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Re:	Call for Comments on Draft 802.16m Evaluation Methodology Document
Abstract	In this contribution, a flexible Doppler behavior model is proposed. The model is obtained by approximating both the directional antenna pattern and the angular distribution of non-isotropic scattering environments by a mixture of von Mises distributions. The proposed model has the closed-form expressions for both spatial/temporal correlations and spectrums, a feature highly desired in real-time correlation-based simulations. It also includes the classical Jakes' model as a special case. In addition, the existing Rayleigh faders can be still used with only slight modifications.
Purpose	For discussion and approval by TGm
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A Flexible Doppler Behavior Model

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1. Introduction

The Doppler behavior depends on the power azimuth spectrum (PAS) [8], which includes the combined effects of both *antenna patterns* and *physical propagation environments*.[1] In the current Draft 802.16m Evaluation Methodology Document [1], the Laplace distribution is used to characterize the PAS in the correlation-based method. In WINNER model [11], the wrapped Gaussian is used to model *physical propagation environments*. In addition, the uniform PAS is recommended as an option to reduce the simulation run time (i.e., the classical Jakes' bathtub-shaped spectrum is used to characterize the Doppler behavior), which, in most cases, assumes isotropic scattering environments and omni directional antennas [2], i.e., the power density is constant over the azimuth angle, $(-\pi, \pi]$.

However, it has been argued [3][5], and experimentally demonstrated [4][5] that the scattering encountered in many environments is non-isotropic. Some typical non-isotropic scattering environments are shown in Fig. 1 [6], and Fig. 4 of [12] (elliptical scattering model).
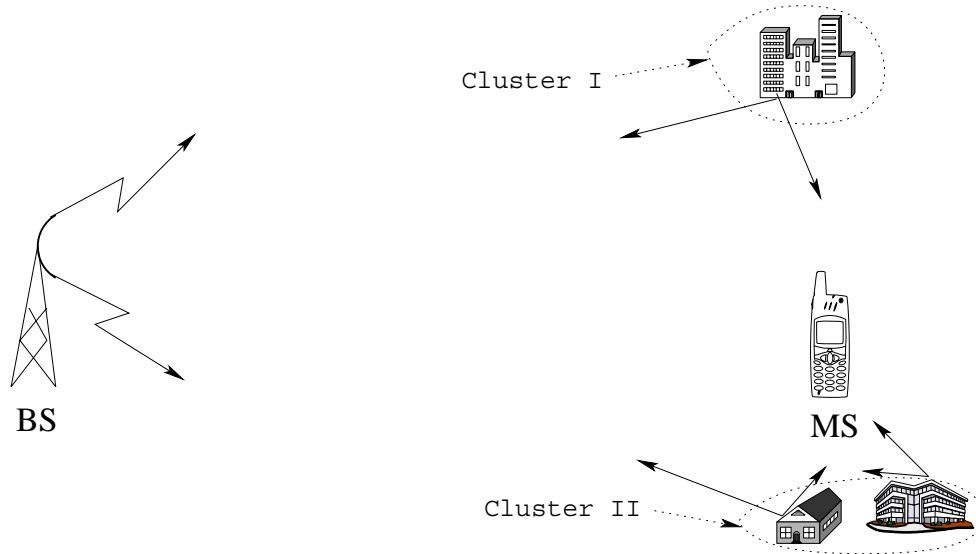
Fig. 1: A typical non-isotropic scattering environment

In addition, the directional antenna will make the received power density non-uniform even in the isotropic scattering environment. One example of the antenna pattern of a direction antenna is shown in Fig. 2 [1].

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[1] In the following text, we use "antenna pattern" for the effect of the antenna radiation pattern, "angular distribution" for the effect of the physical radio propagation scattering environment in the tap/path level, and "PAS" for the combined effect of both antenna pattern and angular distribution in the tap/path level.

Fig. 2: Antenna pattern for 3-sector cells at BS side

Moreover, as bandwidth increases, the Doppler characteristics will not have the classic "bathtub" shape, as stated in [1].

Based on the above description, we observed the following potential problems in the current draft, which may affect the future evaluations of considered techniques.

