Improved Models for the Generation of Multiple Uncorrelated Rayleigh Fading Waveforms

Yahong R. Zheng and Chengshan Xiao, *Member, IEEE*

***Abstract*—In this letter, an improved sum-of-sinusoids simulation model is proposed for Rayleigh fading channels. The new model employs random initial phase, and conditional random Doppler frequency for all individual sinusoids. The second-order statistics of the new simulator match the desired ones exactly even if the number of sinusoids is a single-digit integer. Other key statistics of the new simulator approach the desired ones of Clarke's reference model as the number of sinusoids approaches infinity, while good convergence is achieved when the number of sinusoids is small. Moreover, the new simulator can be directly used to generate multiple uncorrelated fading waveforms; it is also pointed out that a class of 16 different simulators, which have identical statistical properties, can be developed for Rayleigh fading channels.**

***Index Terms*—Channel simulator, fading channels, multipath channels, Rayleigh fading, second-order statistics.**

I. Introduction

In the last three decades, there are many different approaches to the modeling and simulation of mobile radio channels, see [1]–[8] and the references therein. Among them, the well-known mathematical reference model due to Clarke [1] and its simplified simulation model due to Jakes [2] have been widely used for Rayleigh fading channels for about 30 years. However, Jakes' simulator is a deterministic model, and it has difficulty to create multiple uncorrelated fading waveforms for frequency selective fading channels and multiple-input multiple-output (MIMO) channels, therefore different modifications of Jakes' simulator have been reported in the literature [3]–[6]. Despite the extensive acceptance and application of Jakes' simulator, some important limitations of the simulator were determined and discussed in detail recently [8]. It was shown in [8] that Jakes' simulator is wide-sense nonstationary when averaged across the physical ensemble of fading channels. Therefore, an improved simulator was proposed by Pop and Beaulieu in [8] to remove the stationarity problem by introducing random phase shifts in the low-frequency oscillators. However, it was pointed out in [8] that higher order statistics of this improved simulator may not match the desired ones of Clarke's reference model. Consistent with Pop and Beaulieu's caution about higher order statistics of the improved simulator, it was further proved in [9] that second-order statistics of the quadrature components and the envelope do not match the desired ones. Moreover, even in the limit as the number of sinusoids approaches infinity, the autocorrelations and cross-correlations of the quadrature components, and the autocorrelation of the squared envelope of the improved simulator fail to match the desired correlation statistics. Similar drawbacks on these second-order statistics are also retained by Jakes' original simulator and its existing modifications [3]–[6].

Manuscript received January 15, 2002 The associate editor coordinating the review of this letter and approving it for publication was Dr. C.-M. Chang. This work was supported in part by the University of Missouri-Columbia Research Council under Grant URC-02-050.

Y. R. Zheng is with the Department of System and Computer Engineering, Carleton University, Ottawa, ON K1S 5B6, Canada (e-mail: yzheng@sce.carleton.ca).

C. Xiao is with the Department of Electrical and Computer Engineering, University of Missouri-Columbia, Columbia, MO 65211 USA (e-mail: xiaoc@missouri.edu).

Publisher Item Identifier S 1089-7798(02)05800-3.

In this letter, a class of new sum-of-sinusoids statistical simulation models is proposed for Rayleigh fading channels which may contain multiple uncorrelated faders. It is shown that the new Rayleigh fading channel models have the desired statistical properties even when the number of sinusoids is small.

II. New Simulation Models

Consider the frequency nonselective fading process of Clarke's reference model given by [1], [2]

$$g(t)=\sqrt{\frac{2}{N}}\sum_{n=1}^{N}\exp[j(w_d t\cos\alpha_n+\phi_n)] \tag{1}$$

where α_n and ϕ_n are, respectively, the angle of incoming wave and initial phase associated with the nth propagation path, w_d is the maximum angular Doppler frequency occurring when $\alpha_n = 0$.

For large N, the central limit theorem justifies that the quadrature components of $g(t)$ can be approximated as Gaussian random processes, whose statistical properties can be completely described by first-order and second-order statistics. Assuming that α_n and ϕ_n are mutually independent and uniformly distributed on $[-\pi,\pi)$ for all n, and adopting Clarke's 2-D isotropic scattering model, the first-order and second-order statistics of this reference model have been discussed in details in [1], [7] when N approaches infinity.

