Department of Electrical Engineering, IIT Madras

EE5141: Wireless and Cellular Communications

Marks 25* Simulation Assignment **#1** Submit Before: **10:30AM, May 22, 2023**

Note 1: *Hard-copy of the assignment (preferably hand-written, but typed and printed submissions would be accepted after due scrutiny) to be submitted in person at my office (ESB-334B) or to the course TAs, Ms. Prasikaa Shree* ee21d700@smail.iitm.ac.in *or Mr. Shilajeet Banerjee* ee20s004@ee.iitm.ac.in, *on or before **10.30AM on Monday**, May 22, 2023.*

Note 2: *A soft-copy of your commented and working Matlab/C/Python code must be emailed to both of the above TAs. Mark the name of the pdf file* ee5141-SA1-yourrollnumber.pdf.

* *The marks of this assignment would be scaled down to 17.5 marks.*

For all the problems in SA#1 as well as in SA#2, the 512-point FFT based OFDM system and the channel power delay profile given below are to be used.

System Model

Sl #	Attribute	Value / Definition
1.	Subcarrier Bandwidth	f_{sub} = 10KHz = $1/T$ (T is useful symbol duration)
2.	FFT size	$N = 512$
3.	OFDM Signal Bandwidth	W=5.12 MHz
4.	Sampling Rate	$1/T_S = W$ = 5.12 Msps
5.	Cyclic Prefix duration	T_{CP} = 12.5 μsec
6.	Frame duration (S)	S = 5 OFDM blocks (block k =1 to S); The preamble will be the 1st block in the frame. The other 4 blocks carry QAM (and pilot) symbols.
7.	OFDM Symbol duration	$T_{OFDM} = T + T_{CP}$ = 112.5 μsec
8.	Guard Subcarrier (GS) labels	Upper Guard tones: $n \in \{256 \text{ to } 241\}$ DC subcarrier: $n = 0$ Lower Guard tones: $n \in \{-241 \text{ to } -255\}$

Channel Model

Path Gain σ_i^2(in dB scale)	-3	0	-1	-4	-9	-15
Tap Delay m (sample #)	0	7	16	22	39	54

Hint: To normalize average channel gain to unity, in each of these models, rescale the (linear value of) the path variance σ_i^2 to ensure that over the L paths, $\sum_{i=0}^{L-1} \sigma_i^2 = 1$. Each zero-mean path gain a_i, where $E[|a_i|^2] = \sigma_i^2$, is a complex Gaussian random variable with each dimension having a variance of $\sigma_i^2/2$. The impulse-response snap-shot $g[k,m]$ corresponding to a given PDP is obtained by calling L times a circular Gaussian random variable (rv), with the variance of the rv based on the power profile. The frequency response snap-shot $G[k,n]$ is obtained by zero-padding plus FFT (of typically large size to visualize shape easily).

1. [3+5+7=15marks] SC Frequency Sync for Freq. Selective Channel: A preamble symbol is to be designed to ensure that the *entire* frequency offset can be estimated by the Schmidl-Cox (SC) algorithm. Assume that the non-zero subcarriers in the preamble use i.i.d QPSK symbols. The maximum frequency offset seen on the received samples is determined to be $\Delta f = \pm 28.65$ KHz. The samples at the receiver's ADC output can be modeled by $\tilde{y}(k,m) = e^{j2\pi\Delta f m T_s}\,\tilde{r}(k,m)$, where in turn the noisy measurement $\tilde{r}(k,m) = g[k,m] * \tilde{x}(k,m) + v(m)$ ®. Here, "$*$" represents linear convolution, and $\tilde{x}(k,m)$ is obtained by adding the CP to $x(k,m)$, with $\bar{x}(k) = F\bar{d}(k)$ where F is the NxN full DFT (FFT) matrix with scaling factor $1/\sqrt{N}$ to ensure that statistically the average gain of each $\tilde{x}(k,m)$ is unity. Further, assume in ® that $g[k,m]$ is given by the PDP above, and that $v(m)$ is zero-mean, circular Gaussian with variance σ_v^2 . Therefore, the (average) received SNR based on $\tilde{y}(k,m)$ is given by SNR = $1/\sigma_v^2$, which can then be varied by varying the noise variance.

(a) Specify the preamble symbol in the frequency domain and describe this symbol's time-domain properties using a labeled simulated result. What is the maximum frequency offset that it can measure un-ambiguously?

(b) We define the ergodic Mean Square Error (MSE) in the frequency offset estimate by MSE $\triangleq \frac{1}{J}\sum_{j=1}^{J}\left(\Delta f_j - \Delta\hat{f}_j\right)^2$, where $\Delta\hat{f}_j$ is the estimate from the "j^{th}" trial of the SC algorithm with independent signal and noise samples in each trial. Use J =10 for your MSE simulations. Vary the SNR between 0dB and 20dB in steps of 2dB, and measure the MSE in each case. Plot the MSE in the dB scale on the Y-axis, and SNR (also in the dB scale) on the X-axis.

(c) Can you improve the performance of the frequency offset estimator by doing more averaging over the preamble symbol? If possible, explain your approach clearly. Plot the resultant MSE for your approach on the same plot as in part (b). Comment on your result.

2. [3+3=6marks] SC Timing Sync: For the preamble designed for Δf = 28.65KHz in Q.**1** above:

(a) Given that it is a single-tap channel, i.e., h_m =1 in equation ®, provide the plot of the SC correlation output from which the FFT window (timing instant) is derived. Plot this for SNR=6dB, over 2 consecutive frames of size S = 5 OFDM symbols where the first symbol is the preamble in each frame. Use random QPSK data for the other 4 symbols.

(b) Now, replace the h_m in equation ® with the channel model used in Q.**1**. Provide the plot of the SC correlation output from which the FFT window (timing instant) is derived. Plot this for SNR=6 dB, over 2 consecutive frames of size S = 5 OFDM symbols where the first symbol is the preamble in each frame.

3. [4 marks] CP-Corr based Timing Sync for Freq. Selective Channel: For $g[k,m]$ in equation ® Q.**1** with the channel PDP as above, design a CP-correlation based approach to derive timing sync (FFT window). Plot this for SNR=6dB, over 10 consecutive OFDM symbols, since there is no preamble. Describe the CP-corr window you have used, and the number of correlation terms involved. Will this choice be suitable if the PDP of the channel was known to the receiver? Comment.