- In Jakes' model, the maximum Doppler frequency, f_m, fully dictates the channel dynamics. It implies that the larger the f_m, the faster the chann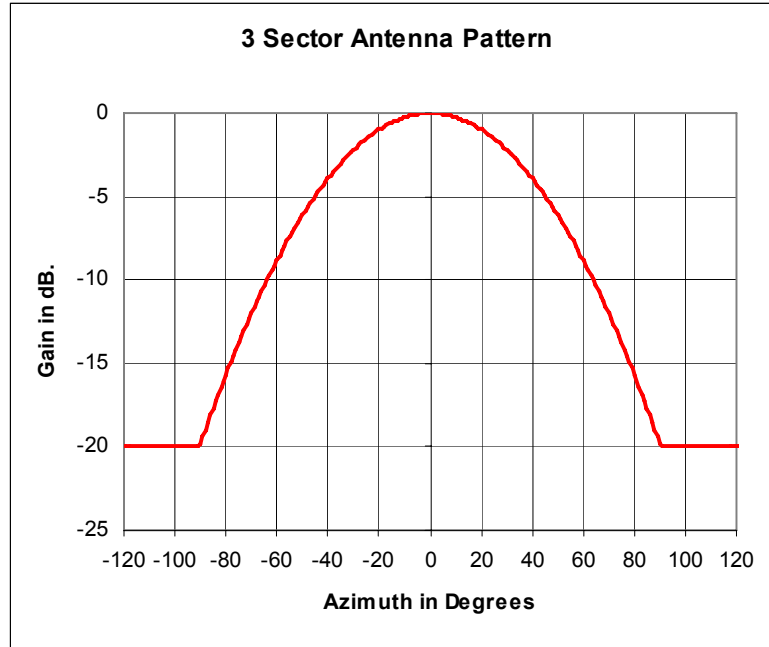el varies, i.e., the shorter the coherence time. However, in the non-uniform PAS scenario, large f_m's do not imply the fast variation of the channel. For example, as shown in Fig. 4, the coherence time of the real-world scenario is much longer than that predicted by the Jakes' model. This may have important impacts on the feedback signaling design.
- In the ray-based method, the defined antenna pattern (r.f. Fig. 2) and wrapped Gaussian angular distribution (r.f. [11]) will not lead to the Laplace PAS, which is assumed in the correlation-based method. Obviously, this will introduce some inconsistence between two implementation methods.

To remedy the above problems, a flexible Doppler spectrum model is proposed in this contribution. We use $h_{m_r,m_t,l}(t)$ to represent the time-varying channel coefficient of the lth path between the m_tth Tx antenna and m_rth Rx antenna, which includes the LOS (line-of-sight) component. We assume the Rician factor to be $K_{m_r,m_t,l}$ and the path power to be $P_{m_r,m_t,l}$, then $h_{m_r,m_t,l}(t)$ can be written as

$$h_{m_r,m_t,l}(t) = \sqrt{\frac{K_{m_r,m_t,l}P_{m_r,m_t,l}}{K_{m_r,m_t,l}+1}}e^{j\varphi_{m_r,m_t,l}} + \sqrt{\frac{P_{m_r,m_t,l}}{K_{m_r,m_t,l}+1}}g_{m_r,m_t,l}(t), \qquad (1)$$

where $\varphi_{m_r,m_t,l}$ is the phase of the LOS component, and $g_{m_r,m_t,l}(t)$ is a circular complex Gaussian random process

with unit power. Since the LOS component (the first item in (1)) is not random and very easy to generate, we consider the Doppler behavior of $g_{m_r,m_t,l}(t)$ only in this contribution. To simplify the notation, we drop off the subscript $_{m_r,m_t,l}$ in $g_{m_r,m_t,l}(t)$. Note that all the incidence angles are relative to the mobile moving direction in this contribution.

2. PAS Model

As stated in the introduction, the Doppler behavior depends on the PAS model. In the current draft [1], the Laplace distribution is used to characterize the PAS. Except for the possible problems described in the above, furthermore, it has the following limitations.

    a. It can not be reduced to the uniform distribution.

    b. It takes the value over (-∞, ∞) instead of (-π, π]. Therefore, it may lack the physical meaning of the underlying angles.

    c. It will not give the closed-form expressions for both spatial and temporal correlations of the channel. To calculate the *spatial* correlation value, we need to resort the numerical integration. As stated in [1], the integration depends on the mean angle of arrival (AoA) and angle of departure (AoD), which makes the channel generation difficult in real time.

If the wrapped Gaussian distribution from the WINNER model is used in the correlation-based method, the above 1 and 3 still apply.