Based on the observation of [9, eqs. (5a)–(5f)], and by removing the aforementioned statistic problems of the improved Jakes' model [8] along with the original Jakes' model [2] and its modifications [3]–[6], we choose $N=4M, \phi_{n+M}=-\varphi_n+(\pi/2), \phi_{n+2M}=-\phi_n, \phi_{n+3M}=\varphi_n+(\pi/2)$, and $\alpha_n=(2\pi n-\pi+\theta)/N$ with θ being a random variable uniformly distributed on $[-\pi,\pi)$. After some algebraic manipulations, the complex-valued low-pass Rayleigh fading process $g(t)$ can be expressed by

$$g(t) = \sqrt{\frac{2}{N}} \left\{ \sum_{n=1}^{M} 2\cos(w_d t \cos\alpha_n + \phi_n) + j\sum_{n=1}^{M} 2\cos(w_d t \sin\alpha_n + \varphi_n) \right\}. \quad (2)$$

It should be pointed out that the choices of α_n and ϕ_n are not unique, however, different choices will lead to different results against (2). Based on (2), we can define a new simulation model as follows.

Definition: The normalized low-pass fading process of a new statistical sum-of-sinusoids simulation model is defined by

$$Z(t) = Z_c(t) + jZ_s(t) \quad (3a)$$

$$Z_c(t) = \sqrt{\frac{2}{M}} \sum_{n=1}^{M} \cos(w_d t \cos\alpha_n + \phi_n) \quad (3b)$$

$$Z_s(t) = \sqrt{\frac{2}{M}} \sum_{n=1}^{M} \cos(w_d t \sin\alpha_n + \varphi_n) \quad (3c)$$

with

$$\alpha_n = \frac{2\pi n - \pi + \theta}{4M}, \qquad n = 1, 2, \ldots, M \quad (4)$$

where ϕ_n, φ_n and θ are statistically independent and uniformly distributed on $[-\pi, \pi)$ for all n.

We now present the statistical properties of the fading signal $Z(t)$ in the following two theorems.

Theorem 1: The autocorrelation and cross-correlation functions of the quadrature components, the autocorrelation functions of the complex envelope and the squared envelope of fading signal $Z(t)$ are given by

$$R_{Z_cZ_c}(\tau) = J_0(w_d\tau) \quad (5a)$$

$$R_{Z_sZ_s}(\tau) = J_0(w_d\tau) \quad (5b)$$

$$R_{Z_cZ_s}(\tau) = 0 \quad (5c)$$

$$R_{Z_sZ_c}(\tau) = 0 \quad (5d)$$

$$R_{ZZ}(\tau) = 2J_0(w_d\tau) \quad (5e)$$

$$R_{|Z|^2|Z|^2}(\tau) = 4 + 4J_0^2(w_d\tau) + \frac{2 + J_0(2w_d\tau)}{M}. \quad (5f)$$

Theorem 2: When M approaches infinity, the envelope $|Z|$ is Rayleigh distributed and the phase $\Psi(t) = \arctan[Z_c(t), Z_s(t)]$[1] is uniformly distributed on $[-\pi, \pi)$, and their probability density functions are given by

$$f_{|Z|}(z) = z \cdot \exp\left(-\frac{z^2}{2}\right), \qquad z \geq 0 \quad (6a)$$

$$f_\Psi(\psi) = \frac{1}{2\pi}, \qquad \psi \in [-\pi, \pi). \quad (6b)$$

Proof: The proof of Theorem 1 is complicated and lengthy, details are omitted here. Since the quadrature components of the fading $Z(t)$ are uncorrelated according to (5c) and (5d), one can prove Theorem 2 by using the procedure provided in [7]. We leave the details to the readers. ∎

[1]The function $\arctan(x, y)$ maps the arguments (x, y) into a phase in the correct quadrant in $[-\pi, \pi)$.

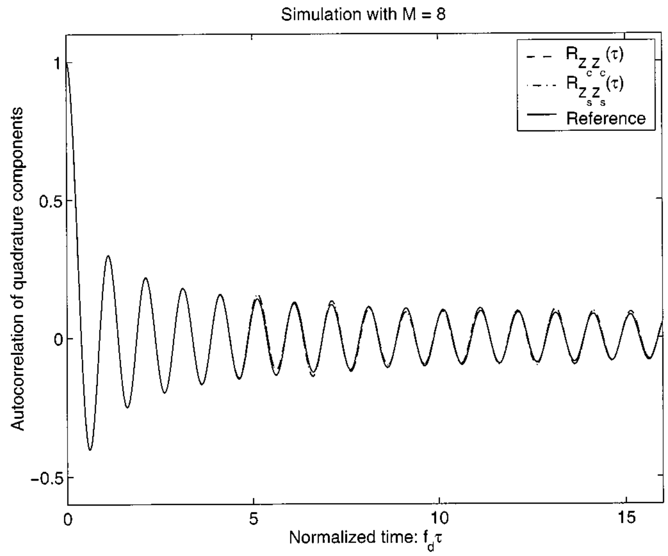

Fig. 1. The autocorrelations of the simulated quadrature components of fading $Z(t)$ and reference $g(t)$.

