Department of Electrical Engineering, IIT Madras

EE5141 : Wireless and Cellular Communications (Jan.-Apr., 2023)

Tutorial #2

Note: Questions marked with "*" are the harder ones; "**" are beyond what was discussed in class; there are purely optional questions.

1. Pilot Design and Overheads: Consider a 20 MHZ OFDM system with FFT size of N=1024. Assuming that the sub-carriers are indexed from 512 to -511, the guard sub-carriers are indexed by (512 to 472) and -472 to -511). The sampling rate is taken to be f_s =20 MHZ. Answer now the following:

(a) What is the sub-carrier band-width Δf_{sub} in KHz?

(b) If the Cyclic Prefix (CP) length N_{cp} is $\frac{1}{4}^{th}$ of the useful symbol duration, what is the duration (in μsec) of the full OFDM symbol?

(c) If QPSK modulation is employed on all the useful subcarriers, what is the "gross" spectral efficiency of this OFDM system (in bits/sec/ Hz)?

(d) This system is deployed to cover a circular area of 3 km radius, where the delay spread is 4 μsec or less. Also, the maximum Doppler frequency is expected to be 400Hz. Generally, it is advisable to have at least 1 or 2 pilot subcarriers (preferably 2 pilots) within a "Coherence band" , and pilots in time as often as every 10% of the " Coherence time" (which is approximately 36^0 phase change). Given this, how will you distribute pilot subcarriers in a 2-D manner (i.e., over frequency and time)? Explain with a figure.

(e)* If 2ppm clocks are used on both the T_X and R_X nodes, how often should the preamble symbol be repeated in time so that between preambles, there is no more than half-a-sample of slip? (Hint: First find the slip-rate in number of sample slips per second.)

(f) Putting (d) and (e) together, make a neat sketch of the OFDM blocks, say drawn over two preamble intervals. The preamble can be assumed to also mark the beginning of a new frame. What is the frame rate in frames/sec?

(g) What is the "nett" spectral efficiency, after accounting for preamble and pilots?

(h) Now, the cell –radius is increased so that the maximum delay spread can at most equal the CP length defined in part (b). Recalculate your answer to part (d), and hence, redo your answer to part (g).

2. Preamble Design for Frequency Offset Estimation: Let us consider a similar OFDM system with FFT of N=1024 as in pbm.1, but with a signal bandwidth of 10 MHz .
Given $f_c = 4GHz$, and that the oscillators (clocks) can have at most 5ppm error on both the T_X and on the R_X side. Assuming that the subcarriers are indexed similar to pbm.1, answer the following:

(a) What is the maximum possible frequency offset f_0 possible at the receiver? (Assume that there is no Doppler, and that f_0 is caused only due to the oscillator clock drift.)

(b) If the preamble symbol uses only every4^{th} subcarrier ($i.e., n, n+4, n+8, \ldots.$), what is the maximum frequency offset Δf that can be corrected (in the time domain) using the Schmidl-Cox method of correlation? Hint: Use the fact that for a spacing of the replicas in the time-domain that are "Q" samples apart, to prevent phase wrapping, we should have $2\pi\Delta f\ QT_s \leq \mp\pi$ radians. Further, use our assumption that $T_S = 1/(N\Delta f_{sub}$) to get the final expression.

(c)* Explain how the remaining (integer) part of the frequency error will be corrected. What are the maximum and minimum values of this integer frequency offset possible?

(d) Now, if it was possible to change the design of the preamble, how will you change it so that there is no integer offset to be estimated (unlike part(c))?

(e) Describe using discrete-time notation the Schmidl-Cox (SC) algorithm that you will use for your choice in part (d). What will be the averaging window?

(f) For your choice as in (d), can you handle some more frequency offset which is induced due to Doppler? If so, how much Doppler (in H_Z) can you handle using this design?