The von Mises distribution (also known as the circular normal distribution) plays a prominent role in statistical modeling and analysis of angular variables [7]. Compared to the Laplace distribution, the von Mises has the following advantages.

    a. It includes the uniform distribution as a special case.

    b. It takes the value over (-π, π] naturally.

    c. It provides closed-form and mathematically-tractable expressions for the channel correlation function and its spectrum, which is highly desired in the real-time correlation-based simulation. In addition, since it includes the uniform distribution, the correlation and spectrum includes the Jakes' model as a special case. It implies that we can include the Jakes' spectrum as an option by changing the underlying parameters only.

We assume the unnormalized PAS to be $q(\theta), \theta \in (-\pi, \pi]$, which includes the combined effect of antenna pattern and angular distribution. The total power of the NLOS component associated with the lth path between the m_tth Tx antenna and m_rth Rx antenna is $\dfrac{P_{m_r,m_t,l}}{K_{m_r,m_t,l}+1} = \displaystyle\int_{-\pi}^{\pi} q(\theta)d\theta$. Therefore, the normalized PAS is given by $p(\theta) = q(\theta) / \displaystyle\int_{-\pi}^{\pi} q(\theta)d\theta$, which is a probability density function (PDF) of the azimuth angle.

As stated in [6], $p(\theta)$ can be written as a mixture of N von Mises distributions, where each of them characterizes one sub-cluster of scatterers.[2]

$$p(\theta) = \sum_{n=1}^{N} p_n \frac{\exp[\kappa_n \cos(\theta - \phi_n)]}{2\pi I_0(\kappa_n)}, \quad \theta \in (-\pi, \pi], \tag{2}$$

where $I_0(z)$ is the zero-th order modified Bessel function of the first kind, ϕ_n is the mean angle of the nth sub-

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[2] Each tap/path corresponds to a cluster, and each cluster may contain several sub-clusters.

cluster, κ_n controls the angle spread of the nth sub-cluster, and p_n represents the contribution of the nth sub-cluster, such that $\sum_{n=1}^{N} p_n = 1, 0 < p_n < 1.$ If $\kappa_n = 0, \forall n,$ the above distribution reduces to

$$p(\theta) = 1/(2\pi), \theta \in (-\pi, \pi], \tag{3}$$

which is the Clarke's 2-D isotropic scattering model. It is easy to show that, if both antenna gain, $G(\theta)$, and angular distribution have a mixture of von Mises in (2), the normalized PAS, $p(\theta)$, will still have a mixture of von Mises. This observation makes it possible for the flexible Doppler model.

We consider the basic case where $N = 1$. If the normalized antenna pattern has the von Mises with ϕ_{ant} and κ_{ant}, and the angular distribution has the von Mises with ϕ_{env} and κ_{env}, the normalized PAS will have the von Mises with $\phi = \arctan\left(\kappa_{\text{env}} \sin\phi_{\text{env}} + \kappa_{\text{ant}} \sin\phi_{\text{ant}}, \kappa_{\text{env}} \cos\phi_{\text{env}} + \kappa_{\text{ant}} \cos\phi_{\text{ant}}\right)$, where $\phi = \arctan(y, x)$ is the four quadrant arctangent,[3] and $\kappa = \sqrt{\kappa_{\text{env}}^2 + \kappa_{\text{ant}}^2 + 2\kappa_{\text{env}}\kappa_{\text{ant}} \cos(\phi_{\text{env}} - \phi_{\text{ant}})}$.