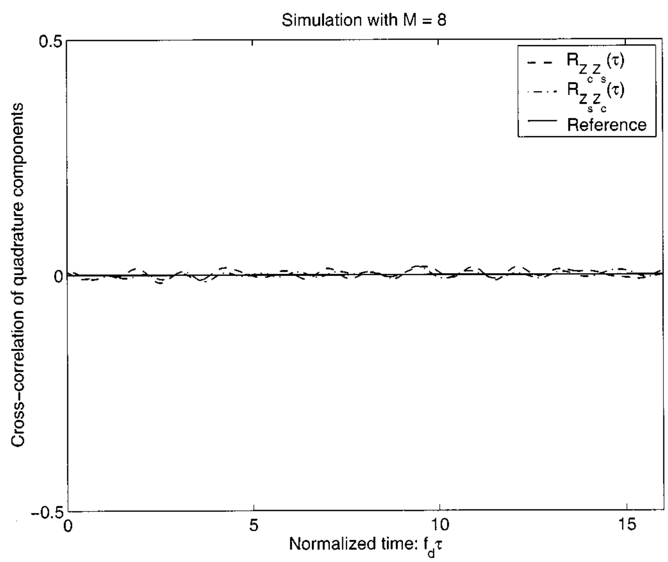

Fig. 2. The cross-correlations of the simulated quadrature components of fading $Z(t)$ and reference $g(t)$.

It should be emphasized here that the autocorrelation and cross-correlation functions given by (5a)–(5e) do not depend on the number of sinusoids M, and they match the desired second-order statistics [7] of Clarke's reference model exactly. This highlights the advantages of the new simulation model over all other existing simulation models. Furthermore, the autocorrelation function of the squared envelope asymptotically approaches the desired autocorrelation of $|g(t)|^2$ [7] as the number of sinusoids M approaches infinity, while good approximation has been observed when M is not less than 8.

The simulation results of the autocorrelations of the quadrature components, the cross-correlations of the quadrature components of the simulator output, the probability density function (PDF) of the fading envelope are shown in Figs. 1–3, respectively. The corresponding theoretically-calculated statistics of

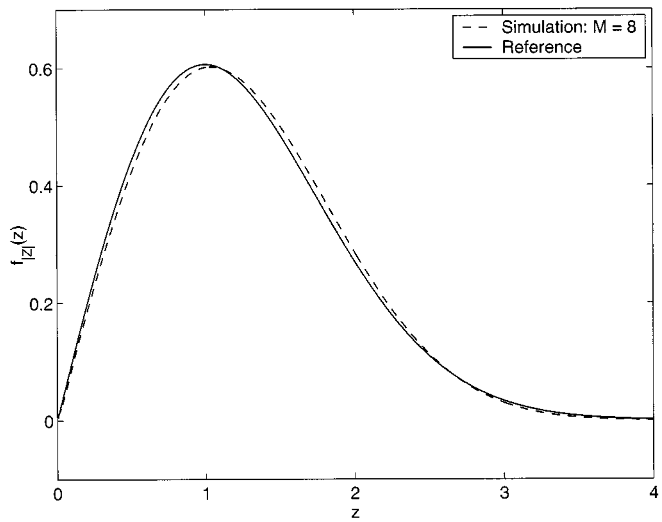

Fig. 3. The PDFs of the simulated fading envelope $|Z(t)|$ and reference $|g(t)|$.

Clarke's mathematical reference model are also included in the figures for comparison purposes. We have also verified the simulation results of the autocorrelations of the complex envelope and the squared envelope, the PDF of the fading phase, the average fader duration (AFD), and the level crossing rate (LCR) comparing with the theoretical results, finding good agreement in all cases.

Before concluding this section, we have two remarks on the new simulation model.

