorrelated, and at the symbol level $E[|s(k)|^2]=1$.



(a) Determine the real scaling term β to ensure that the transmit energy constraint is met at **R**.

(b) Show that the SNR at **D** is given by

$$SNR = \frac{J^2 E[|g|^2] \; E[\,|h|^2]}{(J\, E[|h|^2] + JE[|g|^2] + \sigma_n^2)\, \sigma_n^2}$$

(c) If $E[|g|^2] \gg E[|h|^2]$ and both are large when compared to σ_n^2 , find the approximate expression for received SNR at **R**. For what relationship between h and g will the SNR be maximized? Explain.

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