For example, the antenna pattern in Fig. 2 can be well approximated by the von Mises with $\phi_{\text{ant}} = \pi/4$ and $\kappa_{\text{ant}} = 3.91625$ if the antenna boresight has $\pi/4$ offset to the LOS direction. For the physical scattering environment, we consider the angular PDF in Fig. 6(a) of [12], which can be well approximated by a mixture of two von Mises with $\phi_{\text{env}} = [0, 0]$, $\kappa_{\text{env}} = [5, 1.1]$ and $p_{\text{env}} = [0.45, 0.55]$. Finally, the normalized PAS has a mixture of von Mises with $\phi = [0.3424, 0.6212]$, $\kappa = [8.2480, 4.7581]$ and $p = [0.4945, 0.5055]$. In addition, to see the effect of the approximation errors, we also approximate the angular PDF by one von Mises with $\phi_{\text{env}} = 0$, $\kappa_{\text{env}} = 2.3$. Fig. 3 shows the results, where "Ant. Pat. (exact)" denotes the actual antenna pattern, "Ant. Pat. (mixture, …)" is the von Mises approximation, "Scat. Env. (exact)" denotes the actual angular distribution, "Scat. Env. (mixture, …)" is the von Mises approximation, and "PAS (mixture, …)" is the combined effect of approximated antenna pattern and angular distribution. All the curves are normalized.
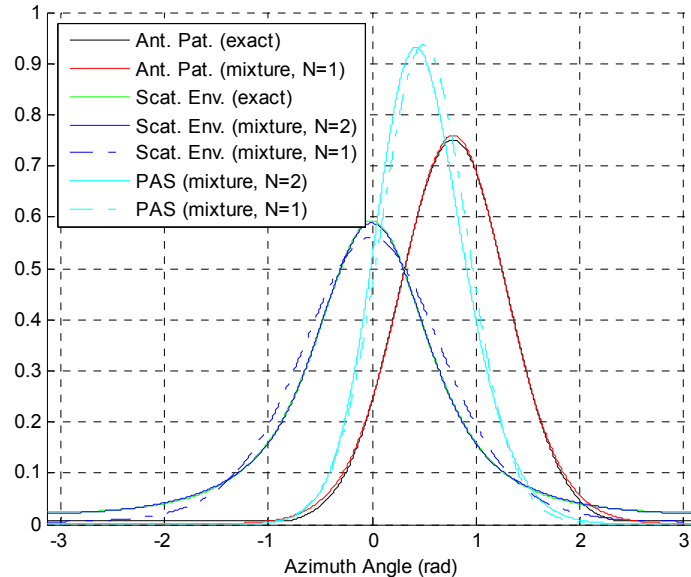

Fig. 3: A PAS example with directional antenna pattern and non-isotropic scattering environment.

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[3] In Matlab, it can be calculated by $\text{atan2}(y, x)$.

3. Channel Temporal Autocorrelation and its Spectrum

With the PAS model in (2), the normalized autocorrelation and the spectrum of the channel, $g(t)$, are respectively given by [6]

$$r_g(\tau) = \mathbb{E}\left[g(t)g^*(t-\tau)\right] = \mathbb{E}_\Theta\left[\exp(j2\pi f_m \tau \cos\Theta)\right]$$

$$= \sum_{n=1}^{N} p_n \frac{I_0\left(\sqrt{\kappa_n^2 - 4\pi^2 f_m^2 \tau^2 + j4\pi\kappa_n f_m \tau \cos\phi_n}\right)}{I_0(\kappa_n)}, \tag{4}$$

And

$$S_g(f) = \mathfrak{F}\left[r_g(\tau)\right] = \sum_{n=1}^{N} p_n \frac{\exp\left(\kappa_n f \cos\phi_n / f_m\right)\cosh\left(\kappa_n\sqrt{1-(f/f_m)^2}\sin\phi_n\right)}{\pi\sqrt{f_m^2 - f^2}\,I_0(\kappa_n)}, \qquad |f| \le f_m, \tag{5}$$

where $\mathfrak{F}[\cdot]$ denotes the Fourier transform, $\cosh(x) = (e^x + e^{-x})/2$ is the hyperbolic cosine function. In addition, $f_m = v/\lambda = vf_c/c$ is the maximum Doppler frequency shift in Hertz, v is the moving speed of the mobile station, λ is the wavelength, f_c is the carrier frequency, and c is the speed of light in a vacuum. If $\kappa_n = 0, \forall n$, i.e., the 2-D isotropic scattering, (4) and (5) reduce to the classical Jakes' correlation and Doppler models, given by

$$r_g(\tau) = J_0\left(2\pi f_m \tau\right), \tag{6}$$

and

$$S_g(f) = \frac{1}{\pi\sqrt{f_m^2 - f^2}}, \qquad |f| \le f_m, \tag{7}$$

We call Eq. (5) the *von Mises' Doppler spectrum*, and the model governed by (4) and (5) the *von Mises' model*.

