Department of Electrical Engineering, IIT Madras

EE5141: Wireless and Cellular Communications

Marks 25* **Simulation Assignment #2** **Submit Before: 10:30AM, May 26, 2023**

Note 1: *Hard-copy of the assignment (preferably hand-written, but typed and printed submissions would be accepted after due scrutiny) to be submitted in person at my office (ESB-334B) or to the course TAs, Ms. Prasikaa Shree* ee21d700@smail.iitm.ac.in *or Mr. Shilajeet Banerjee* ee20s004@ee.iitm.ac.in, *on or before* ***10.30AM on Friday, May 26, 2023****.*

Note 2: *A soft-copy of your commented and working Matlab/C/Python code must be emailed to both of the above TAs. Mark the name of the pdf file* ee5141-SA2-yourrollnumber.pdf.

* *The marks of this assignment would be scaled down to 17.5 marks.*

In SA#2, the same 512-point FFT based OFDM system and channel power delay profile as in SA#1 are to be used; they are given again below for your convenience.

System Model

Sl #	Attribute	Value / Definition
1.	Subcarrier Bandwidth	f_{sub} = 10KHz = $1/T$ (T is useful symbol duration)
2.	FFT size	N = 512
3.	OFDM Signal Bandwidth	W=5.12 MHz
4.	Sampling Rate	$1/T_S = W$ = 5.12 Msps
5.	Cyclic Prefix duration	T_{CP} = 12.5 μsec
6.	Frame duration (S)	S = 5 OFDM blocks (block k =1 to S); The preamble will be the 1st block in the frame. The other 4 blocks carry QAM (and pilot) symbols.
7.	OFDM Symbol duration	$T_{OFDM} = T + T_{CP}$ = 112.5 μsec
8.	Guard Subcarrier (GS) labels	Upper Guard tones: $n \in \{256 \text{ to } 241\}$ DC subcarrier: n = 0 Lower Guard tones: $n \in \{-241 \text{ to } -255\}$

Channel Model

Path Gain σ_i^2(in dB scale)	-3	0	-1	-4	-9	-15
Tap Delay m (sample #)	0	7	16	22	39	54

[3+3+6+5+3+5=25 marks] OFDM Channel Estimation: The OFDM data symbols following the preamble symbol carry embedded pilots. The N_P pilot sub-carriers carry (known) QPSK symbols, and have a frequency spacing of 9 subcarriers (n, n-9, n-18,…) starting from subcarrier # 239. Recall that the DC-subcarrier must not be used in any case. Assuming perfect timing and frequency synchronization, we have IBI and ICI free measurements which translate after FFT at the receiver to the scalar measurement model given by

$$Y[k,n] = X[k,n]G[k,n] + V[k,n].$$

The corresponding vector measurement model on the N_P equi-spaced pilot subcarriers is given by

$$\mathbf{Y}[n] = \mathbf{X}[n]\mathbf{G}[n] + \mathbf{V}[n]$$

where $\mathbf{X}$[n] is N_P x N_P matrix and has all the pilot symbols placed on the diagonal. Simulate J=100 independent channel realizations (Monte-Carlo trials) using the above PDP. For each realization of CIR $\mathbf{g}[k]$, let the corresponding CFR be $\mathbf{G}[k]$. The aim is to estimate this CFR using some of the well known channel estimation algorithms, and compare their performance.

Given the N_Ux1 error vector $\mathbf{e}[k] = \mathbf{G}[k] - \widehat{\mathbf{G}}[k]$, (where N_U is the number of useful "data-carrying" subcarriers), the Mean Square Error (MSE) using the 100 Monte-Carlo runs is approximated as follows:

$$MSE \triangleq \frac{1}{J}\sum_{k=1}^{J} \mathbf{e}[k]^H \, \mathbf{e}[k]$$

The MSE performance curve is obtained by plotting 10 $\log_{10}(MSE)$ in the Y-axis versus 10 $\log_{10}$(SNR) on the X-axis, where both are in dB scale. Let the SNR vary in 3 dB steps from 0dB to 30dB. *All the below MSE curves must be plotted on the* ***same*** *graph, for the same set of channel realizations.*

(a) What is the MSE curve of the Zero-Forcing CFR estimate $\widehat{\mathbf{G}}_{ZF}$ measured only on the pilot locations?
(b) If the ZF estimate is linearly interpolated to the remaining subcarriers, what is the MSE performance? *Bonus*: Instead of linear interpolation, you are welcome to try any other frequency-domain interpolation technique. Describe the technique and provide the MSE curve.
(c) Instead of the schemes in (b), use the FFT-based interpolator discussed in the class. Use any other method including windowing to enhance the MSE performance of this $\widehat{\mathbf{G}}_{FFT}$, and plot the same.
(d) Now, develop the modified Least Square (mLS) estimator, where the estimator assumes only that the CIR is confined to be less that N_{CP}. Plot the MSE of this $\widehat{\mathbf{G}}_{mLS}$.
(e) Do you need to regularize the pseudo-inverse computation for the mLS estimate? If so, do an appropriate diagonal loading, and again compute the MSE performance. Comment.
(f) If the receiver knows the knowledge of the delay profile (i.e., multipath delay locations), can the estimation of the corresponding $\widehat{\mathbf{G}}_{mLS}$ be improved? Describe your method clearly and provide the performance curve.