Remark 1: The proposed simulation model is not unique. There are other 15 different models which can be derived from (1) with different assumptions on ϕ_{M+i} and α_{M+i} for $i > 0$. These models have different combinations of cosine and sine functions for the quadrature components. Here we list one of them as an example

$$\bar{Z}(t) = \bar{Z}_c(t) + j\bar{Z}_s(t) \tag{7a}$$

$$\bar{Z}_c(t) = \sqrt{\frac{2}{M}} \sum_{n=1}^{M} \cos(w_d t \cos\alpha_n + \phi_n) \tag{7b}$$

$$\bar{Z}_s(t) = \sqrt{\frac{2}{M}} \sum_{n=1}^{M} \sin(w_d t \sin\alpha_n + \varphi_n) \tag{7c}$$

where α_n, ϕ_n, and φ_n are the same as those of $Z(t)$. It is noted that all these 16 models form a new class of Rayleigh fading channel simulators which have identical statistical properties.

Remark 2: The new simulation model can be directly used to generate uncorrelated fading waveforms for frequency selective fading channels, MIMO channels and diversity combining scenarios. Let $Z_k(t)$ be the kth Rayleigh fader given by

$$Z_k(t) = \sqrt{\frac{2}{M}} \left\{ \sum_{n=1}^{M} \cos\left[w_d t \cos\left(\frac{2\pi n - \pi + \theta_k}{4M} \right) + \phi_{n,k} \right] + j \sum_{n=1}^{M} \cos\left[w_d t \sin\left(\frac{2\pi n - \pi + \theta_k}{4M} \right) + \varphi_{n,k} \right] \right\} \tag{8}$$

where $\theta_k, \phi_{n,k}$ and $\varphi_{n,k}$ are mutually independent and uniformly distributed on $[-\pi, \pi)$ for all n and k. Then, $Z_k(t)$ retains all the statistical properties of $Z(t)$ defined by (3), and $Z_k(t)$ and $Z_l(t)$ are uncorrelated for all $k \neq l$.

III. Conclusion

In this letter, a new class of sum-of-sinusoids statistical simulation models is proposed for Rayleigh fading channels. Comparing with Jakes' sum-of-sinusoids deterministic model and its modifications, the new models re-introduces the randomness to Doppler frequency and initial phase of the sinusoids to have nondeterministic simulators with desired statistical properties. It has been proved that the autocorrelations of the quadrature components, the cross-correlations of the quadrature components, the autocorrelation of the complex envelope of the new simulators match the desired ones exactly even if the number of sinusoids used to generate the channel fading is as small as a single-digit integer. It has also been shown that the autocorrelation of the squared envelope, the probability density functions of the fading envelope and the phase of the new simulators approach those of Clarke's mathematical reference model as the number of sinusoids approaches infinity, while good convergence can be reached even when the number of sinusoids is as small as 8. All these statistical properties of one of the new simulators have been evaluated by extensive simulation results with excellent agreement in all cases. Moreover, it has been pointed out that the new simulation models can be directly used to generate multiple uncorrelated fading waveforms for frequency selective channels, MIMO channels, and diversity combining scenarios.

Acknowledgment

The authors wish to thank Prof. N. C. Beaulieu for his fruitful comments on the original manuscript. The authors are also grateful to the anonymous reviewers and the Associate Editor, Dr. C.-M. Chang, for their helpful criticisms.

References

[1] R. H. Clarke, "A statistical theory of mobile-radio reception," *Bell Syst. Tech. J.*, pp. 957–1000, July–Aug. 1968.
[2] W. C. Jakes, *Microwave Mobile Communications*: Wiley, 1974. reprinted by IEEE Press in 1994.
[3] P. Dent, G. E. Bottomley, and T. Croft, "Jakes fading model revisited," *Electron. Lett.*, vol. 29, no. 13, pp. 1162–1163, June 1993.
[4] M. Patzold and F. Laue, "Statistical properties of Jakes' fading channel simulator," *Proc. IEEE VTC'98*, pp. 712–718, 1998.
[5] Y. X. Li and X. Huang, "The generation of independent Rayleigh faders," *Proc. IEEE ICC'00*, pp. 41–45, 2000.
[6] Y. B. Li and Y. L. Guan, "Modified Jakes model for simulating multiple uncorrelated fading waveforms," *Proc. IEEE ICC'00*, pp. 46–49, 2000.
[7] G. L. Stuber, *Principles of Mobile Communication*, 2nd ed. New York: Kluwer Academic , 2001.
[8] M. F. Pop and N. C. Beaulieu, "Limitations of sum-of-sinusoids fading channel simulators," *IEEE Trans. Commun.*, vol. 49, pp. 699–708, Apr. 2001.
[9] C. Xiao, Y. R. Zheng, and N. C. Beaulieu, "Second-order statistical properties of the WSS Jakes' fading channel simulator," *IEEE Trans. Commun.*, vol. 50, pp. 888–891, June 2002.