If the bandwidth is large, each cluster only has one sub-cluster. Therefore, $N=1$ is enough to characterize the PAS. This reduces, respectively, (2), (4) and (5) to

$$p(\theta) = \frac{\exp[\kappa\cos(\theta - \phi)]}{2\pi I_0(\kappa)}, \qquad \theta \in (-\pi, \pi], \tag{8}$$

$$r_g(\tau) = \frac{I_0\left(\sqrt{\kappa^2 - 4\pi^2 f_m^2 \tau^2 + j4\pi\kappa f_m \tau \cos\phi}\right)}{I_0(\kappa)}, \tag{9}$$

and

$$S_g(f) = \frac{\exp\left(\kappa f \cos\phi / f_m\right)\cosh\left(\kappa\sqrt{1-(f/f_m)^2}\sin\phi\right)}{\pi\sqrt{f_m^2 - f^2}\,I_0(\kappa)}, \qquad |f| \le f_m. \tag{10}$$

Figs. 4 and 5 compare the Jakes' model and one real-world scenario in terms of temporal correlation and Doppler spectrum, respectively. From Figs. 4 and 5, we can conclude that the Jakes' model is far away from the real-world, and may not describe the non-uniform PAS environments. However, the proposed Doppler model can describe most practical PASs, including the uniform one (corresponding to the Jakes' model). In addition, according to Figs. 4 and 5, the difference between two approximations in both correlation and spectrum is marginal, although Fig. 3 shows obvious gaps, i.e., both correlation and spectrum are immune to small approximation errors. This further demonstrates that $N=1$ is good enough to characterize the PAS in practice.
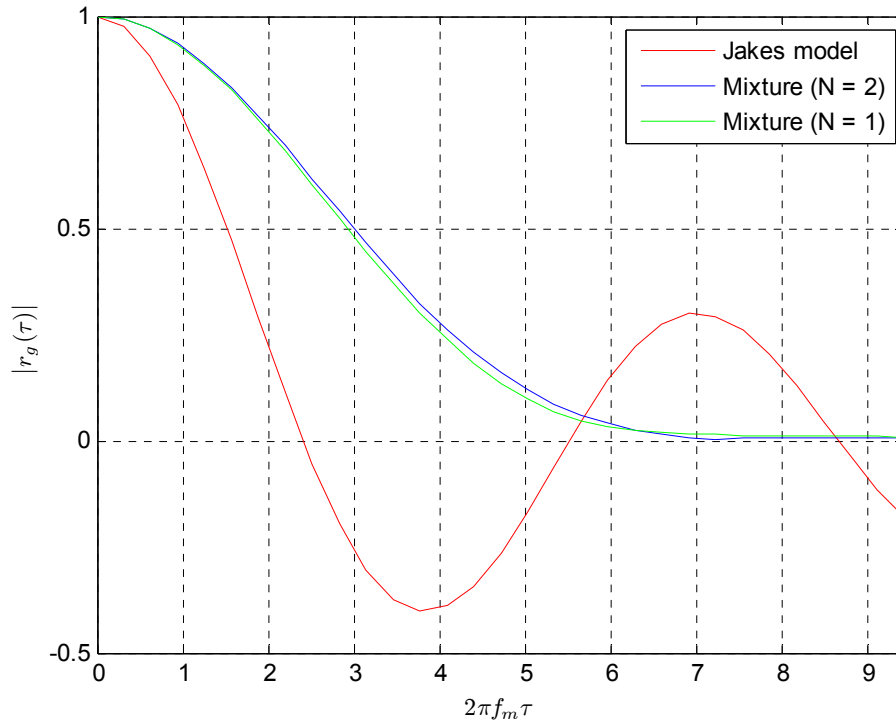
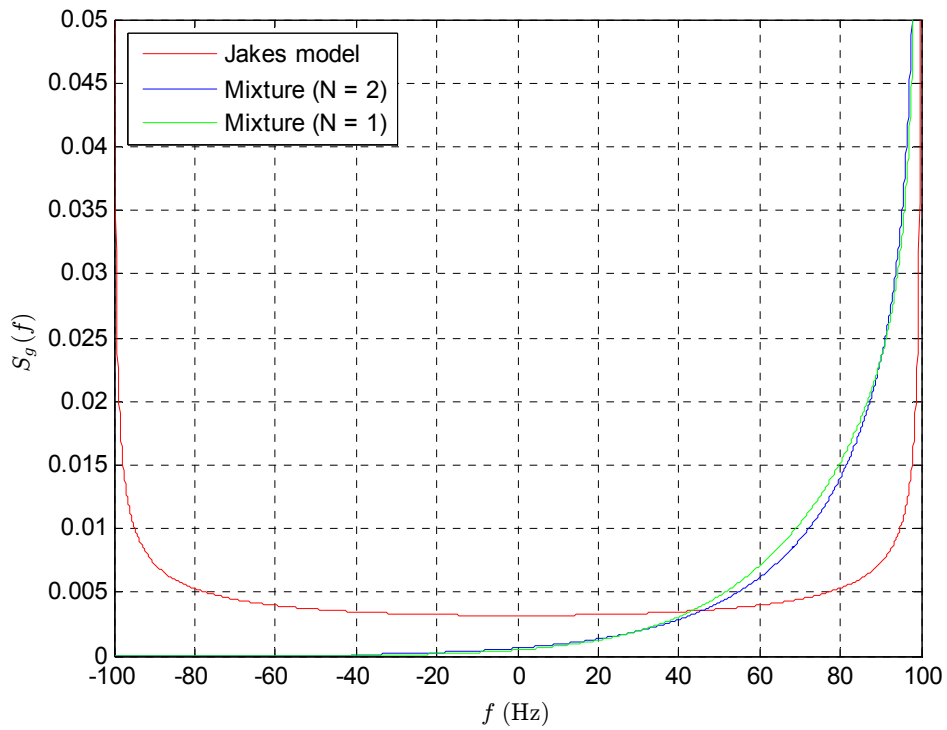Fig. 4: Temporal correlation of the example in Fig. 3.