3. IBI and ICI * : Recall that the N-point (inverse) DFT to obtain the time-domain samples can be given by $x[k,m] = \frac{1}{N}\sum_{n=-\frac{N}{2}+1}^{N/2} X[k,n]e^{j\frac{2\pi}{N}mn}$ where $m = -N_{CP}, -N_{CP}+1, \ldots, -1,0,1,2,\cdots, N-1.$, and to make the index go form 1 to $N + N_{CP}$ we can define $z[k(N + N_{CP})T_S + N_{CP}T_S + mT_S] = x[k,m]$ where the sampling time is T_S.

(a)Now, if we would like to instead use the summation from $n = 0\ to\ N-1$ for the I-DFT above, then how does the index for m, as well as the definition of z[.], change?

(b) Take an example where $N = 8$, and $N_{CP} = 1$. Let the channel $h[mT_S] = h[m] = \delta[m]$. For the following choices of timing offset Δ , determine the expressions for the signal term, and the IBI,ICI, and the IBI- induced ICI terms with reference to the DC subcarrier (i.e., as measure on the subcarrier n=0). Also, what is the signal to interference ratio (SIR) expression in each case, where all the 8 sub-carriers (including the DC subcarrier) are using QPSK modulation with symbol energy E_S . Assume that all the QPSK symbols are i.i.d. Choices of Δ are: (i) $\Delta = 0$; (ii) $\Delta = 1$; (iii) $\Delta = -2$; and (iv) $\Delta = 5$.

4. ** **(Optional) OFDM Power Allocation and Rate:** An 8-point FFT is used to define a OFDM system which can use $P_T = 2$ Joules every symbol. After excluding the tones used for spectral shaping at the two band-edges (namely H(4) and H(-3), and the DC subcarrier corresponding to H(0), the magnitude frequency response over the other 5 tones are given as follows: $|H(3)|$=0.1; $|H(2)| = 0.8$; $|H(1)| = 0.3$; $|H(-1)| = 0.4$; $and, |H(-2)| = 0.1$.

(a) Following the development in the book by *Cho et.al*, what is the expression for the total (Hartley-Shannon) channel capacity in terms of *H(n)* if the noise variance is No, and all tones use the same power?

(b) If $N_0 = 0.10$, then what is the value of the above capacity?

(c) Now, water-pouring is enabled by feeding back the values of *H(n)* to the transmitter. Set up the constrained optimization problem, and solve using the Language multiplier technique. What is the expression for the *P(n),* the power that be allocated to the n^{th}subcarrier (in terms of the Lagrangian λ, *H(k),* and No)?

(d) Solve for λ by substituting back in the power-constraint equation. What is this value of λ ?

(e) Using this value of λ and the expression for P(n) you derived in part (c), calculate the values of power (energy) P(n) allocated to each of the 5 useful sub-carriers.

(f) What is the channel capacity with this water-pouring? Compare your answer with (b) and comment.

(h) With uniform power allocation, what will be the rate? Compare with (f).

(i) With zero-forcing power allocation, what will be the rate? Compare with (f) and (h).

5. (Optional) LMMSE Channel Estimation:** Given and IBI free and ICI free measurement model on the n^{th} pilot subcarrier of an OFDM receiver, namely $Z(n) = X(n)H(n) + V(n)$, where *V(n)* is AWGN with noise variance $\sigma_V{}^2$, derive the expression for the linear MMSE based channel estimator $\widehat{H}_{L-MMSE}(n)$ for this scalar model (which neglects frequency correlation). Assume that the pilot symbols *X(n)* have variance $\sigma_x{}^2$.

6. ** (Optional) SIMO and MISO Capacity: Following the development in Chuiueh & Tsai's book, recall the expressions for Ergodic channel capacity for **the** $1 \times N$ SIMO and the $N \times 1$ MISO measurements in fading channels. Let the received signal to noise ratio be given by $SNR = 1/\sigma_V{}^2$ where the noise variance $\sigma_V{}^2 = 0.1$. Using the following inequality, namely $E[\log(1+x)] \leq \log(1+E(x))$, complete the SIMO and MISO capacities in bps/Hz for N=10. Use other suitable approximations if required (but clearly state the approximations made) to get your answer.