Fig. 5: Doppler spectrum of the example in Fig. 3.

With the von Mises distribution, the spatial correlations in [1] can be easily evaluated in the closed form without the numerical difficulties described in [1]. For example, the spatial correlation between the pth and qth antennas at the BS is given by

$$r_{n,\text{BS}}(p,q) = \int_{-\pi}^{\pi} p(\theta)\exp\left[j\frac{2\pi d_{\text{BS}}}{\lambda}(p-q)\sin(\text{AoD}_n + \theta)\right]d\theta = \frac{I_0\left(\sqrt{\kappa_n^2 - d_{pq}^2 + j2\kappa_n d_{pq}\sin\text{AoD}_n}\right)}{I_0(\kappa_n)}, \qquad (11)$$

where $d_{pq} = 2\pi d_{\text{BS}}(p-q)/\lambda$ and $\kappa_n \approx \text{AS}_{\text{BS,path}}^{-2}$ for small angle spreads[4] [5]. Note that both AoD_n and $\text{AS}_{\text{BS,path}}$ are in radian. Eq. (11) is also valid for the spatial correlation at the MS by replacing corresponding parameters. It is clear to see that it will be much easier to generate the spatial correlation in the real-time simulation using the closed-form expression (11) instead of the numerical integration.

4. Modification in Existing Faders

It is straightforward to update some existing Rayleigh fading generators with the von Mises' model, i.e., replacing the Jakes' model with the von Mises' model. Specifically,

- o If the classic correlation, $J_0(\cdot)$ in (6), is used to generate the Rayleigh fading channel, we replace it with the new correlation function shown in (4).
- o If the classic spectrum in (7) is used, we replace it with the new spectrum shown in (5).
- o If the sum-of-sinusoid (SoS) method is used, we replace the uniform distribution in (3) with the von Mises distribution in (2). The generation of von Mises random variables is discussed in [7] and [10].

As stated in Section 3, $N=1$ is enough to characterize the PAS in most practical cases. Therefore, (8)-(10) can be used instead of (2), (4) and (5).

5. Conclusions

In this contribution, we proposed a flexible Doppler behavior model. It can remedy the potential problems in the current draft. In addition, it includes the classical Jakes' model as a special case, and gives the *closed-form* expressions for both the autocorrelation and spectrum of the channel, which is highly desired in the real-time simulation. In addition, it only has *very minor* updates in the existing Rayleigh fading generators. Therefore, *we propose that the von Mises' model be used to characterize the Doppler behavior for the evaluation methodology document, and von Mises distribution be used to model the AoA and AoD instead of the Laplace for the evaluation methodology document*.

6. References

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[4] The exact value can be calculated according to $\kappa = \sqrt{\kappa_{\text{env}}^2 + \kappa_{\text{ant}}^2 + 2\kappa_{\text{env}}\kappa_{\text{ant}}\cos(\phi_{\text{env}} - \phi_{\text{ant}})}$, as shown in Section 2.

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